Project Number: 60451

Mechanics of Bubbles in Sludges and Slurries: Modeling Studies of Particulate Materials

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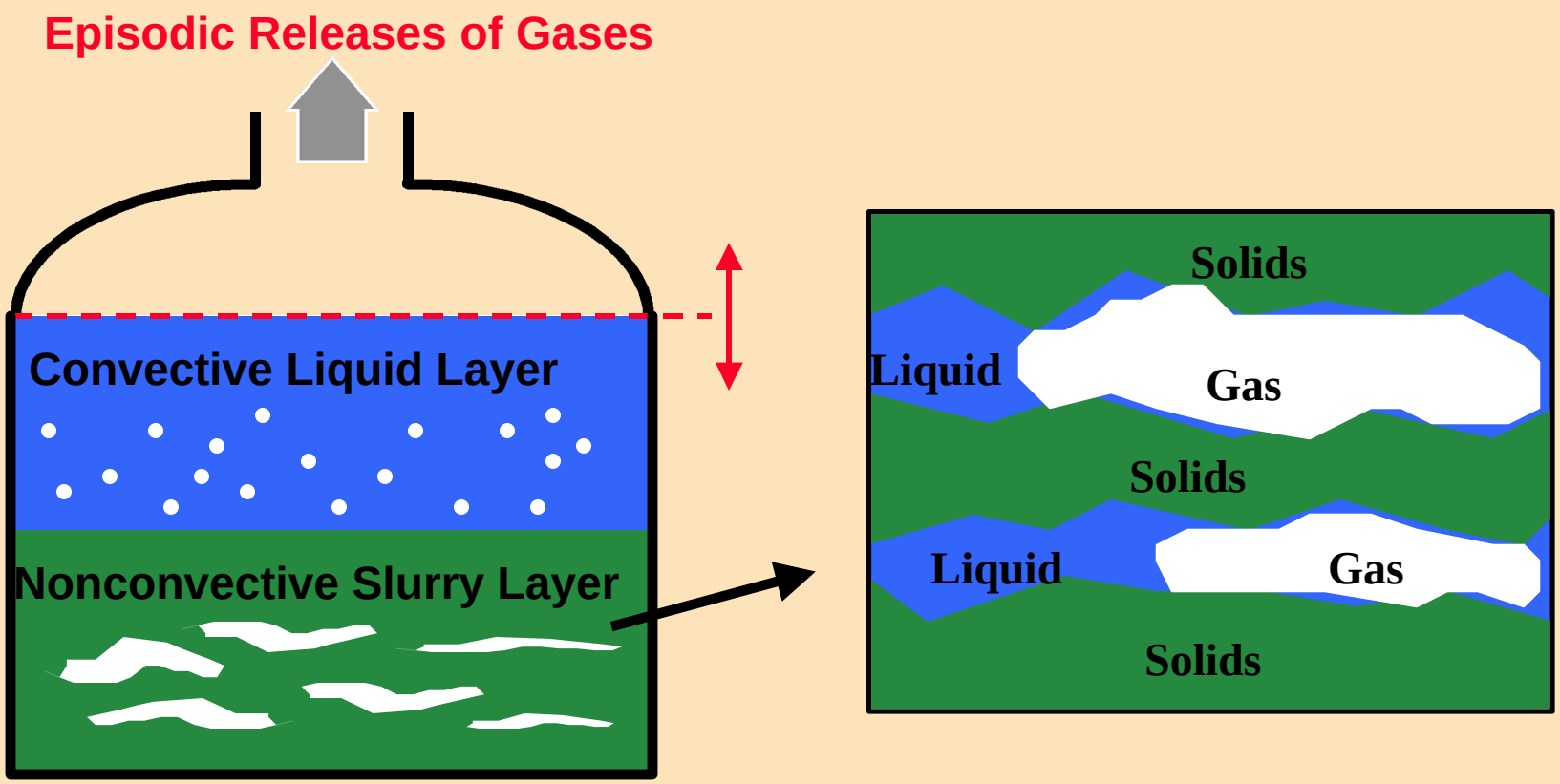
Objectives

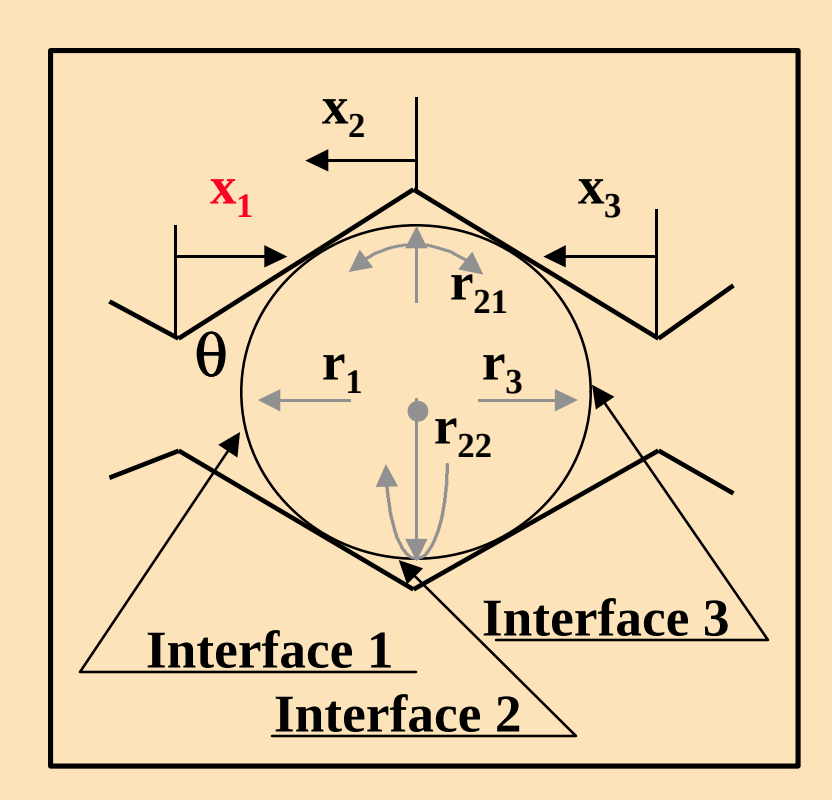
- **Effective compressibility** of gases trapped in an porous medium :
  - two different models
    - » 1D biconical pore network
    - » spherical pore geometry with 3D connectivity
  - fitting to the field measurement
    - » liquid level change in response to barometric pressure
- **Rheological properties** of gas/liquid/solids system
  - yield stress
  - compressibility

