A report to the Department of Energy on

Studies of Complexity in Fluid Systems

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1 Principal Investigators

All of the PI's are at the University of Chicago, Chicago, IL, 60637.

Co-PI	Local Address	Telephone
Leo P. Kadanoff	Research Institutes, L109	312 702 7189
Peter Constantin	Ryerson Laboratory, 362A	312 702 7399
Todd F. Dupont	Ryerson Laboratory, 167	312 702 3485
Sidney Nagel	Research Institutes, L115	312 702 7190

2 Principal Project Personnel

2.1 Leo P. Kadanoff

Role in Project:
Interpretating experimental results, deriving mathematical models, and conducting numerical studies.

Principal Areas of Research Expertise:
Theoretical condensed matter physics.

Percent of Time on Project: About 10%.

Born: January 14, 1937.

Education:
A.B.: Harvard, 1957
M.A.: Harvard, 1958
Ph.D.: Harvard, 1960

Honors and Professional Service:
Alfred P. Sloan Fellow, 1962-67.
Fellow, American Physical Society.
Buckley Prize American Physical Society, 1977.
Member, National Academy of Sciences.
The Wolf Foundation Award, 1980.
Fellow, American Academy of Arts and Sciences.
Fellow, American Association for the Advancement of Science.
Elliott Cresson Medal, The Franklin Institute, 1986.
Boltzmann Medal - IUPAP, 1989.
Quantrell Award, University of Chicago, 1990.
Centennial Medal, Harvard University, 1990.

Member, Advisory Committee, Institute for Theoretical Physics (Santa Barbara), 1978-81.
Member, Advisory Committee, Schlumberger Doll Research Laboratory Statistical Physics Program 1981-86.
Director, Material Research Laboratory, University of Chicago, 1981-84.
Vice-Chairman, Scientific and Technical Advisory Committee, Argonne National Laboratory, 1983-84.
Member, Board of Physics and Astronomy, National Research Council, 1983-86.
Member, Board of Governors, Argonne National Laboratory, 1983-84.
Member, Advisory Committee, Institute for Theoretical Physics University of Minnesota, 1986-present.
Member, Advisory Board of the Miller Institute for Basic Research in Science, 1988-91.
Member Editorial Board, Journal of Statistical Physics, 1974-78.
Member Editorial Board, Science Year, 1975-79.
Member Editorial Board, Nuclear Physics, 1980-present.
Member Editorial Board, Annals of Physics, 1982-present.
Associate Divisional Editor, Physical Review Letters, 1984-87
Member Editorial Board, Complex Systems, 1987-present.

Employment:

Postdoctoral Researcher, Bohr Institute for Theoretical Studies, 1960-62.
Assistant Professor, University of Illinois, 1962-63.
Associate Professor, University of Illinois, 1963-65.
Professor, University of Illinois, 1965-69.
Visiting Professor, Cambridge University, 1965.
University Professor of Physics, Brown University, 1969-78.
Professor of Engineering, Brown University, 1971-78.
Professor of Physics, University of Chicago, 1971-present.
John D. and Catherine T. MacArthur Distinguished Service Professor of Physics and Mathematics, 1982-present.

Selected Publications:

- *Passive Scalars, 3D Volume Preserving Maps and Chaos* (with M. Feingold and O. Piro), J. Stat. Phys. **50**, p. 529 (1988).
- *Singularities in Complex Interfaces* (with P. Constantin), Phil Trans R Soc London **A333**, p. 379 (1990).
- *Dynamics of a Complex Interface* (with P. Constantin), Physica D, **47**, p. 450 (1991).
- *Interface Dynamics and the Motion of Complex Singularities* (with W.-S. Dai and S.-M. Zhou), Phys. Rev. A, **43**, p. 6672 (1991).

- *Transitions in Convective Turbulence: the Role of Thermal Plumes* (with I. Procaccia, E. Ching, P. Constantin, A. Libchaber and X.-Z. Wu), Phys. Rev. A **44**, p. 8091 (1991).
- *Beyond all Orders: Singular Perturbations in a Mapping* (with C. Amick, S.C.E. Ching and V. Rom-Kedar), Journal of Nonlinear Science **2**, p. 9 (1992).
- *Droplet Breakup in the Hele-Shaw Cell?* (with P. Constantin, T. Dupont, R. Goldstein, M. Shelley and S.-M. Zhou) submitted to Phys. Rev. A, April 1992.

