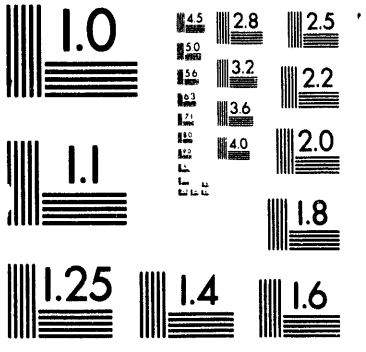
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An Improvement in the Modified Finite Element Procedure for Underwater Shock Analysis

S. K. Chan
Westinghouse Electric Corporation
Bettis Atomic Power Laboratory
West Mifflin, PA 15122

Abstract

The modified finite element procedure for underwater shock analysis decomposes the total pressure field into the incident, reflected and radiated pressure. The incident pressure is calculated by using a closed form solution. The reflected and radiated pressure are calculated in two separate finite element analyses. Artificial damping is added in the finite element analyses. Since these two pressures are generated from the fluid-structure interface and mostly propagate away from the interface, the artificial damping has no significant effect on the result. The modified finite element procedure was developed using a displacement formulated finite element to model the fluid region. In the paper, a pressure formulated finite element is used to model the fluid region which has a potential of saving 90 percent of the computer time. In doing so, the reflected and radiated pressure are calculated in one analysis which greatly simplifies the analysis procedure and saves more computer time. Two verification examples are given.

INTRODUCTION

When computing the response of a structure submerged in a fluid body and subjected to an underwater explosion, the shock wave is usually assumed to be a spherical wave. In the analysis, the incident wave which propagates through the fluid medium has to be calculated. It is a kind of wave propagation problem; however, the response of the structure is of the utmost interest.

Generally speaking, it is difficult to analyze transient wave propagation problems with numerical methods, Ref[2], because the discretization of the continuum gives

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spurious reflections and tends to obscure the results. Since, the major interest is the response of the structure, Ref[1] introduced the modified finite element procedure (MFE) for this kind of analysis.

The MFE procedure for underwater shock analysis using displacement formulated finite elements decomposes the total pressure field into the incident, reflected and radiated pressure. The incident pressure is calculated by using a closed form solution. The reflected and radiated pressure are calculated in two separate finite element analyses. Artificial damping is added in the finite element analyses. Since these two pressures are generated from the fluid-structure interface and mostly propagate away from the interface, the artificial damping has little effect on the result of the structure response.

In Ref[1], a displacement formulated finite element was used to demonstrate the MFE procedure because the idea of computing the reflected pressure was based on applying the negative of the displacement due to the incident pressure on the fluid-structure interface and the availability of existing computer codes which have the capability of adding the artificial damping. The MFE procedure can be improved by using a pressure formulated finite element. This will result in greatly reduced computer run time because the pressure formulated finite element has one degree of freedom (DOF) per node, whereas the displacement formulated finite element has three DOF's per node.

In the process of investigating how the pressure formulated finite element would be used to solve the underwater shock problem, it was found that the pressure field can be decomposed into two parts namely the incident pressure (as before) plus a scattered pressure. The scattered pressure is the sum of the reflected and radiated pressure which were defined in Ref[1]. This in turn will require only one finite element analysis step, namely that required to compute the scattered pressure. The ability to calculate the result in one analysis saves more computer time and greatly simplifies the analysis procedure because it avoids the work of obtaining the reflected pressures from one finite element analysis and then applying them to another finite element analysis which calculates the radiated pressure. This paper describes the improved procedure and gives two verification examples.

THEORY

To analyze underwater shock, if the shock pressure is not very large, the motion of the water particle is small, such that the equilibrium and the constitutive equations can be written as

$$\nabla P + \rho \ddot{u} = 0 \tag{1}$$

$$P = -B \nabla \cdot u \tag{2}$$

where P is the pressure field, u the displacement field, ρ the density, B is the bulk modulus, and the double dot indicates the second time derivative. Substituting Equation (2) into the gradient of Equation (1) gives

$$\rho\ddot{P} = B\nabla^2 P \tag{3}$$

Using the Galerkin procedure and Green's theorem, Equation (3) yields

$$\int_V \left[\delta P \rho \ddot{P} + B\nabla\delta P \cdot \nabla P\right] dV + \int_S \delta P \left(Bn \cdot \nabla P\right) dS = 0 \tag{4}$$

where δP is an arbitrary variational pressure and the integrations are carried over the fluid volume or the surface of the fluid volume; n is the outward normal vector on the structure surface.