Effective Compressibility (1D biconical model)



Modeling Bubble Growth

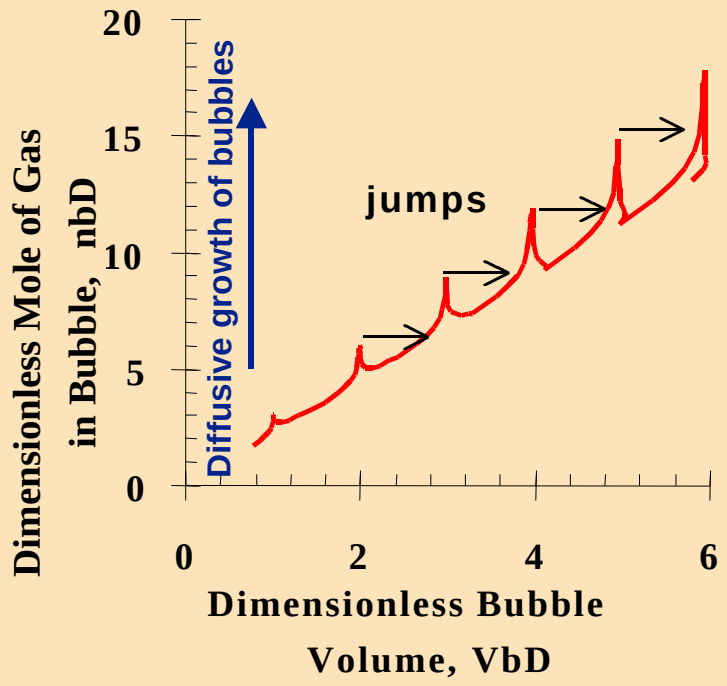
- Pore Geometry
  - All pores are biconical and identical.
  - Three Interfaces
  - Position x_1 determines radii & positions of other interfaces (capillary equilibrium)
- Three time scales
  - long term : bubble growth
  - short term : bubble volume responding to pressure change
  - very-short term : interface jumps