2.2 Peter Constantin

Role in Project:
To analyze the relationships between mathematical properties of the models of fluid flow that arise from other parts of the project.

Principal Areas of Research Expertise:
Mathematical aspects of Navier-Stokes turbulence: estimates of bulk disspation, decay of the power spectrum; scaling exponents..

Percent of Time on Project: Approximately 33%.

Born: August 29, 1951.

Education:
B.A.: University of Bucharest, 1974
M.A.: University of Bucharest, 1975.
Ph. D.: The Hebrew University of Jerusalem, 1980.

Honors and Professional Service:
Editorial Board of *Nonlinearity*, January 1992-present.
Collaborator, Los Alamos National Laboratories, 1985-present.
Alfred P. Sloan Research Fellow, 1986-90.

Employment:
Assistant (instructor), The Hebrew University of Jerusalem, 1976-80.
Visiting Assistant Professor, Indiana University, 1982-1983.
Visiting Member, Courant Institute of Mathematical Sciences, 1983-85.
Visiting Member, Math. Sciences Research Institute, Berkeley, 1984.
Visiting Assistant Professor, Indiana University, 1985.
Assistant Professor, University of Chicago, 1985-88.
Professor of Mathematics, University of Chicago, 1988-present.

Selected Publications:

- *On the possibility of soft and hard turbulence in the complex Ginzburg-Landau equation*, (with M. Bartuccelli, C. Doering, J. D. Gibbon and M. Gisselfalt), Physica D **44** (1990), 421 - 444.
- *Navier-Stokes equations and area of interfaces*, Commun. Math. Phys. **129** (1990), 241 - 266.

- *Remarks on the Navier-Stokes equations*, in **New Perspectives in Turbulence**, L. Sirovich Ed., Springer-Verlag, New York.
- *Dynamics of a complex interface*, (with L.P. Kadanoff), Physica D **47** (1991), 450 - 460.
- *Singularities in complex interfaces*, (with L.P. Kadanoff), Phil. Trans. R. Soc. London A **333** (1990), 379 - 389.
- *Transitions in convective turbulence: the role of thermal plumes*, (with E. Ching, L.P. Kadanoff, A. Libchaber, I. Procaccia and X-Z Wu), Phys.Rev **A44**, 1991, 8091.
- *Fractal geometry of isoscalar surfaces in turbulence: theory and experiment*, (with I. Procaccia and K.R. Sreenivasan), Phys. Rev. Lett. **67** 13(1991), 1739-1742.
- *Energy dissipation in shear driven turbulence*, (with Ch. Doering), Phys.Rev.Lett. **69** (1992), 1648-1651.
- *Global regularity for vortex patches*, (with A. Bertozzi), accepted Commun. Math. Phys.
- *The dimension of the carrier of dissipation in turbulence: intermittency in fluid mechanics*, (with I. Procaccia), accepted, Phys.Rev. A

2.3 Sidney Nagel

Role in Project:
Conducting experiments in three areas:
Ultrafast Photography of drop snap-off
Flow of granular fluids
Scaling of susceptibility near glass transition

Principal Areas of Research Expertise:
Condensed matter experimentalist, with experience in studying granular flows and relaxation at liquid/glass transition.

Percent of Time on Project: Approximately 10%.

Born: September 28, 1948.

Education:
B.A.: Columbia University, 1969.
M.A.: Princeton University, 1971.
Ph.D.: Princeton University, 1974.

Honors and Professional Service:
Phi Beta Kappa, 1969.
NSF Trainee, 1969-70.
Alfred P. Sloan Research Fellowship, 1979-81.
Fellow, American Physical Society, 1988.

Director, Materials Research Laboratory, University of Chicago, 1987-91.

Member, Scientific and Technical Advisory Committee for Argonne National Laboratory, 1985-90.

