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ESTIMATION OF LIGHTNING STROKE PEAK CURRENT AS A FUNCTION OF PEAK ELECTRIC FIELD AND THE NORMALIZED AMPLITUDE OF SIGNAL STRENGTH: CORRECTIONS AND IMPROVEMENTS

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ABSTRACT

Corrections and improvements have been made to published equations relating return stroke peak current and peak electric field for cloud-to-ground lightning. The original equations were derived from measurements made at the Rocket-Triggered-Lightning Program facility at the Kennedy Space Center in Florida. Some previous articles include errors in the conversion of equations from units of volts per meter, for peak electric field, into Lightning Location and Protection units, representing normalized signal strength amplitude. This article presents the proper conversion procedure and the resultant corrected equations. Furthermore, equations are presented that were produced by converting other published relationships. Finally, we combined 1985-1988 RTLP data to produce equations based on more data than has been used previously; the resulting equation for peak current as a function of normalized signal strength amplitude supports previous evidence that the slope should be near 0.2 for a zero intercept. All relationships are tabulated, and three are graphically compared against the data.

1. INTRODUCTION

The production of nitrogen oxides (NO_x) by lightning is a source term of some uncertainty in global NO_x budgets and thus a recent focus of lightning research. Recent field investigations indicate that lightning may contribute to a large portion of the global atmospheric concentrations of NO_x [*Liaw et al.*, 1990; *Franzblau and Popp*, 1989; *Drapcho et al.*, 1983], even though recent global chemistry modeling studies assume that the contribution of lightning is an order of magnitude lower than that of anthropogenic sources [*Lyons et al.*, 1994]. The production of NO_x by lightning is proportional to the amount of energy dissipated in lightning flashes. *Plooster* [1971] showed that the energy dissipation per unit length of a lightning return stroke channel is proportional to the peak current raised to the 1.2 power. For much of the United States, an estimate of NO_x production by lightning can be made from lightning location and protection network information, which provides values of peak current in kiloamperes and normalized signal strength amplitude in Lightning Location and Protection (LLP) units. It is desirable to convert equations relating experimentally measured peak current and peak electric field in volts per meter, to corresponding equations in LLP units.

Willett et al. [1989] and *Orville* [1991] produced regression equations relating lightning return stroke peak current and the peak electric field using simultaneous measurements made at the Rocket-Triggered-Lightning Program facility (RTLP) at the Kennedy Space Center in Florida during 1985 through 1988. Measurements of peak electric field were adjusted for the distance (referred to as range) of the lightning

flash from the point of measurement. *Orville* [1991] and *Rakov et al.* [1992] modified the equations by converting peak electric field into normalized signal strength amplitude. However, they made errors in making the conversion of peak electric field from volts per meter into LLP units, thereby producing relationships of peak electric current and LLP units that are incorrect. We show the proper conversion procedure and the resultant corrected equations, include conversions of relationships from *Rakov et al.* [1992], and we combine data from four field seasons (1985-1988) at RTLP [*Orville*, 1991; *Willett et al.*, 1988; *Willett et al.*, 1989] to produce new relationships based on more data than used by previous investigators.

2. EQUATIONS RELATING PEAK CURRENT TO PEAK ELECTRIC FIELD

2.1 *Published Equations And Corrections To Them*

Willett et al. [1989] used simultaneous field measurements of peak current and peak electric field to determine a linear regression equation. The measurements made in 1987 from twenty-eight return strokes, produced by seven rocket-triggered lightning flashes at RTLP. The following relationship was determined,

$$I = -3.9 \times 10^{-2}\, D\, E - 2.7 \times 10^{3}, \qquad (1)$$

where I is return stroke peak current in positive amperes, E is return stroke peak electric field in negative volts per meter,

and D is the range (distance of the flash from the measurement point) in meters. The range and peak electric field are not independent of each other; they were multiplied together to become the dependent term in the linear regression. Peak current was used as the independent term in the regression, the results of which were rearranged by *Willett et al.* [1989] to yield (1). *Rakov et al.* [1992] reexamined the same set of data, using range times peak electric field as the independent variable, and obtained

$$I = 1.5 - 0.037\ D\ E, \tag{2}$$

where I is in negative kiloamperes, D is in kilometers, and E is in positive volts per meter. Regressions that we performed on the same data seem to indicate that *Rakov et al.* [1992] rounded the last significant digit down when the digit to the right of it was five or less, to obtain some of the regression coefficients in his table. The second and third columns of equations in Table 1 include coefficients that have been rounded up when the digit past the last significant one was five or greater. Therefore, the intercept that we obtained in regressing the data used in (2) was 1.6.