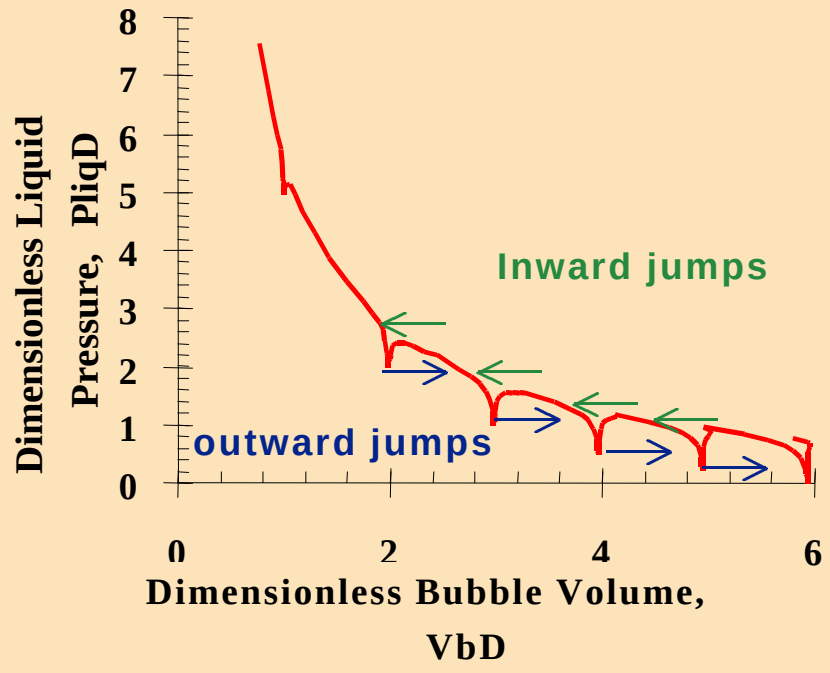


Single Bubble

Diffusive growth of bubble

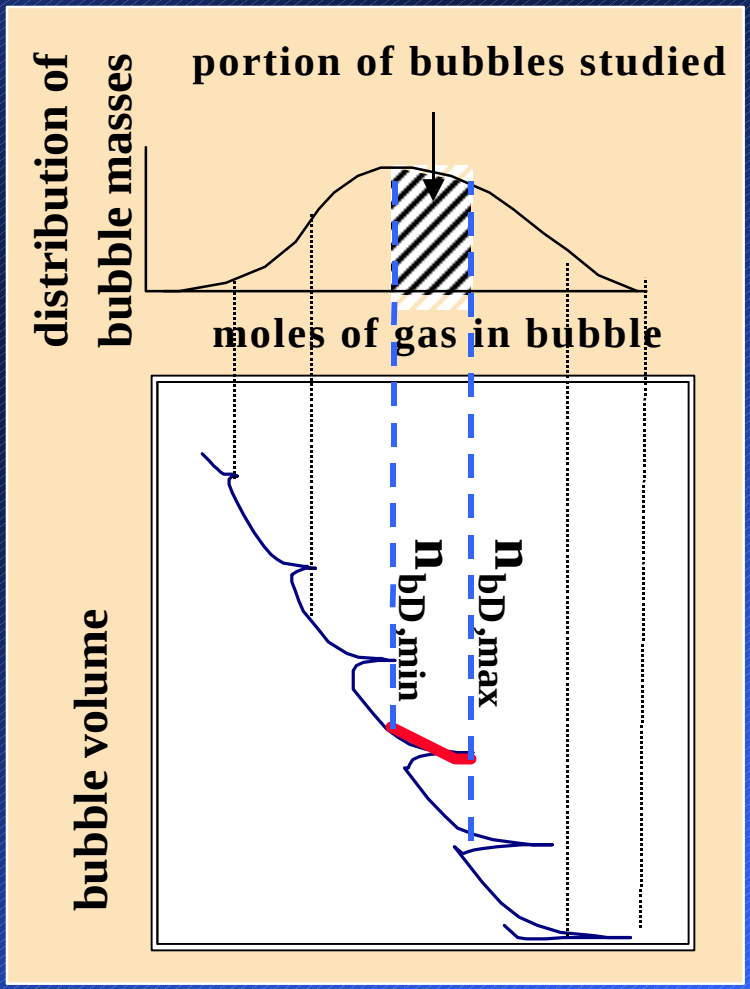
Bubble volume change in response to liquid pressure change at fixed bubble mass





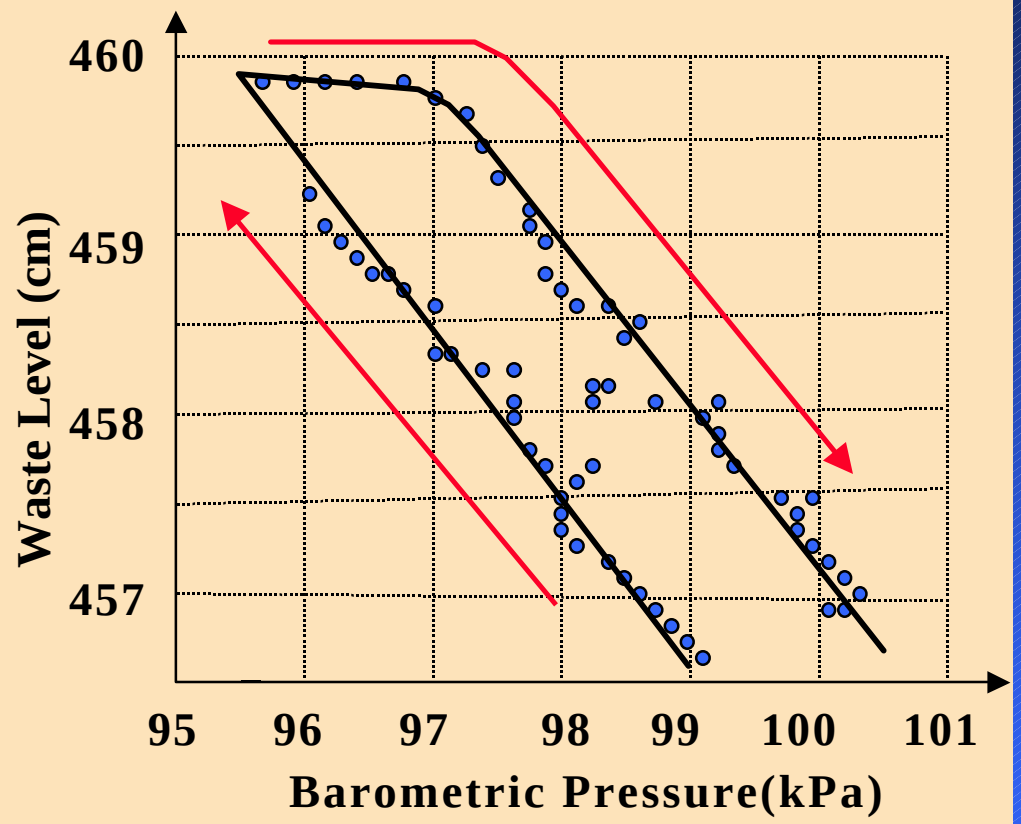
Population Of Bubbles

- How to average population of bubbles?
- Assume bubbles have masses uniformly distributed between consecutive jumps .
- When bubbles grow large, they are all poised near pore throats.



Field Measurement

- During pressure decrease
  - Liquid level increases linearly with decrease in pressure.
- During subsequence pressure increase
  - Liquid level stays during small increase in liquid pressure.
  - Then, liquid level decreases linearly with increase in pressure.

