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A Note on Cosmic (p, q, r) Strings

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The spectrum of (p, q) bound states of F- and D-strings has a distinctive square-root tension formula that is hoped to be a hallmark of fundamental cosmic strings. We point out that the BPS bound for vortices in $\mathcal{N} = 2$ supersymmetric Abelian-Higgs models also takes the square-root form. In contrast to string theory, the most general supersymmetric field theoretic model allows for (p, q, r) strings, with three classes of strings rather than two. Unfortunately, we find that there do not exist BPS solutions except in the trivial case. The issue of whether there exist non-BPS solutions which may closely resemble the square-root form is left as an open question.

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INTRODUCTION

There has been recent interest in the cosmic rehabilitation of fundamental strings [1]. This has led to the exciting possibility that the properties of a cosmic network made of fundamental strings may have observationally distinct signatures from more mundane solitonic objects. There are two smoking guns. The first is the probability of reconnection which scales as $P \sim g_s^2$ for fundamental strings [2], while it is essentially unity for abelian vortices [3, 4]. Recently it was shown that non-abelian vortices may give rise to a probability $P < 1$ but the velocity dependence differs substantially from the fundamental string case [5].

The second smoking gun [6] is the existence of both F- and D-strings in warped IIB compactifications which form a distinctive spectrum of bound states with tension

$$\mu_{(p,q)} = \sqrt{p^2\,\mu_F^2 + q^2\,\mu_D^2}. \tag{1}$$

A simple consequence of these bound states is the existence of 3-string junctions, with angles dictated by charge conservation and the tension formula (1). Apects of network formation and gravitational lensing of such junctions was studied in [7, 8].

It is rather simple to construct field theories which admit bound states of vortices and the corresponding 3-string junctions. Examples include vortices charged under discrete symmetries [9] and multiple abelian gauge groups [10]. However, none of the field theoretic models studied so far reproduce the stringy spectrum (1). The purpose of this short note is to show that the general Bogomolnyi bound in gauge theories with multiple $U(1)$ gauge groups includes the string spectrum (1). In fact, we shall see that there is a maximum of three different types of supersymmetric vortices, with the tensions bounded by

$$\mu = \sqrt{k_1^2\mu_1^2 + k_2^2\mu_2^2 + k_3^2\mu_3^2} \tag{2}$$

where k_i are integer gauge winding charges. If we choose a field theory without the third type of vortex, then this mimics the IIB string theory spectrum of cosmic strings. Although we find that no nontrivial BPS solutions exist which have this square-root spectrum, it is possible that non-BPS solutions exist which could closely resemble it.

THE BOGOMOLNYI BOUND

In theories with $\mathcal{N} = 2$ supersymmetry, the real D-term and complex F-term are unified into a triplet, transforming under an $SU(2)_R$ R-symmetry. This existence of this triplet is responsible for the three different tensions appearing in (2). Recall that matter lives in a hypermultiplet, consisting of two complex scalars ϕ and $\tilde{\phi}$ transforming in conjugate representations of the gauge group. For a single scalar charged under a single $U(1)$ gauge group, the D- and F-terms in the scalar potential [12] are fixed by $\mathcal{N} = 2$ supersymmetry to be,

$$V = \frac{e^2}{2}(|\phi|^2 - |\tilde{\phi}|^2 - r_3)^2 + \frac{e^2}{2}|2\tilde{\phi}\phi - r_1 - ir_2|^2. \tag{3}$$

Here e^2 is the gauge coupling constant. There are three vacuum expectation values r_1, r_2 and r_3 allowed by supersymmetry (often referred to as a Fayet-Iliopoulos parameters). The $SU(2)_R$ symmetry of this potential can be made manifest by defining the doublet $\omega^T = (\phi, \tilde{\phi}^\dagger)$ and writing

$$V = \frac{e^2}{2}(\omega^\dagger \vec{\sigma}\,\omega - \vec{r})^2 \tag{4}$$

where $\vec{r} = (r_1, r_2, r_3)$ and $\vec{\sigma}$ are the triplet of Pauli matrices.

Consider now a $U(1)^N$ gauge theory with gauge coupling $e_a^2, a = 1, \ldots, N$. We couple N hypermultiplets ω_i with integer charges $Q^i{}_a$ under the ath gauge group. The covariant derivatives are given by $\mathcal{D}\omega_i = \partial\omega_i - i(\sum_{a=1}^N Q^i{}_a A_a)\omega_i$. The energy functional for static