FERMILAB-PUB-06-479-AD

Coherent Motion with Linear Coupling

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February 14, 2007

Abstract

Coupling of the transverse degrees of freedom modifies analysis of beam coherent motion. A general, simple and effective rule to do that is derived.

1 Introduction

When the two fractional tunes of a storage ring are equal, $\{\nu_x\} = \{\nu_y\}$, the beam stays at a coupling resonance. This line in the tune space is of a special attraction: staying there maximizes tune area free from the dangerous resonances. That is why so many machines stayed, stay or plan to be there. Near the coupling resonance, even a small skew quadrupole or solenoid may result in a significant change of the beam optics, making it strongly coupled. If so, a conventional uncoupled 2D optical formalism cannot be used; instead, a 4D analysis has to be applied. Thus, any beam issue underlain by the optics has to be revisited, assuming that eigenmodes do not describe planar vertical and horizontal motion any more. One of these issues is a problem of the beam transverse coherent motion. This problem was considered in Refs. [1], [2], and discussed in [3]. Here, we suggest our view of the problem, and come to a solution, which is general and simple at the same time. The leading idea is that the classical mechanics is invariant over the canonical transformations. In a basis of the eigenmodes, the beam motion gets to be uncoupled, and formally similar to conventional $x - y$ uncoupled case. There is though a single difference between the $x - y$ space and the space of the normal modes. This difference relates to wake functions or impedances, which are given in the $x - y$ space. Thus, to solve the problem, the wakes and impedances have to be properly projected on the eigenvectors.

2 Eigenmodes perturbations

For arbitrary coupling, the beam optics can be described in terms of 4D eigenvectors. Hereafter, a parametrization suggested in [4] is used, where the 4