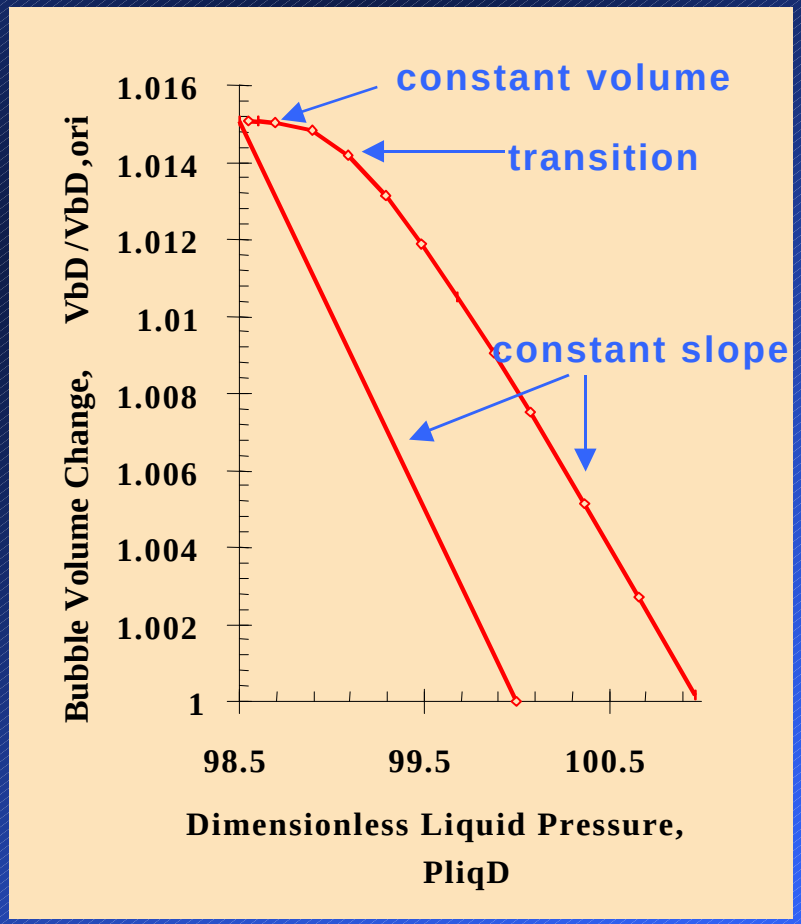


(After Whitney et al., 1996)

Fitting To Field Measurement

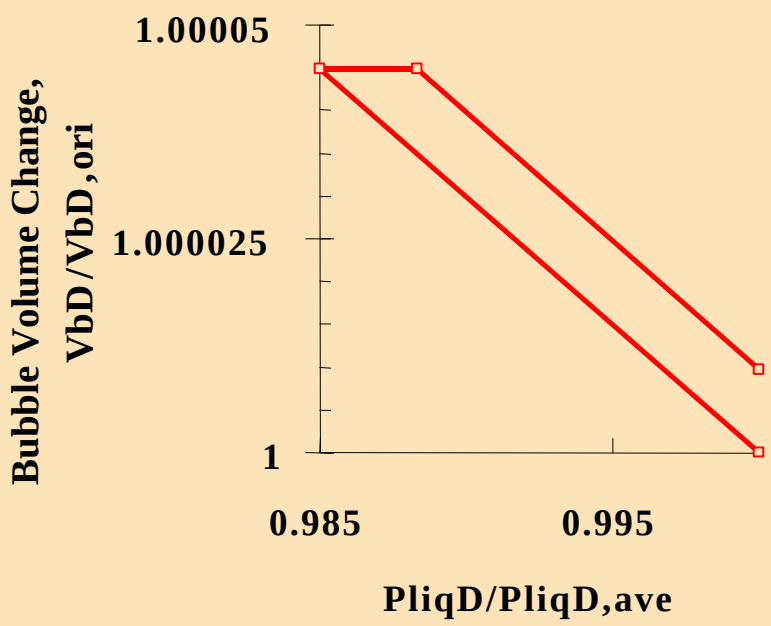
($R_b/R_t = 1.5$, $N_m = 10000$, $P_{liq}/P_c^e = 100$)

- Constant-volume region
  - interfaces at pore throats at the beginning of pressure increase. --> *N_m large (bubbles many pores long)*
  - contribution of liquid film at pore corners should be small-->*R_{bD} (i.e., R_b/R_t) small*
- Transition region: interface movement along throat to body
- Two constant-slope regions
  - compressibilities are comparable--> *P_{liqD} (i.e., P_{liq}/P_c^e) large*

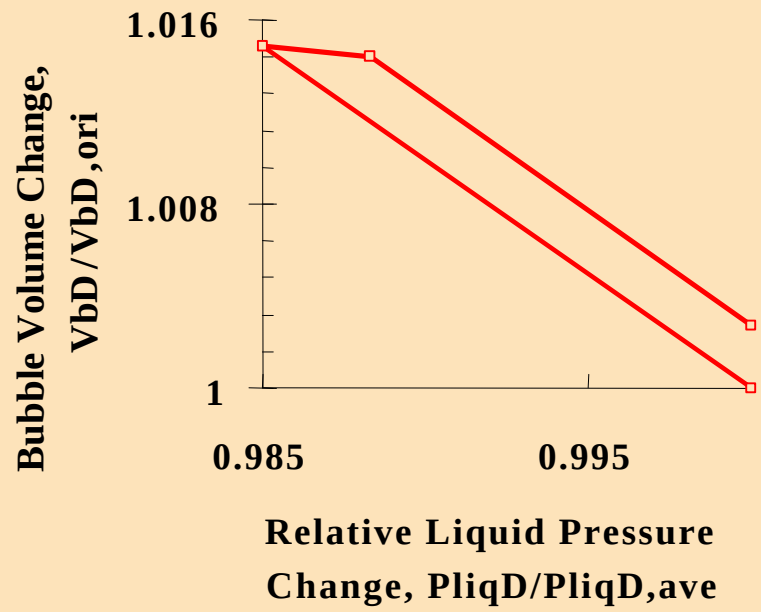


Ambiguity in Model Fit

- P_{liqD} ($\equiv P_{liq} / P_c^e$) cannot be determined uniquely.
  --> ambiguity in determining capillary effect.