Substituting Equation (1) into the above equation gives

$$\int_V \left[\delta P \rho \ddot{P} + B\nabla\delta P \cdot \nabla P\right] dV - \int_S \delta P \left(\rho Bn \cdot \ddot{u}\right) dS = 0. \tag{5}$$

This equation is commonly used for the finite element discretization of the fluid domain of a fluid-structure interaction problem. The equation for the structure is:

$$\int_V \delta\epsilon : \sigma dV + \int_V \rho_s \delta u \cdot \ddot{u} dV + \int_S P\delta u \cdot n dS = \int_S \delta u \cdot t dS \tag{6}$$

where ϵ and σ are the strain and stress dyadics, the ':' is the double inner product such that the first term is the virtual strain energy. Damping is ignored for simplicity.

The pressure field is decomposed into the incident pressure, P_{inc}, and the scattered pressure, P_{scat}, as

$$P = P_{inc} + P_{scat}. \tag{7}$$

The incident pressure will be calculated by using a closed form solution, and it also satisfies Equation (4), that is

$$\int_V \left[\delta P \rho \ddot{P}_{inc} + B\nabla\delta P \cdot \nabla P_{inc}\right] dV = -\int_S \delta P \left(Bn \cdot \nabla P_{inc}\right) dS. \tag{8}$$

Substituting Equations (7) and (8) into Equations (5) and (6) yields

$$\int_V \left[\delta P \rho \ddot{P}_{scat} + B\nabla\delta P \cdot \nabla P_{scat}\right] dV - \int_S \delta P \left(\rho Bn \cdot \ddot{u}\right) dS = \int_S \delta P \left(Bn \cdot \nabla P_{inc}\right) dS \tag{9}$$

and

$$\int_V \delta\epsilon : \sigma dV + \int_V \rho_s \delta u \cdot \ddot{u} dV + \int_S P_{scat}\delta u \cdot n dS = \int_S \delta u \cdot t dS - \int_S P_{inc}\delta u \cdot n dS. \tag{10}$$

Equations (9) and (10) define the underwater shock problem. These two equations are similar to the equations for regular fluid-structure interaction equations, Equation (5) and (6); except that there are two extra terms with P_{inc} in the right hand side. The finite element formulation will be the same as the regular finite element formulation, except for calculating these two extra terms as a load vector. Computing the extra term in Equation (10) is straight forward. The extra term in Equation (9) involves the incident pressure gradient and it can be easily computed in terms of the incident pressure and its temporal derivative. As an example for a spherical incident pressure wave:

$$\nabla P_{inc} = -\hat{r}\left(P_{inc}/R + \dot{P}_{inc}/C\right)$$

where $\hat{r}$ is the unit radial vector pointing from the center of the incident pressure to the point on the surface, R is the distance from the pressure center to the point on the surface, and C is the speed of sound in the fluid.

Therefore, in using the approach outlined in this paper, the finite element formulation for the underwater shock problem is the same as for any fluid-structure interaction problem with the specification of an extra load vector at the fluid structure interface. This load vector can either be calculated separately by a pre-processor or included as part of the formulation for the fluid-structure interface element. The latter method was used for the examples in this paper.

EXAMPLES

Underwater shock problems are very complicated. Not many accurate solutions are available, and the most accurate solutions available have the geometry of a circular cylinder, a sphere, or are one dimensional. The published cylinder results have only the hoop stress in the mid-plane of the cylindrical shell for which the short time approximation can give good comparison. Therefore, the cylindrical problem is not considered here. A one dimensional problem and a spherical problem will be analyzed here. These problems were analyzed with the same type of artificial damping as that used by Ref[1] but half the amount. A little less damping seems gives a little better results.

One-Dimensional Example

A one-dimensional example is examined here. This simple problem can reveal many wave propagation characteristics. A one-dimensional semi-infinite long rod extending from $x = 0$ to $x = \infty$ is being analyzed; the rod is free to move at $x = 0$. A 3 lb/ft^2 step pressure wave is traveling from right to left. Since it is impossible

to model a semi-infinite long rod, the finite element mesh is arbitrarily stopped at $x = 1$ foot with no reflection element at this end. The material of the rod has the velocity of sound, $C = 4$ ft/sec, and mass density $\rho = 0.5 slug/ft^3$. The reference time, $t = 0$ second, is when the front of the pressure pulse reaches $x = 0$.

