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Coherent Synchro-Betatron Resonance

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Abstract

Coherent synchro-betatron resonances can present a serious limit for low-energy synchrotrons with strong space charge. Here, an excitation of a dipole transverse mode is considered at resonance condition.

1 Introduction

Rapid cycling sinchrotrons normally deal with so intense beams, that the space charge tune shift can reach or even exceed 0.5. Therefore, to prevent integer resonances, the machine tunes have to be set just a little below of integer numbers. Due to high acceleration rate, these machines have rather high value of the synchrotron tune at injection energy. Small (non-relativistic) injection energy results in the incoherent tune shifts (mainly due to beam space charge) to be smaller the coherent (due to image charges). All these circumstances can lead to an excitation of low-order coherent synchro-betatron resonance (CSBR). Due to the high space charge tune shift, the coherent and incoherent spectra are effectively separated; thus, there is no Landau damping, otherwise stabilizing the resonant mode. CSBR is in a way paradoxical phenomenon: a coherent mode is excited by external perturbations. Growth of this mode does not depend on the beam intensity, as soon as the incoherent spectrum is separated from the coherent line by the space charge, which easily happens at relatively low beam current.

Some indications show that CSBR is an important phenomenon for the Fermilab Booster. This paper does not discuss observations though, leaving that for future reports, and being limited to the very concept of CSBR.

2 Dipole Resonances

Condition of dipole CSBR is $\nu_b + l\nu_s = n$, with $\nu_b = \omega_b/\omega_0$, $\nu_s = \omega_s/\omega_0$ as the coherent betatron and synchrotron tunes, and l, n as integer numbers. When this condition is satisfied, any dipole perturbation generally drives the resonance mode. To understand main features of the phenomenon, a longitudinal distribution is taken here as the air-bag one: all the particles have the