(a) R_{bD} = 1.0001 , P_{liqD} = 0.003



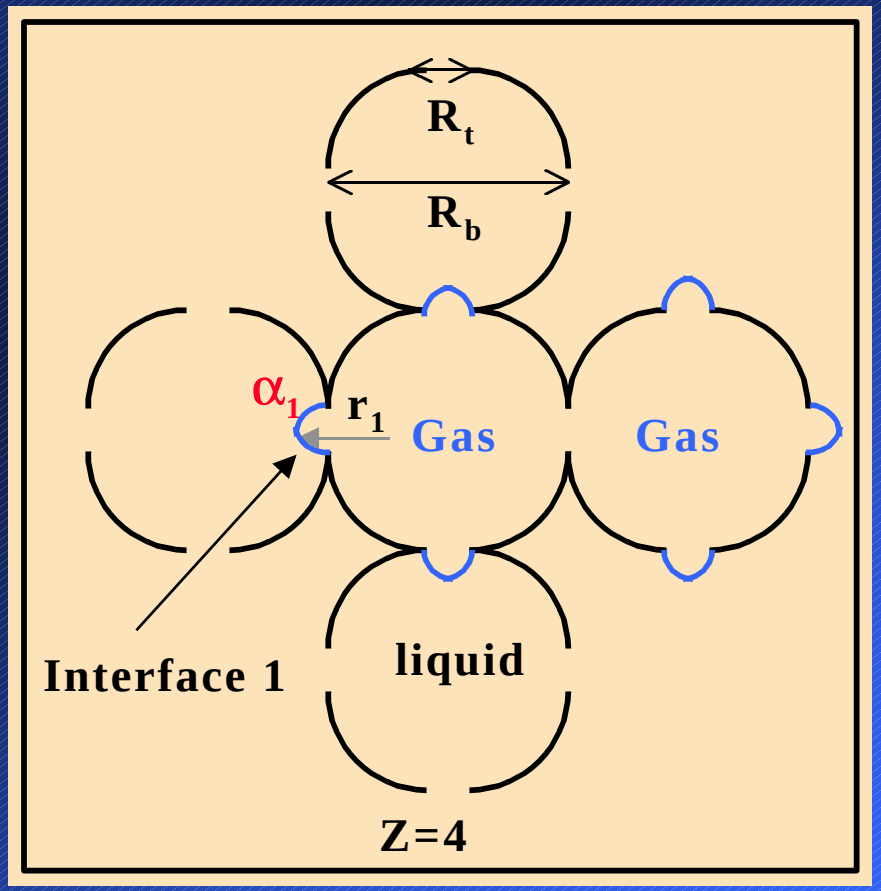
(b) R_{bD} = 1.5, P_{liqD} = 100

Conclusions (biconical model)

- The mechanics of bubble movement in the slurry layer in a Hanford waste tank, responding to liquid pressure change, were studied using a 1D biconical-pore network model.
- The model shows that there is a hysteresis in effective compressibility, between pressure decreases and increases. Impulsive jumps occur from pore throat to (near) pore throat during pressure decreases, and from pore body to (near) pore body during pressure increases.
- The model can fit tank data from the Hanford reservation with good agreement provided that N_m is large and R_{bD} is relatively small; however, the relative magnitude of capillary effects cannot be determined unambiguously.

Effective Compressibility (spherical-pore model)

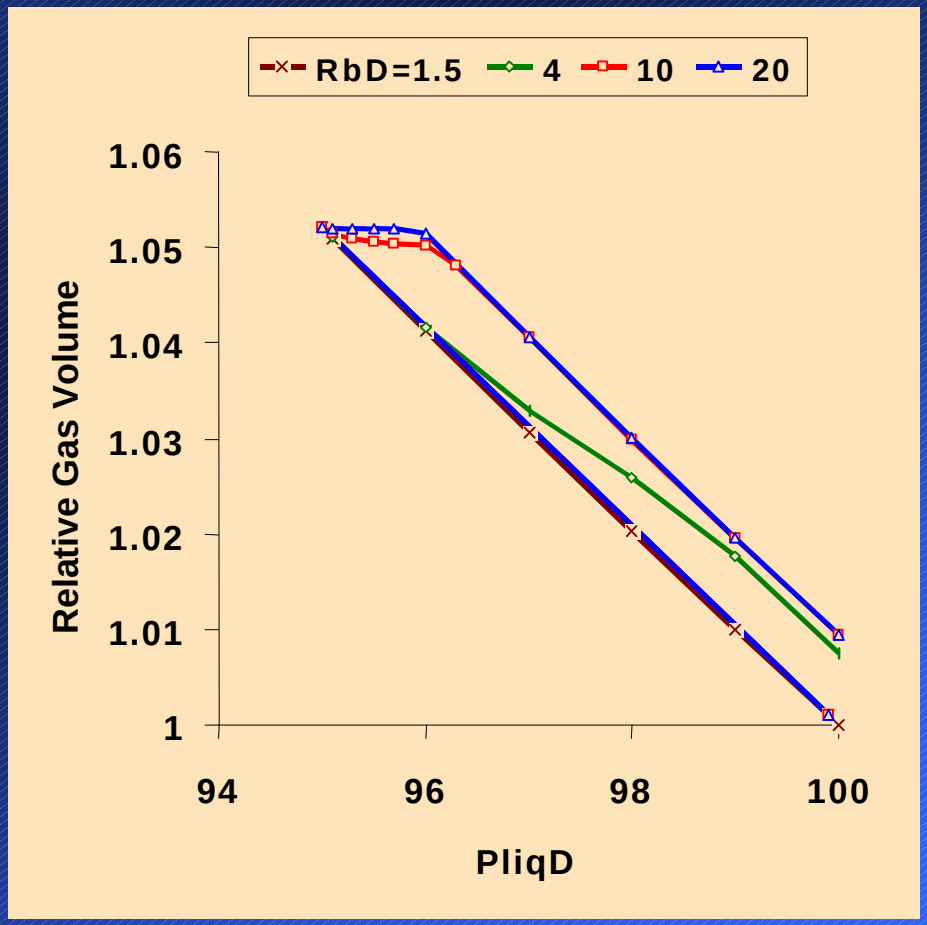
- **Pore Geometry**
  - All pores are spherical and identical.
  - No interface at the pore corner (no pore corner)
  - Able to consider the effect of connectivity (coordination number Z)