The two dimensional finite element mesh used is as shown in Figure 1. At the free end, a mass of 0.5×10^{-09} slugs is added to remove the singularity of the displacement DOF of these nodes. The incident pressure is calculated with the equation: $P(x,t) = P(x + 4t)$; the incident pressure history at $x = 0$, $P_{inc}(0,t)$ is shown in Figure 2. The resulting free end velocity history is shown in Figure 3 with the exact solution. The finite element result is close to the exact solution except the result of the first time step which the velocity has a discontinuous jump from zero to 1.5. This is a limitation of most numerical methods; however, if more accurate results are required, smaller elements and time step size can be used.

A Unit Sphere Subjected To Plane Step Shock Wave

Dimensionless values are used in this problem. The spherical shell has a unit radius, a shell thickness of 0.02, and it is subjected to a unit step plane pressure wave as shown in Figure 4. The properties of the water are $\rho = 1.0$, and $C = 1.0$. The properties of the spherical shell material are: density 7.7873, Poisson's ratio 0.3 and modulus of elasticity 96.992. This is a two dimensional axisymmetric problem, and the finite element mesh of the sphere is shown in Figure 5. The shell deformation results obtained are compared with the available series results, Ref[3]. One-half of the relative displacement between the points (R=1, $\theta = 0$) and (R=1, $\theta = \pi$) is plotted in Figure 6 along with the series result. The time histories of the radial velocity of the points (R=1, $\theta = 0$) and (R=1, $\theta = \pi$) are plotted in Figure 7.

In order to evaluate if the computed solution has converged, the mesh in Figure 5 was refined by doubling the number of elements in both the radial and angular directions, increasing the radius of the outer boundary by 50%, and using one half of the previous the time step size. These results are compared with the original results in Figure 8 and 9. The change due to the mesh and time step size refinement is small which indicates that the results are very much converged.

The stiffness matrix of this example has a root mean square wavefront size of 32.1 and uses 2.56 seconds of a workstation to perform matrix triangularization. The example in Ref[1] which uses a displacement formulated element, has a root mean square wavefront size of 56.0 and used 6.281 seconds of the same machine to perform matrix triangularization for the radiated pressure calculation. In addition, about the same amount of computer time is needed for the reflected pressure calculation.

CONCLUSION AND DISCUSSION

Implementation of the modified finite element method of Ref[1] can be greatly improved by using pressure formulated finite elements. The finite element formulation for the underwater shock problem is the same as for any fluid-structure interaction problem with the specification of an extra load vector at the fluid structure interface. The extra load vector generated for the underwater shock problems can be easily programed into a pre-processor computer code or into a fluid-structure interaction finite element. For the two dimensional sphere example, using a pressure formulated finite element requires half the time needed by the displacement formulated finite element to perform matrix triangularization. Theoretically, for large three dimensional problems such that the equation solving uses most of the computer time, more than 95 percent of the computer time can be saved by using this fluid-structure interface element and the pressure formulated element instead of the displacement formulated element.

ACKNOWLEDGMENTS

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[3] Huang, H., "Transient Interaction of Plane Acoustic Waves with a Spherical Elastic Shell" J. of Acoustical Society of American, Vol. 45, No. 3 1969.

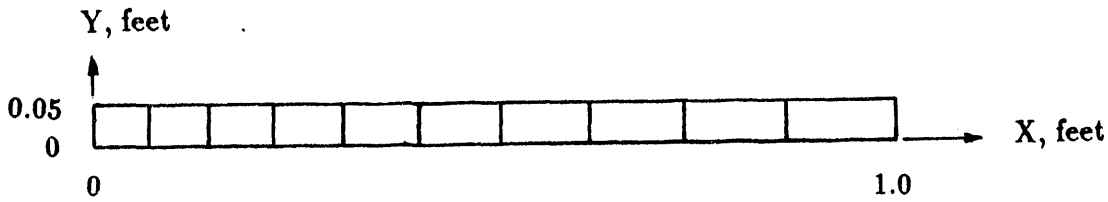

Figure 1: The Finite Element Mesh of a Rod.