Member, Editorial Board, Physical Review B, 1989 - 1991.

Organizer, Workshop on "Self-Organized Criticality" Santa Fe Institute 1991.

Co-Organizer, Symposium on "Disordered Materials: Fractals, Scaling, and Dynamics", MRS Meeting Boston (1992).

Employment:

Research Associate, Brown University, 1974-1976.

Research Associate, Assistant Professor, University of Chicago, 1976-77.

Assistant Professor, University of Chicago, 1977-81.

Associate Professor, University of Chicago, 1981-84.

Professor, University of Chicago, 1984-present.

Selected Publications:

- *Relaxation at the Angle of Repose*, H. M. Jaeger, C.-H Liu and S. R. Nagel, Phys. Rev. Lett. **62**, 40 (1989).
- *Scaling and Universality in Avalanches*, L. P. Kadanoff, S. R. Nagel, L. Wu and S.-M. Zhou, Phys. Rev. A **39**, 6524 (1989).
- *Avalanches in Three and Four Dimensions*, K. P. O'Brien, L. Wu and S. R. Nagel, Phys. Rev. A **43**, 2052 (1991).
- *Friction in Granular Flows*, H. M. Jaeger, C.-h. Liu, S. R. Nagel and T. A. Witten, Europhysics Letters 11, **7** 619 (1990).
- *Scaling in the Relaxation of Supercooled Liquids*, P. K. Dixon, L. Wu, S. R. Nagel, B. D. Williams, and J. P. Carini, Phys. Rev. Lett. **65**, 1108 (1990).
- *Search for a Correlation Length in a Simulation of the Glass Transition*, R. M. Ernst, S. R. Nagel and G. S. Grest, Phys. Rev. B **43**, 8070 (1991).
- *Implications of a Conservation Law for the Distribution of Earthquake Sizes*, G. L. Vasconcelos, M. de Sousa Vieira and S. R. Nagel, Phys. Rev. A **44**, R7869 (1991).
- *Instabilities in a Sandpile*, S. R. Nagel, Rev. Mod. Phys. **64**, 321 (1992).
- *Physics of the Granular State*, H. M. Jaeger and S. R. Nagel, Science **255**, 1523 (1992).

- *Sound in Sand*, C.-h. Liu and S. R. Nagel, Phys. Rev. Lett. **68**, 2301 (1992).

- *Phase Transitions in a Spring-Block Model of Earthquakes*, G. Vasconcelos, M. de Sousa Vieira and S. R. Nagel, Physica A (in press).

2.4 Todd F. Dupont

Role in Project:
Construction of software for approximating the solutions of various mathematical models that arise in this study.

Principal Areas of Research Expertise:
Numerical solution of partial differential equations

Percent of Time on Project: About 10%.

Born: August 29, 1942.

Education:
B.A.: Rice University, 1963.
Ph.D.: Rice University, 1968.

Honors and Professional Service:
Associate editor SIAM J. Numer. Anal 1976-86.
Associate editor Math. Comp. 1977-84.
Associate editor Numer. Math. 1981-84.
Reviewer Math. Reviews 1975-84.
Principal in DREM (formerly Dupont-Rachford Engineering Mathematics Company) 1969-92.
Senior Technical Advisor, Stoner Associates Inc., 1992-present.

Employment:
Mathematician, Esso Production Research Co., 1967-68.
Instructor, University of Chicago, 1968-69.
Assistant Professor, University of Chicago, 1969-72.
Associate Professor, University of Chicago, 1972-75.
Professor of Mathematics, 1975-present.
Professor of Computer Science, 1985-present.

Selected Publications:

- *Mesh modification for evolution equations*, Math. Comp. **36** (1982), 85-107.

- *Hierarchical basis multigrid method*, with R. E. Bank and H. Yserentant, Numer. Math. **52** (1988), 427-458.

- *The rate of convergence of the modified method of characteristics for linear advection equations in one dimension*, with C. N. Dawson and M. F. Wheeler, Mathematics for Large-Scale Computation (J. Diaz, ed.), Marcel-Dekker, New York, 1989, 115-126.