Fitting To Field Measurement

(N_m = 10000, P_{liq}/P_c^e = 100, Z=6)

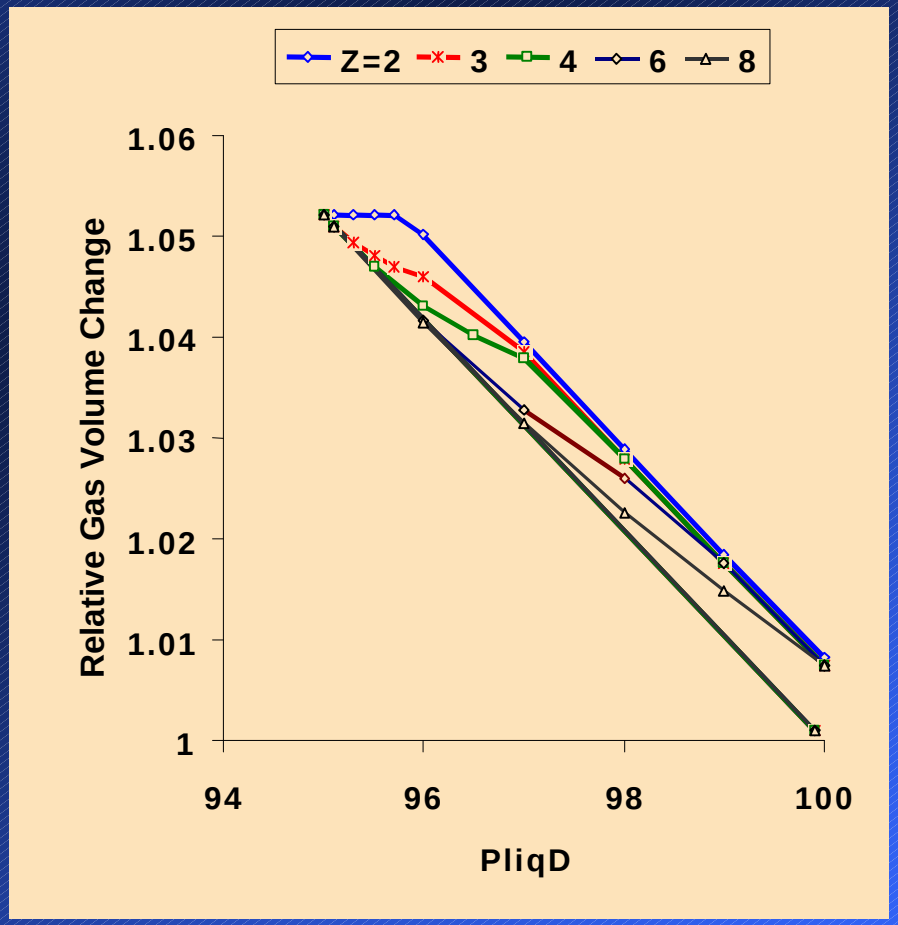
- As with 1D biconical model, large N_m and P_{liqD} (i.e., P_{liq}/P_c^e) are required.
- Effect of R_{bD}
  - Contrary to 1D biconical model, large R_{bD} (i.e., R_b/R_t) is required to fit model.
  - Physically, same implication : small volume change at the beginning of pressure increase!



Fitting To Field Measurement

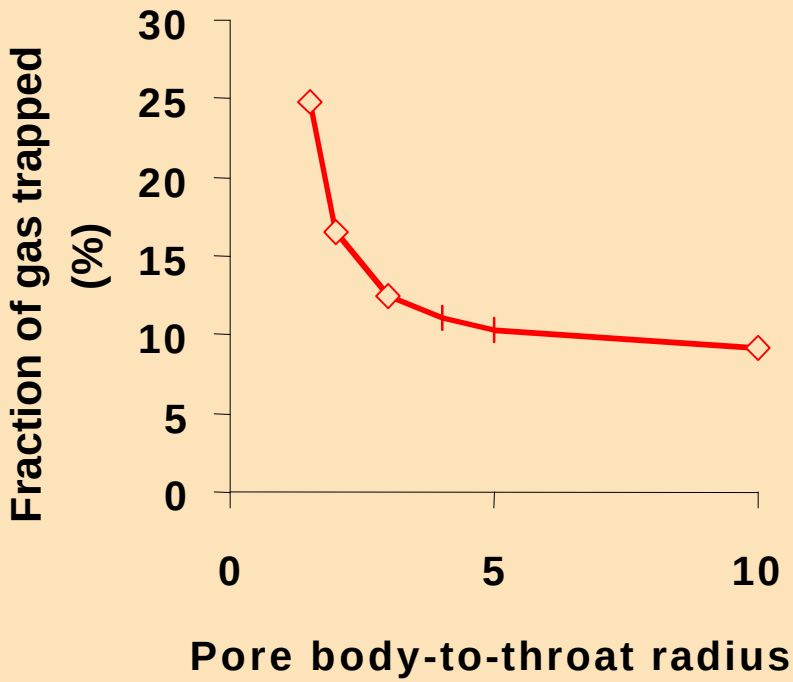
(N_m = 10000, P_{liq}/P_c^e = 100, R_b/R_t = 4)