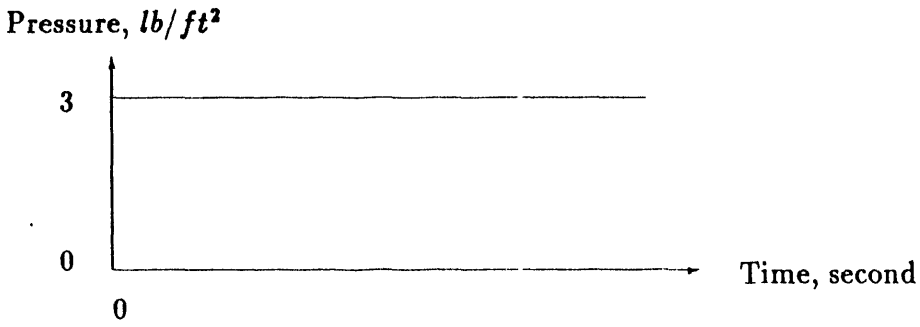

Figure 2: The Incident Pressure History at $x = 0$.

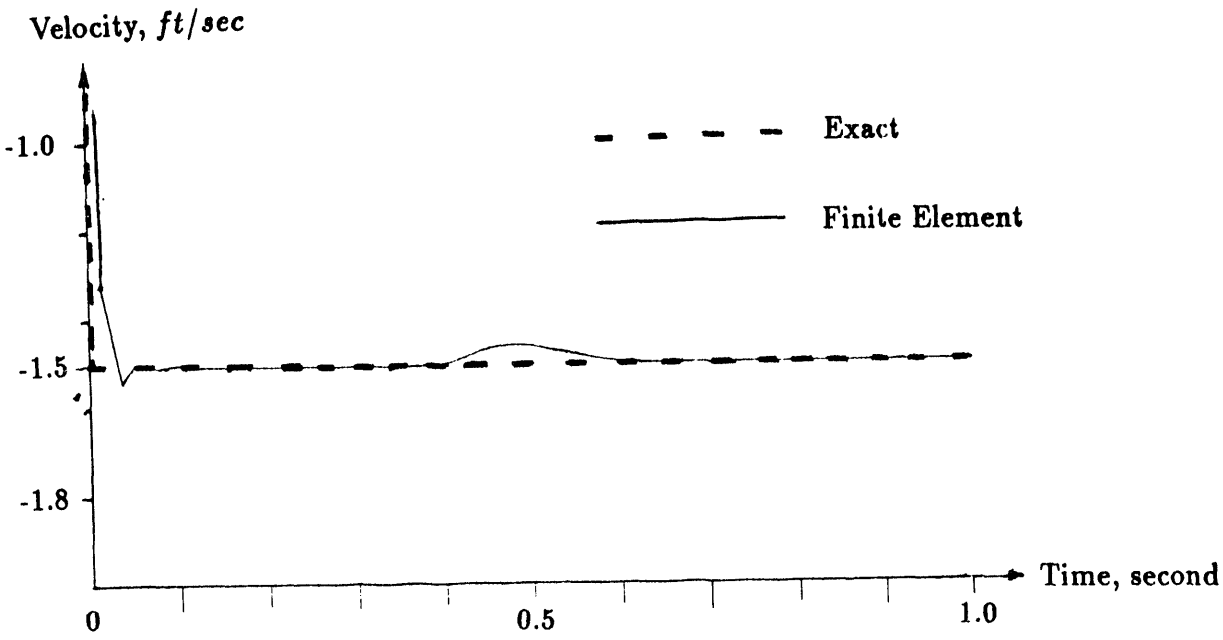

Figure 3: The Resulting Free End Velocity at $x = 0$.