- *Mixed finite element methods for time-dependent problems: application to control*, with R. Glowinski, W. Kinton, and M. F. Wheeler, Element Analysis in Fluids (T. J. Chung, ed.), 1052-1065, Hemisphere Publishing, 1992.
- *Explicit/implicit, conservative, Galerkin domain decomposition procedures for parabolic problems*, with C. N. Dawson, Math. Comp. **58** (1992), 21-34.
- *Droplet Breakup in the Hele-Shaw Cell?* submitted to Phys. Rev. A, April 1992, with P. Constantin, T. Dupont, R. Goldstein, M. Shelley and S.-M. Zhou.
- *Drop Formation in a One-Dimensional Approximation of the Navier-Stokes Equation*, with Jens Eggers, submitted Jour. Fluid Mechanics, October, 1992.

3 Additional Project Personnel

The personnel mentioned in this section all hold faculty appointments at the University of Chicago.

Robert Almgren was appointed Assistant Professor in the Department of Mathematics in fall 1992. His has had postdoctoral research positions at The Courant Institute of Mathematical Sciences and at University of Paris 7. His Ph.D. was awarded in 1989 from Princeton University where his advisor was A. Majda. His expertise is in the analysis of models of physical processes, the design and analysis of numerical algorithms, and in computing solutions of systems governed by partial differential equations. In this project he will be studying the solutions of model equations numerically and theoretically.

Andrea Bertozzi is starting her second year as a Dickson Instructor in Mathematics. She received her Ph.D. in 1991 from Princeton University, advisor A. Majda. She has an N.S.F. Postdoctoral Fellowship. She has studied the properties of solutions of partial differential equations theoretically and numerically, and that is the role in which she will participate in this project.

Fausto Cattaneo is a Senior Research Associate in the Department of Astronomy. He is accomplished in computational fluid dynamics, particularly the simulation of highly convective flows, and his expertise in this area will be valuable in this project. He received his Ph.D. from Darwin College, University of Cambridge, in 1984, advisor M. R. E. Proctor.

David Grier is an Assistant Professor in the Physics Department, and he got his Ph.D. from the University of Michigan. Experimental condensed matter physics is his area of interest. He plans to conduct some Hele-Shaw experiments which will be used in studying mathematical models of such situations. His expertise in the use of video image manipulation will play a role in several other experiments as well.

Heinrich Jaeger is an Assistant Professor in Physics. He got his Ph.D. in Physics from the University of Minnesota in 1987. A large fraction of his work deals with thin superconducting materials. Experimental study of the flow of granular fluids is the research area in which he will be interacting with the others involved in this project.

4 Project Overview

The Objective of this project is to bring together researchers from several disciplines who share an interest in studying the development of complexity in fluid systems. Researchers involved have expertise in mathematics, numerical computation, and theoretical and experimental physics.

This project itself has a very short history; it started nine months ago on 1 May, 1992. However, it is building on considerable previous success, as can be seen form the lists of selected publications given in section 2. In the area of turbulence there is the the work of Constantin, Kadanoff and others on transitions in convective turbulence; the results of Constantin and Doering on energy dissipation in turbulence; and the fractal isoscalar surface paper of Constantin, Procaccia and Sreenivasan. In fluid flow singularities there is the study of droplet breakup in Hele-Shaw cells, carried out by several of the people listed here (another manuscript is in preparation) and the work on drop formation by Dupont and Eggers. In the physics of granular flow there is a large collection of applications that Nagel and his coworkers have investigated. With Jaeger he looked at avalanches in real sand piles; with Kadanoff and others he looked at simulations of avalanches in cellular models (self-organized criticality); and he found that the propagation of sound in granular systems is significantly different from that found in other forms of matter.

In joint work with Ch. Fefferman, Constantin is studying the possible scaling exponents in Navier-Stokes turbulence. Departure from the K41 theory is possible only if certain generalized skewness ratios have Reynolds number dependence. Connections between the scaling exponents of the structure functions and the scaling exponents of these skewness ratios are established.