Orville [1991] used data from eighteen triggered lightning return strokes at RTLP during 1985, 1986, 1987, and 1988 to calibrate the local lightning location and protection system. One of the *Orville* [1991] observations duplicates one in the *Willett et al.* [1989] data set. His regression equation for estimating the peak current from the measured signal strength amplitude, normalized to a range of 100 km, is

$$I = 2.3 + 0.19\ LLP, \tag{3}$$

where I is in positive kiloamperes, and the normalized signal strength amplitude is in positive LLP units (the units for peak current and the units for normalized signal strength amplitude remain the same for all succeeding equations in this article). Normalized signal strength amplitude was used as the independent variable in the regression. *Orville* [1991] compared the *Willett et al.* [1989] equation (1) with his equation (3) by converting the peak electric field in (1) to normalized signal strength amplitude. The transformed equation on page 17,138 of *Orville* [1991] is

$$I = 2.7 + 0.18\ LLP. \tag{4}$$

However, the conversion factor used to obtain (4) was incorrect. Based on the calibration information provided by *Orville* [1991] (which was derived from medians of measured data presented by *Berger et al.* [1975]), a 20 kiloamperes peak current produces a normalized signal strength amplitude of 100 LLP units at 100 kilometer range. Equation (4) yields a peak current of 20.7 kiloamperes for 100 LLP units, a value that is only slightly greater than that suggested by the calibration information. Using the *Orville* [1991] calibration information in (1), peak electric field is found to be -5.82 volts per meter and (1) is correctly converted to

$$I = -2.7 + 0.227\ LLP. \tag{5}$$

Rakov et al. [1992] pointed out the error in the constant term (+2.7) of (4), but did not recognize that the slope is also in error. Equations (3) and

(5) yield somewhat different results, having been produced from different data sets. Furthermore, their forms differ; the intercepts have different signs.

R*akov et al.* [1992] converted (2) to normalized signal strength amplitude, obtaining,

$$I = -1.5 + 0.17\ LLP. \tag{6}$$

This equation appears on page 11,532 of *Rakov et al.* [1992]. Unfortunately, this conversion is also in error and results in (6) yielding a peak current of only 15.5 kiloamperes for 100 LLP units. From (2), and using our intercept of 1.6, peak electric field is determined to be 5.84 volts per meter, using, in a similar manner as before, a peak current of -20 kiloamperes, a range of 100 kilometers, and a normalized signal strength amplitude of 100 LLP units. The 5.84 volts per meter value is consistent (in absolute terms) with that determined from (1) earlier. Correctly converted, (2), using our intercept, becomes

$$I = -1.6 + 0.216\ LLP. \tag{7}$$

Note that (3) yields a peak current that is considerably greater than 20 kiloamperes for 100 LLP units, and has a positive intercept. These features make it unique, and inconsistent with the other similar equations in this article. Therefore, (3) and (7) yield values of peak current that are somewhat different, as is the case for (3) and (5).

Orville [1991] obtained regression equations using both normalized

signal strength amplitude as the independent variable and peak current as the independent variable. Equation (3) is based on the regression relationship plotted in figure 3 on page 17,138 of *Orville* [1991]. The regression relationship plotted in figure 2 of *Orville* [1991] uses peak current as the independent variable and is

$$LLP = 15.9 + 4.3\,I. \tag{8}$$

Equation (8) can be rearranged as,

$$I = -3.7 + 0.233\,LLP. \tag{9}$$

Equation (9) produces peak currents that are consistent with, although slightly smaller than, the results of (5), are somewhat different from (7), and are greatly different from (3). Equations (3) and (9) delineate the extremes of slope and intercept of the equations relating peak current and normalized signal strength amplitude in this article. Understandably, *Orville* [1991] used (3) in the text of his article because peak current would normally be considered to be the dependent variable. However, the form and results of (9) are consistent with those from regressions on the other data sets using normalized signal strength amplitude as the independent variable; the other equations are therefore preferred over (3).