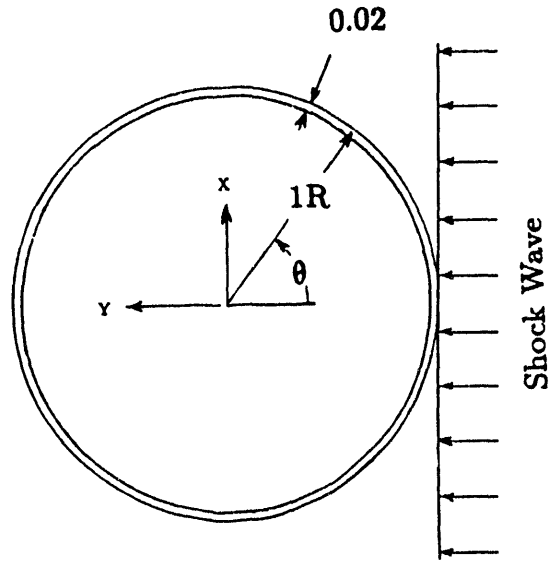

Figure 4: Cross Section of a Sphere and Shock Wave

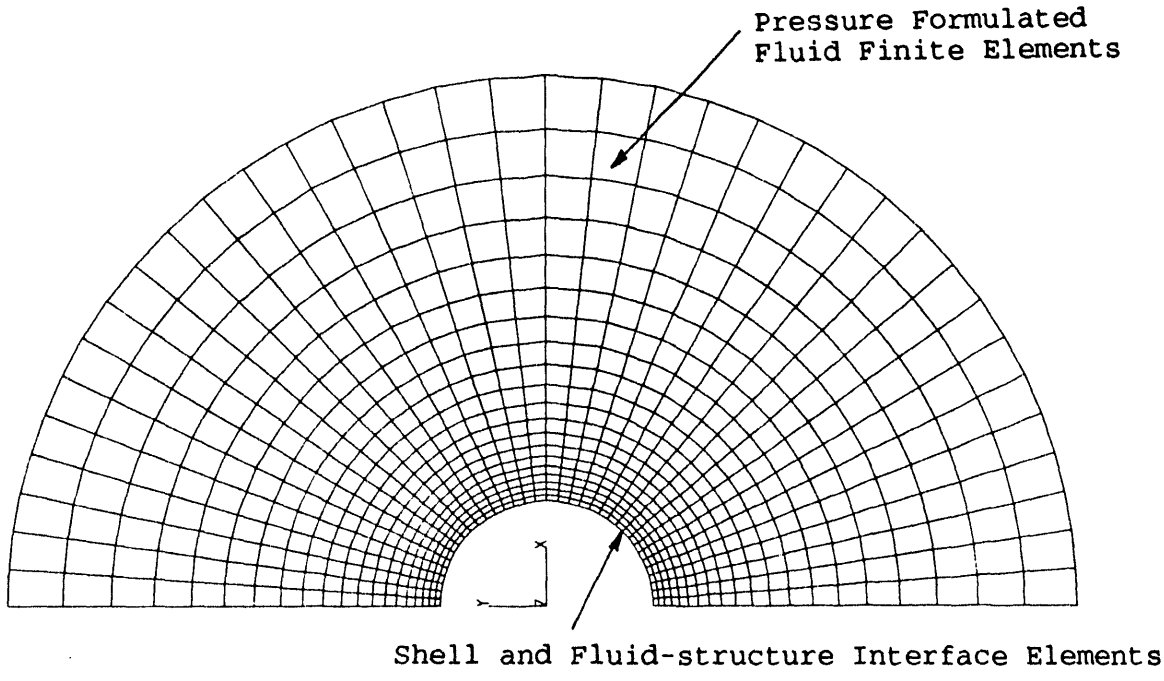

Figure 5: Finite Element Mesh