Work is in progress on the development of singular interfluid interfaces on several fronts, with frequent interaction: there is experimental, simulational, and theoretical effort actively being applied. Striking variations in droplet formation can be observed in both physical experiments and simulations based on simple models, and it is expected that study of the models will give a clearer understanding of the effects involved. We are currently taking pictures of a small liquid drop which is in the process of breaking into droplets. These high-speed photographs will be used to compare with the shapes that have been predicted by the theoretical models.

We are continuing our experimental studies of granular materials. We are currently studying the flow properties, i.e., convection, that occurs when a granular material is vibrated. This effect, originally discovered by Faraday, is still not understood. In conjunction with this project there is a related one in which we study the effect of shaking on the spontaneous demixing of a granular material according to the size of the particles. It is often, but not always, found that larger particles tend to rise in a sea of smaller ones when the whole system is shaken. It is not yet known what experimental parameters are necessary and sufficient for this behavior to occur.

The relation to the DOE mission seems quite clear. The 1991 Summary Report of the Office of Basic Energy Sciences states that the "objectives of the AMS subprogram are to advance the knowledge of mathematical, computational, and computer sciences required for understanding complex physical ...systems ..." and that the "underlying philosophy" of the AMS program is "collaboration of interdisciplinary groups of scientists ...". The complexity that arises in fluid flow is inherent in many systems, and this project seeks to use DOE funds to encourage collaboration between researchers from quite different fields.

5 Scientific and Technical Content

The three main subjects of study are the following:

- The mathematical and phenomenological approaches to turbulence.
- The development of singularities in fluid flow, most particularly in the motion of interfaces. Mathematical and numerical studies of viscous droplet generation.
- The experimental study of relaxation phenomena in a variety of fluid flows. Studies of the scaling properties of highly viscous liquids and the nature of avalanche dynamics in granular flows. Connections between the experiments and simulation have been particularly emphasised.

The approach taken in studying singularities in fluid flow is to look at a simple, but nontrivial, experimental situation; build a simplified mathematical model; and study the properties of the model theoretically and numerically. In turbulence the approach is primarily analytic in nature: use the PDE. In the area of physics of granular systems the approach has been experimental, using actual sand piles and idealized models.

We are concerned with obtaining a better understanding of how complexity manifests itself in the world around us, and thereby perhaps better understanding of the degree of predictability in some specific situations. Our main objects of study will be fluid and granular materials, but we hope that the physical and mathematical methods which we develop might have an even broader applicability. In addition, we hope to train a number of students and postdocs who will be able to work on interdisciplinary projects on the borderline between math and physics.

6 Project Output

One real accomplishment produced during the nine months of this grant's life is the success we have had in bridging the gaps between researchers who work in widely different areas, so that they can put significant effort on a set of problems which interest them in very different ways. Some indication of the progress to date is given in the papers that are included with this report.

Two of the paper listed below, describing work of Constantin-Procaccia and Constantin-Fefferman, give new insights in the scaling properties of Navier-Stokes turbulence.

The Eggers-Dupont paper on droplet breakup shows that even up to the develop: .ent of the singularity a one-dimensional model gives very good agreement with experimental results. Since the partial differential equations in the one-dimensional model are much simpler than three-dimensional Navier-Stokes, we stand a much better chance of being able to develop a full understanding of its behavior.

The Dupont-Goldstein-Kadanoff-Zhou and the Bertozzi- Brenner-Dupont-Kadanoff works describe the nature of and mechanism for singularity formation in one-dimensional interface motion. This represents a major piece of work completed under DOE sponsorship.

Constantin's paper represents in part a description of older material related to xxx and in part a major new departure which one might hope will lead to insight into the nature of the solutions to Euler and Navier Stokes equations.

In addition there has been considerable experimental work. This falls into two areas: granular materials and singularity formation in droplets. In the area of granular materials we have made progress both on understanding the convection and segregation phenomena in vibrated materials as well as in understanding the nature of sound propagation in these media. In the area of droplet formation we have been able to study with photographic techniques how some of the parameters of the liquid influences the droplet shape. A brief summary of each of these research topics is given below.