Table 1 summarizes the equations referenced or determined in this article. Included in the table are two regression equations for peak current as a function of range times peak electric field, determined by *Rakov et al.* [1992] with data other than that used for (2). The two data

sets that he used are: seventeen return strokes obtained during the 1985 field season at RTLP (a data set originally presented in *Willett et al.* [1988]), and forty-five return strokes from a combination of the 1985 and 1987 RTLP data. Our regressions suggest that *Rakov et al.* [1992] rounded down the intercept on the equation for the latter data set. We have rounded the intercept up in our corresponding equation in the middle equation column of Table 1. Our regression equations require peak electric field to be 6.63 volts per meter for the 1985 data alone and 6.14 volts per meter for the combined 1985 and 1987 data to yield a peak current of 20 kiloamperes for a range of 100 kilometers. The 6.63 volts per meter value for the 1985 data is quite different from the 5.82 to 5.84 volts per meter determined earlier for the 1987 data. Using the 1985 and 1987 volts per meter factors, we converted the two *Rakov et al.* [1992] equations into our consistent units of peak current and normalized signal strength amplitude:

$$I = -3.2 + 0.232\ LLP, \tag{10}$$

for the 1985 data, and

$$I = -1.5 + 0.215\ LLP, \tag{11}$$

for the 1985 and 1987 combined data. The factors were also used to convert the 1985 and 1987 peak electric field data from volts per meter to normalized signal strength amplitude in LLP units, so that it could be displayed in Figure 1. Notice that (7) and (11) are nearly the same.

Equations (5), (9), and (10) are very similar and yield similar results of peak current.

Table 1 includes information about the data associated with the listed regression equations. Equations are shown as originally published in the *Orville* [1991], *Rackov et al.* [1992], and *Willett et al.* [1988, 1989] references, and in formats (peak current as a function of electric field, peak current as a function of normalized signal strength amplitude) that emphasize consistent signs and units, allowing the reader to more easily see the differences between the data sets and the resulting regression equations. The correlation coefficient value (R) in Table 1, when squared, indicates how well the regression equation explains the variance in the data used to produce it. Some of the correlation coefficient values squared indicate that less than 85% of the variance is explained by the associated regression equation.

2.2 *Equations From Combined Data*

By combining data sets from four field seasons (1985-1988) at RTLP [*Orville*, 1991; *Willett et al.*, 1988; and *Willett et al.*, 1989] and eliminating four duplicate data points, a new regression equation based on fifty-nine return strokes of rocket-triggered-lightning was obtained,

$$I = -0.6 + 0.034\ DE. \tag{12}$$

where D is the range in kilometers, and E is in positive volts per meter. Using the standard calibration of 20 kiloamperes peak current for a

100 kilometer range, peak electric field is calculated from (12) to be 6.09 volts per meter. The regression on peak current and normalized signal strength amplitude yields,

$$I = -0.6 + 0.204\ LLP. \tag{13}$$

Using the 6.09 volts per meter factor to convert from (12) to LLP units yields only a slightly different slope (0.207) than was found from the regression that produced (13). The correlation coefficient for both regression equations is 0.93. This coefficient reveals that there is some scatter in the data, which is perhaps caused by varying experimental or natural conditions at the RTLP over the several years of measurements. Equation (13) is an improvement over previously derived relationships. The intercept is very small and the slope is very near the value of 0.2 suggested by the analyses of *Berger et al.* [1975]. Equation (13) yields peak currents that are within five percent of what would result from a zero intercept and a 0.2 slope for greater than 40 LLP units, and within two percent for 100 LLP units or more. Equations (12) and (13) are included in the bottom row of Table 1.

To produce (12), we had to convert the normalized signal strength amplitude data of *Orville* [1991] into units of volts per meter. This was not a trivial task, as a consequence of the intercept for the two equation forms having to be the same, as has been assumed for all of the calculations in this article. The factor (5.84 volts per meter) used to convert the *Orville* [1991] data to volt per meter units had to also result in (12) and (13) having the same intercept.