of the Sphere and Water Around It

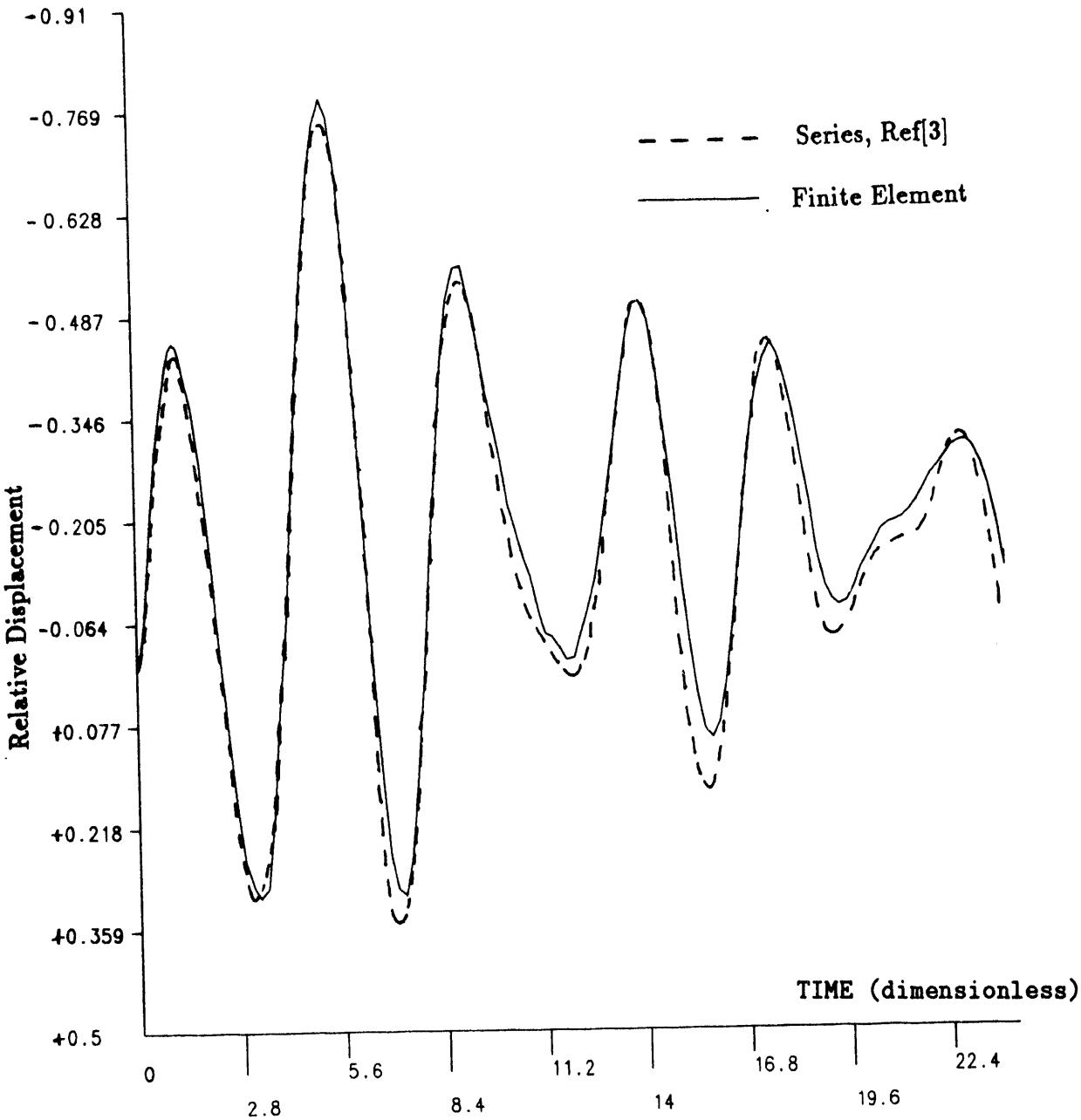

Figure 6: Time History of Half the Relative Displacement Between $\theta = 0$ and π