Vibration-Induced Size Separation in Granular Media: the Convection Connection J. B. Knight, H. M. Jaeger and Sidney R. Nagel

We have investigated the rise of a single large glass bead through a vibrated cylindrical column of smaller beads. We find that vibration-induced size separation in this geometry arises from convective processes rather than from local rearrangements or avalanches as had been proposed previously. A symmetric convection cycle, rising in a wide column in the middle of the cell and dropping in a thin stream along the walls of the container, was responsible for all particle movement. Particles larger than the width of the thin downward convection zone are carried to the top of the column and then trapped, resulting in size segregation. When we prevented the convection cycle by varying the surface preparation of the walls, we observed no relative particle movement and no size separation.

Giant Sound-Conductance Changes Caused by Minute Movements of Single Sand Grains Chu-heng Liu and Sidney R. Nagel

We have investigated how a small disturbance of a single grain of sand can affect the transmission of sound in an unconsolidated granular material. The thermal expansion of a single grain due to a 1 K temperature rise (2000A) can significantly change the transmission even though this expansion is minute compared to either the grain diameter (0.5cm) or the sound wavelength (1cm). The extreme sensitivity is unique to sand and diffusive-wave theory cannot account for this behavior without including further postulates about the nature of the scattering centers. By installing 50 heaters inside a sand pile we have also investigated the spatial pattern of the wave propagation. The disturbance due to the heaters is very irregular in space: two adjacent heaters can give very different responses. This suggests that the medium is very inhomogeneous. The average over many configurations of the rms disturbance decays with distance from the source or detector.

Bifurcation of Falling Liquid Droplets X. D. Shi, D. G. Grier and Sidney R. Nagel

A drop falling under gravity and snapping off from a thin pipette represents singular behavior developing from smooth initial conditions. At the point where the liquid splits into two separated drops a singularity occurs. We have performed experimental studies of this situation using an ultra-high-speed camera and a video camera with a high-speed shutter to image the shape of the falling droplets. We have varied the viscosity of the liquid (using water and glycerol mixtures) and the dimensions of the pipette to see how these parameters influence the droplet dynamics. As the viscosity increases, the length of the thin connecting tube between the pipette and the droplet increases significantly. We find that the length of this neck varies approximately linearly with the viscosity. For pure water, where the viscosity is low, the shape of the droplet is in good agreement with the predictions of Eggers and Dupont.

Bibliography of DOE-supported papers

- *Scaling in fluid turbulence: a geometric theory*, P. Constantin and I. Procaccia, submitted to Physics of Fluids, September, 1992.
- *Drop Formation in a One-Dimensional Approximation of the Navier-Stokes Equation*, J. Eggers and T. Dupont, submitted to Jour. Fluid Mechanics, October, 1992.
- *Finite Time Singularity Formation in the Hele-Shaw System*, T. Dupont, R. Goldstein, L. Kadanoff, and S. Zhou, submitted to Phys Rev E, February, 1993.

- *Scaling in fluid turbulence: some analytical results*, P. Constantin and Ch. Fefferman, in preparation.
- *Vibration-Induced Size Separation in Granular Media: the Convection Connection*, J. B. Knight, H. M. Jaeger and Sidney R. Nagel, preprint.
- *Giant Sound-Conductance Changes Caused by Minute Movements of Single Sand Grains*, Chu-heng Liu and Sidney R. Nagel., preprint.
- *Bifurcation of Falling Liquid Droplets*, X. D. Shi, D. G. Grier and Sidney R. Nagel, preprint.
- *Singularities and Similarities in Fluid Flow*,
- *Bifurcation of Falling Liquid Droplets*, X. D. Shi, D. G. Grier and Sidney R. Nagel, preprint.
- *Singularities and Similarities in Fluid Flow*, Andrea Bertozzi, Michael Brenner, Todd F. Dupont, and Leo P. Kadanoff, to be published in Springer Verlag Applied Mathematics Centenial Volume.
- *The fractal geometry of the level sets of a contaminant dispersed by chaotic surface waves*, Peter Constantin and I. Procaccia, Euro Phys. Lett, submitted
- *Non-Kolmogorov scaling exponents and the geometry of high Reynolds number turbulence*, Peter Constantin and I. Procaccia, submitted

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