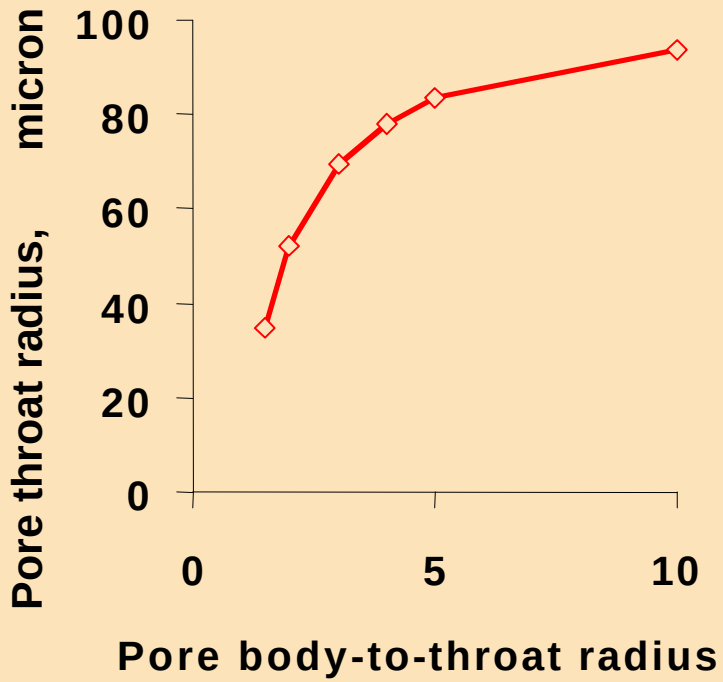
- Effect of coordination number *Z*
  - Smaller Z fits data better.
  - Physically, same implication : small volume change at the beginning of pressure increase!



Estimating Trapped Gas Volume

- With pore-geometry information, can estimate trapped-gas fraction



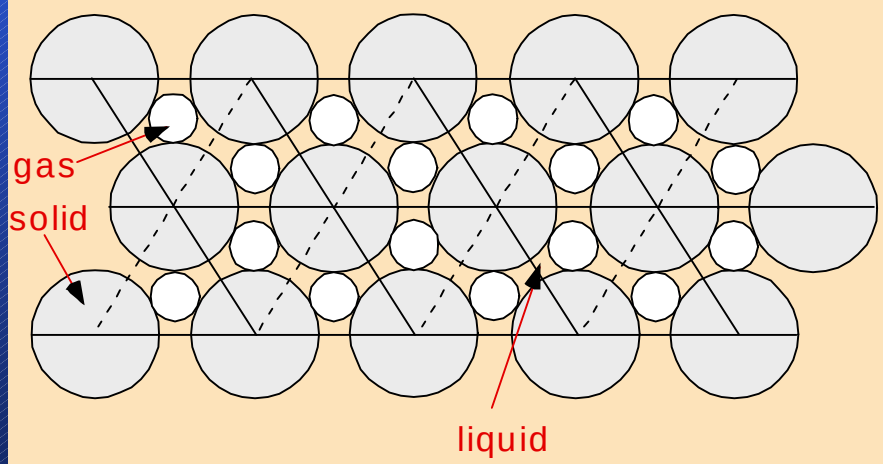


Conclusions (spherical-pore model)

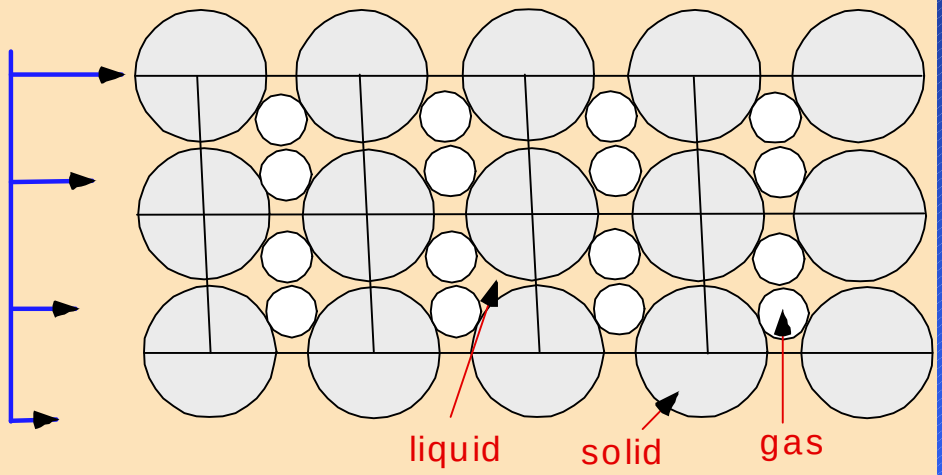
- Simple model using spherical pore geometry is considered to see the effect of different pore geometry and connectivity.
- There exist steady/unsteady movements for the population of bubbles and a hysteresis in effective compressibility, between pressure decreases and increases as found in 1D biconical pore network model.
- The model can fit field data with large N_m and P_{liqD}. Larger R_{bD} and smaller Z improve fit.
- This model suggests this model also requires information on pore size to estimate trapped-gas volume.