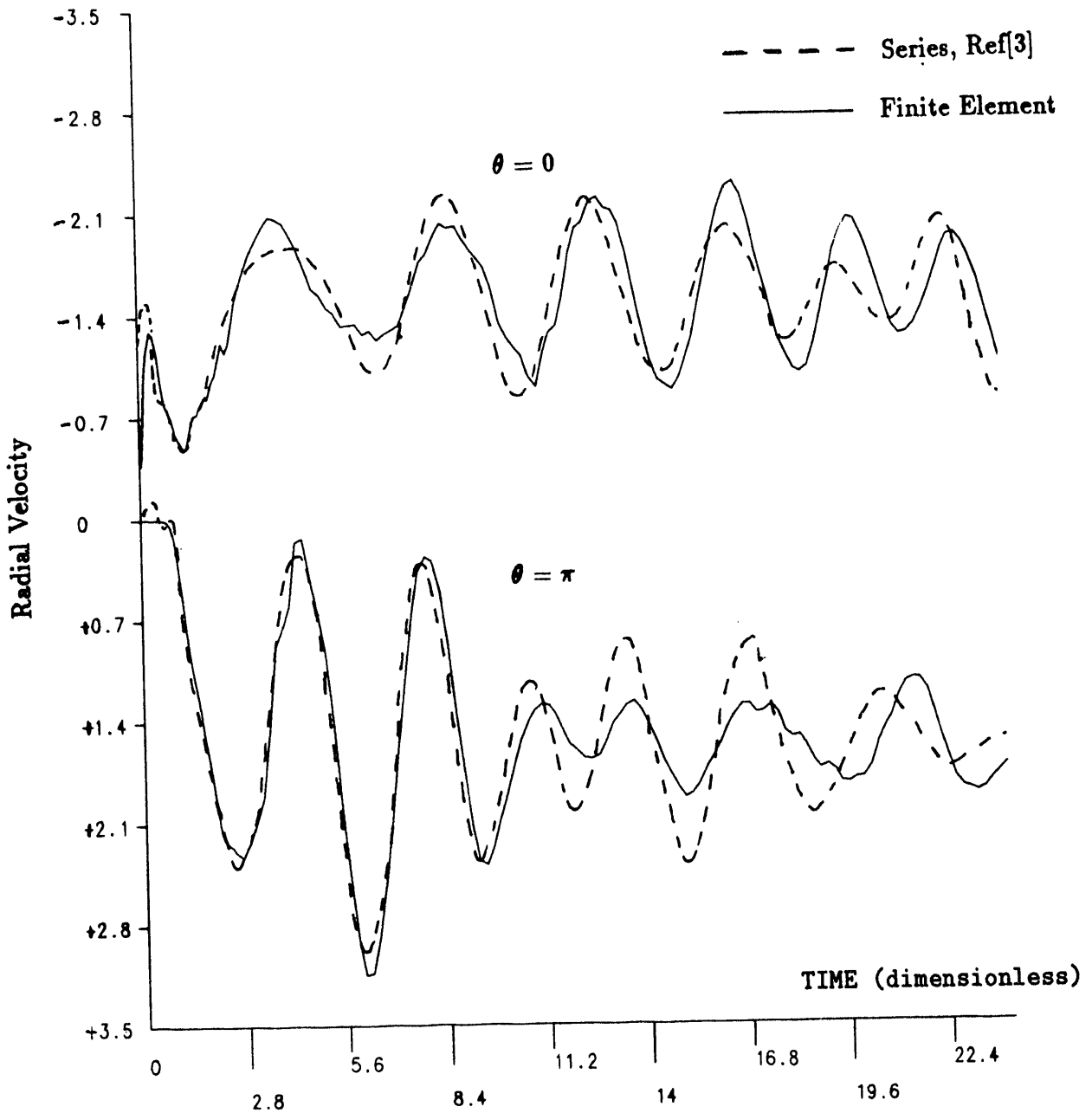

Figure 7: Time History for the Radial Velocity at $\theta = 0$ and π

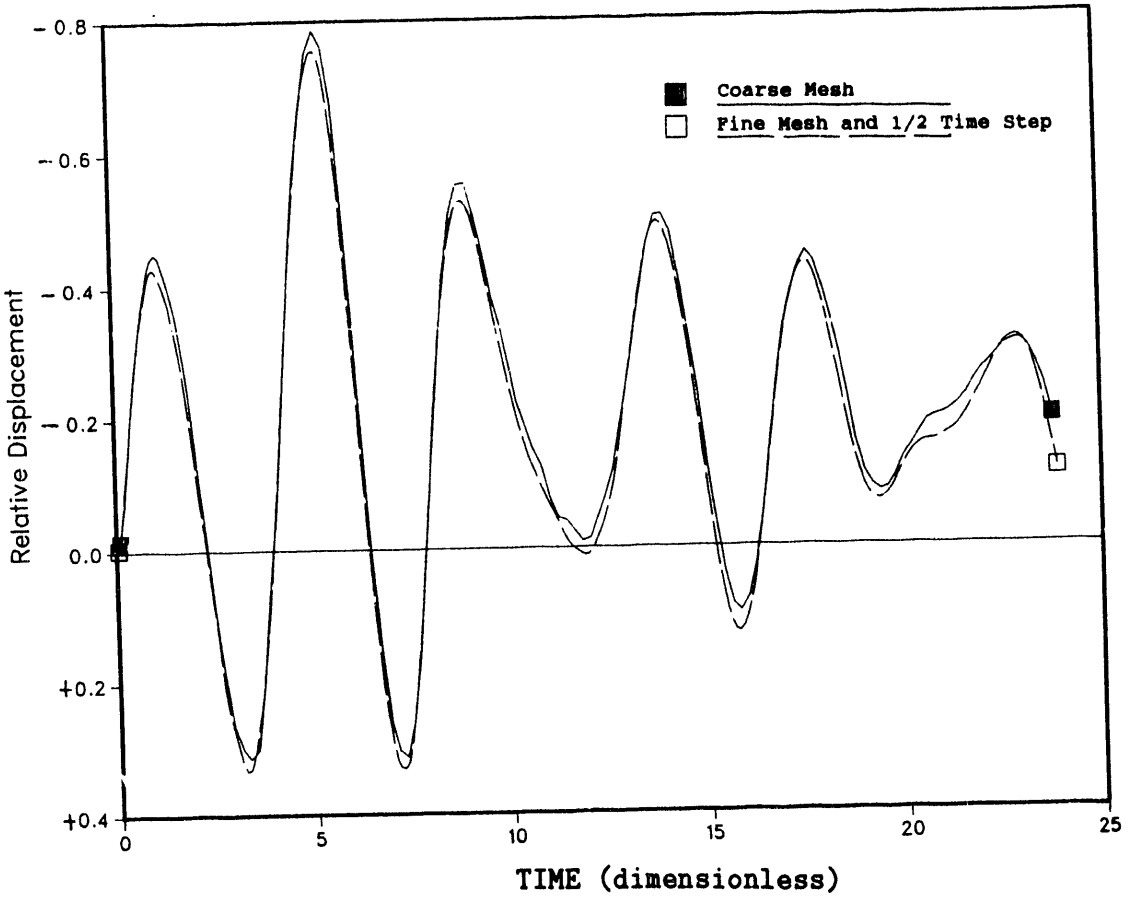

Figure 8: Relative Displacement Between $\theta = 0$ and π Using a Coarse Mesh and a Fine Mesh with Reduced Time Steps