Regression equations in terms of peak electric field, corresponding to (3) and (9), are shown in the second column of equations of Table 1. Because of accumulated rounding errors in conversions and calculations, and because only two significant digits are used in the calculations, application of the *Orville* [1991] calibration information to these two equations requires that peak electric field be 5.53 and 5.93 volts per meter to accomplish conversion to (3) and (9), respectively. A change of only 0.001 in the slope would bring the volts per meter factors to near 5.84 in the first case, and to less than 5.84 in the second. The factor used to convert the *Orville* [1991] data could be changed by 0.5 positive or negative and produce the same slopes, within 0.001. Irregardless, the evidence presented in this article seems to infer a volts per meter factor near 6.0 and a slope of 0.333 for peak current as a function of range times peak electric field, with zero intercept.

Figure 1 includes the fifty-nine data points used to produce (12) and (13), and three regression lines. The regression lines are based on three of the equations shown in Table 1, and are plotted as peak current as a function of the normalized signal strength amplitude in LLP units (using our consistent signs and units). Two of the plotted lines represent the extremes of regression equation slope and intercept, as represented by (3) and (9), with the third, and central, line being determined from (13). Figure 1 presents a comparison of the three equations for a wide range of peak current and normalized signal strength amplitude. The greatest differences in peak currents produced by the equations occur at low and at high normalized signal strength amplitudes. Most of the equations of peak

current as a function of normalized signal strength amplitude in the far right column of Table 1 adequately represent the plotted data; (3), in the second row of Table 1, is the exception. Equation (13) appears to best represent the data.

3. SUMMARY OF RESULTS AND CONCLUSIONS

Orville [1991] and *Rakov et al.* [1992] made errors in their conversions of regression equations relating lightning-return-stroke peak current and peak electric field, to equations relating peak current and normalized signal strength amplitude. Therefore, the equations produce inconsistent values of peak current. Even after correctly converting the regression equations, they still do not yield consistent values of peak current. We obtained another relationship for peak current as a function of normalized signal strength amplitude, using a regression equation that was presented in figure 2 of *Orville* [1991] but not in the text. This equation gives peak current values that are more consistent with those resulting from the properly converted *Rakov et al.* [1992] equation. Furthermore, we converted two equations derived by *Rakov et al.* [1992], from two years of data, into equations of peak current as a function of normalized signal strength amplitude. These equations are similar to the other, properly converted, equation of *Rakov et al.* [1992] previously mentioned.

Most importantly, we obtained a new linear regression equation for peak current as a function of normalized signal strength amplitude, using combined data (59 data points) from four field seasons at RTLP. Being

based on more data than has been used by previous investigators, our equation appears to best represent the linear relationship between peak current and normalized signal strength amplitude. Our combined data equation supports previous evidence that the slope of a linear relationship between peak current and normalized signal strength amplitude should be close to 0.2 for a zero intercept. Equation (13) yields peak currents that are within five percent of what would result from a zero intercept and a 0.2 slope for greater than 40 LLP units, and within two percent for 100 LLP units or more.

