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Integrals for IBS and Beam Cooling

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Abstract. Simulation of beam cooling usually requires performing certain integral transformations every time step or so, which is a significant burden on the CPU. Examples are the dispersion integrals (Hilbert transforms) in the stochastic cooling, wake fields and IBS integrals. An original method is suggested for fast and sufficiently accurate computation of the integrals. This method is applied for the dispersion integral. Some methodical aspects of the IBS analysis are discussed.

Keywords: electron cooling, stochastic cooling, intra-beam scattering
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INTRODUCTION

Simulation of beam cooling usually requires intensity effects to be taken into account. For stochastic cooling, computation of the beam response requires Hilbert transformation of the distribution function being repeated many times for the simulation process. If the beam is intense enough, intra-beam scattering (IBS) and instabilities normally have to be taken into account both for stochastic and electron cooling scenarios.

From a computational point of view, intensity effects require integral transformations of the evolving distribution function to be repeated every time step or so, which puts a heavy burden on the CPU. The cooling scenario is characterized by many parameters. Some of these parameters have to be optimized, other have to be below some tolerances. One of the main purposes of the modeling is to find these optimal points and limits. Solving problems like that requires numerous cut-and-try runs of the modeling program for various points in a multi-dimensional space of parameters. That is why the program has to be sufficiently fast, taking not longer than a few minutes per a single run. To meet this goal, all the mentioned composites of the beam evolution have to be computed at a proper compromise between the duration and accuracy. This paper describes some ideas which the author found useful in his computations for electron cooling projects of antiprotons in the Recycler, FNAL [1] and heavy ions in RHIC, BNL [2].

GUESS METHOD OF INTEGRAL TRANSFORMATION

General Description

Imagine that you have to calculate an integral like

$$c(u) = \int_{-\infty}^{\infty} f(v)K(u,v)dv, \qquad (1)$$

where ***f(v)*** is a distribution function, or its derivative, or something related to the distribution function, and ***K(u,v)*** is a given kernel, which might have an integrable singularity, usually at $u = v$. Such integrals appear in stochastic cooling as the dispersion integrals (Hilbert transformations), and for intra-beam scattering as the friction force and diffusion coefficient. These singular integrals have to be calculated numerically at every point of the phase space (for every u), and at every (or so) time step, which puts a heavy burden on the CPU and could result in unacceptably long run time. If so, a proper approximate method of taking the integral is required.

The result of integration depends on the specifics of the distribution function, and could vary a lot from one distribution to another. However, for many problems, the distribution function has some a priori known features. For example, let's take it as an even function with a single maximum at zero argument.