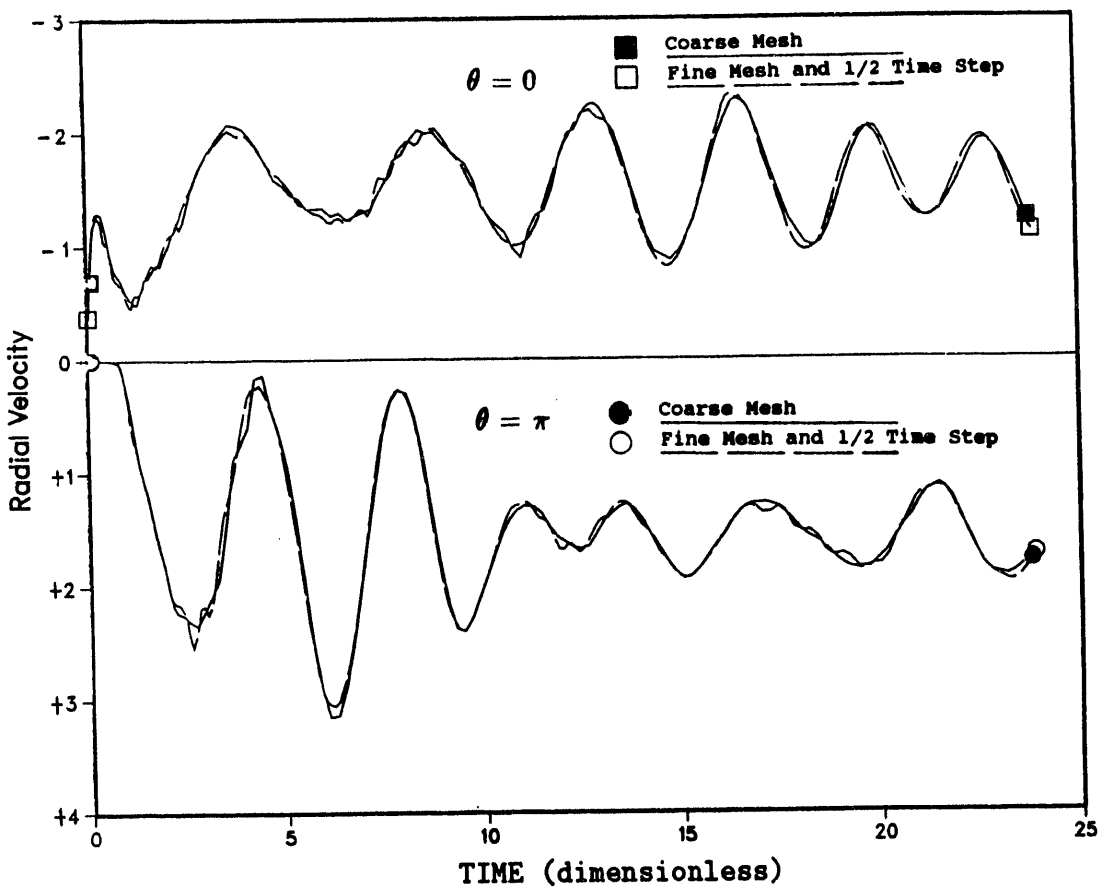

Figure 9: Radial Velocity at $\theta = 0$ and π Using a Coarse Mesh and a Fine Mesh with Reduced Time Steps

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