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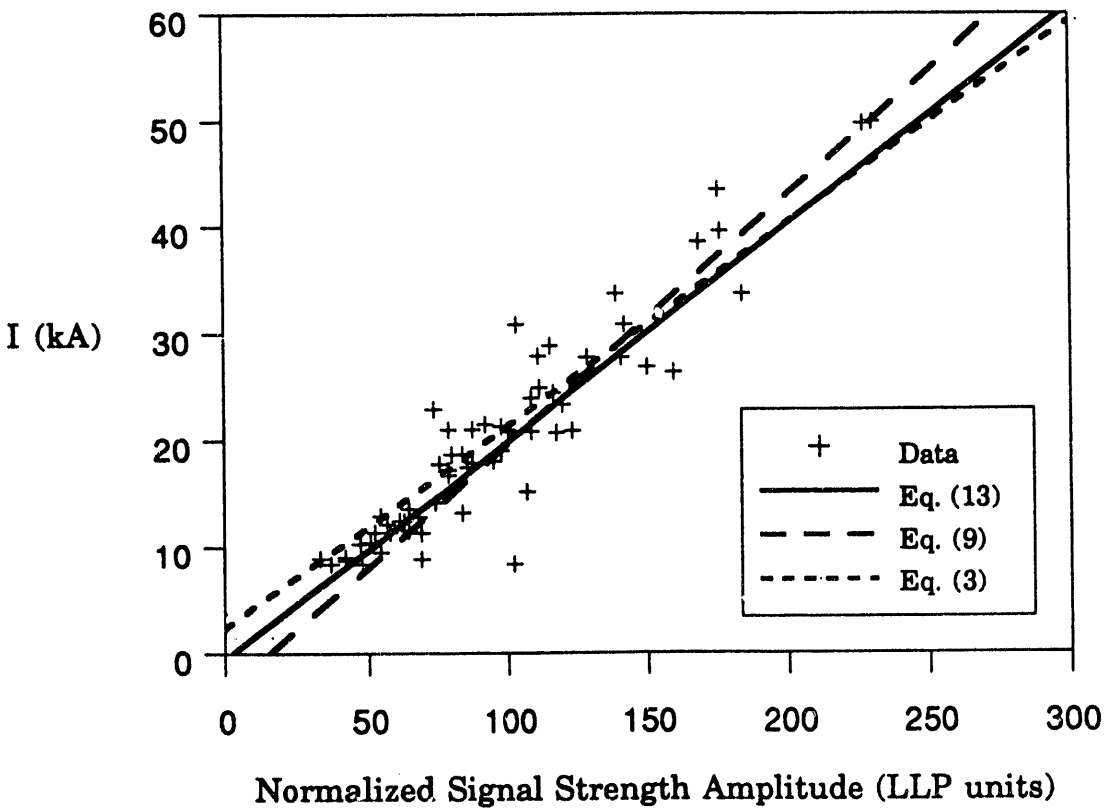

Fig. 1. Comparison of three linear regression equations for wide ranges of peak current and normalized signal strength amplitude.

TABLE 1. Linear Regression Equations and Information About the Associated Data, Including Equations As Originally Published In the References, and In Formats That Emphasize Consistent Signs and Units.

Reference	Data	[a] No.	[b] R	Equation As In Reference					*I* versus *E* (Same Sign and Units)					*I* versus *Normalized Amplitude*			
				Sign		Units			Sign		Units			Sign		Units	
				I	*E*	*I*	*D*	*E*	*I*	*E*	*I*	*D*	*E*	*I*	*LLP*	*I*	*Amplitude*
Willett et al. [1989]	1987	28	0.97	+	-	A	m	V/m	+	-	kA	km	V/m	+	+	kA	LLP
				$I=-3.9\times10^{-2}DE-2.7\times10^{3}$					$I=-2.7-0.039DE$					[c] $I=-2.7+0.227LLP$			
Orville [1991] (figure 3)	1985 - 1988	18	0.89	*I*	*LLP*	*I*	*Amplitude*		$I=2.3-0.032DE$					$I=2.3+0.19LLP$			
				+	+	kA	LLP										
				$I=2.3+0.19LLP$													
Orville [1991] (figure 2)	1985 - 1988	18	0.89	*I*	*LLP*	*I*	*Amplitude*		$I=-3.7-0.04DE$					$I=-3.7+0.233LLP$			
				+	+	kA	LLP										
				$LLP=15.9+4.3I$													
Rakov et al. [1992]	1987	28	0.97	*I*	*E*	*I*	*D*	*E*	$I=-1.6-0.037DE$					[d] $I=-1.6+0.216LLP$			
				-	+	kA	km	V/m									
				$I=1.5-0.037DE$													
Rakov et al. [1992]	1985	17	0.97	*I*	*E*	*I*	*D*	*E*	$I=-3.2-0.035DE$					$I=-3.2+0.232LLP$			
				-	+	kA	km	V/m									
				$I=3.2-0.035DE$													
Rakov et al. [1992]	1985 and 1987	45	0.96	*I*	*E*	*I*	*D*	*E*	$I=-1.5-0.035DE$					$I=-1.5+0.215LLP$			
				-	+	kA	km	V/m									
				$I=1.4-0.035DE$													
This Article	1985 - 1988	59	0.93						$I=-0.6-0.034DE$					$I=-0.6+0.204LLP$			

[a] Number of data points.

[b] Correlation coefficient of linear regression.

[c] This equation was incorrectly presented on p. 17138 in *Orville* [1991].

[d] This equation was incorrectly presented on p. 11532 in *Rakov* [1992].

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