Yield Stress of Three-phase Mixtures

- Three phases are present in the parallelogram.
- Contact angle is assumed to be zero.
- Surfactant stabilizes liquid films between bubbles.



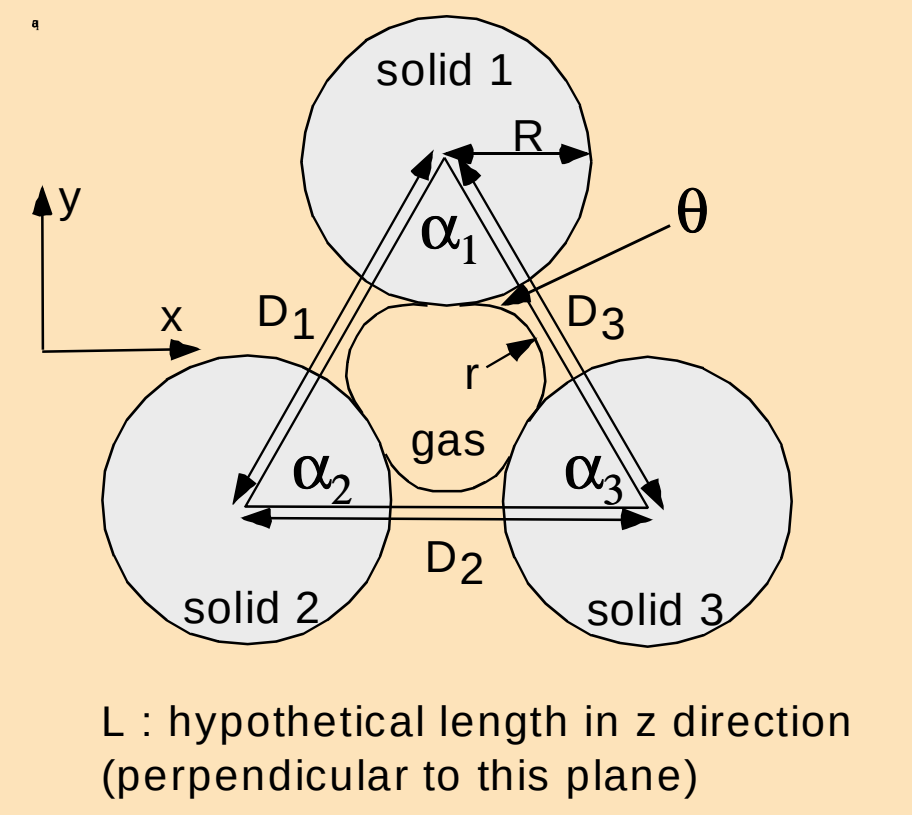
(a) Structure of foamy sands at rest (all phases are evenly distributed in the unit cell)



(b) Structure of foamy sands in the presence of finite shear stress (all phases are still distributed in the unit cell)

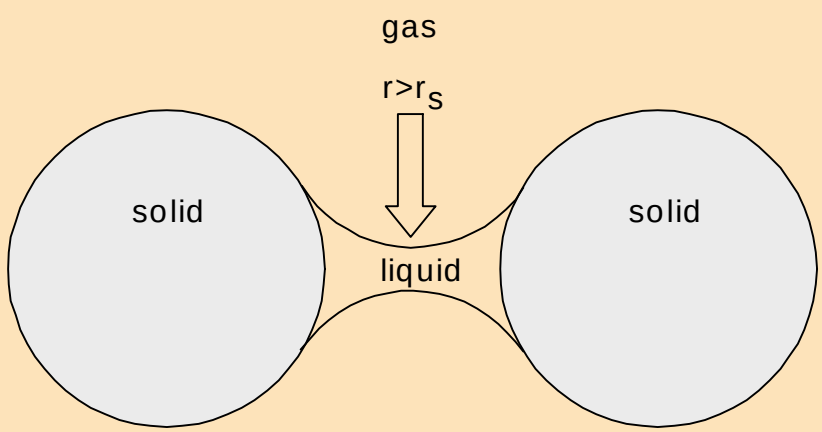
Input Parameters

- Fraction of three phases in the mixture gives unique geometric distribution in the triangular unit cell.
- Bubble deforms as solid 1 is displaced; volume of each phase constant.

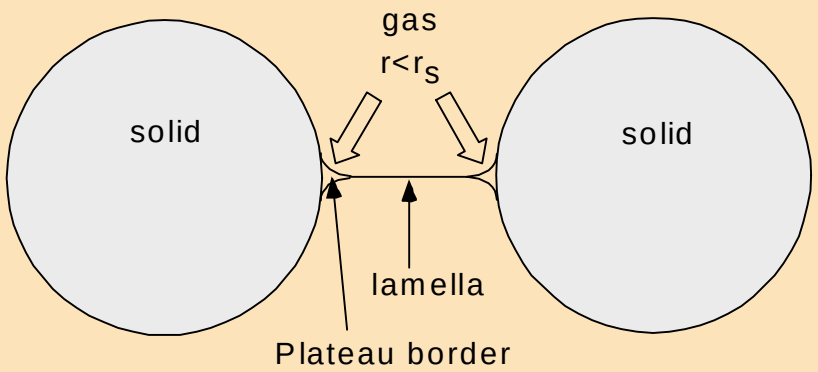


Existence Of Two Modes

- **There exists two modes depending on the gas fraction**

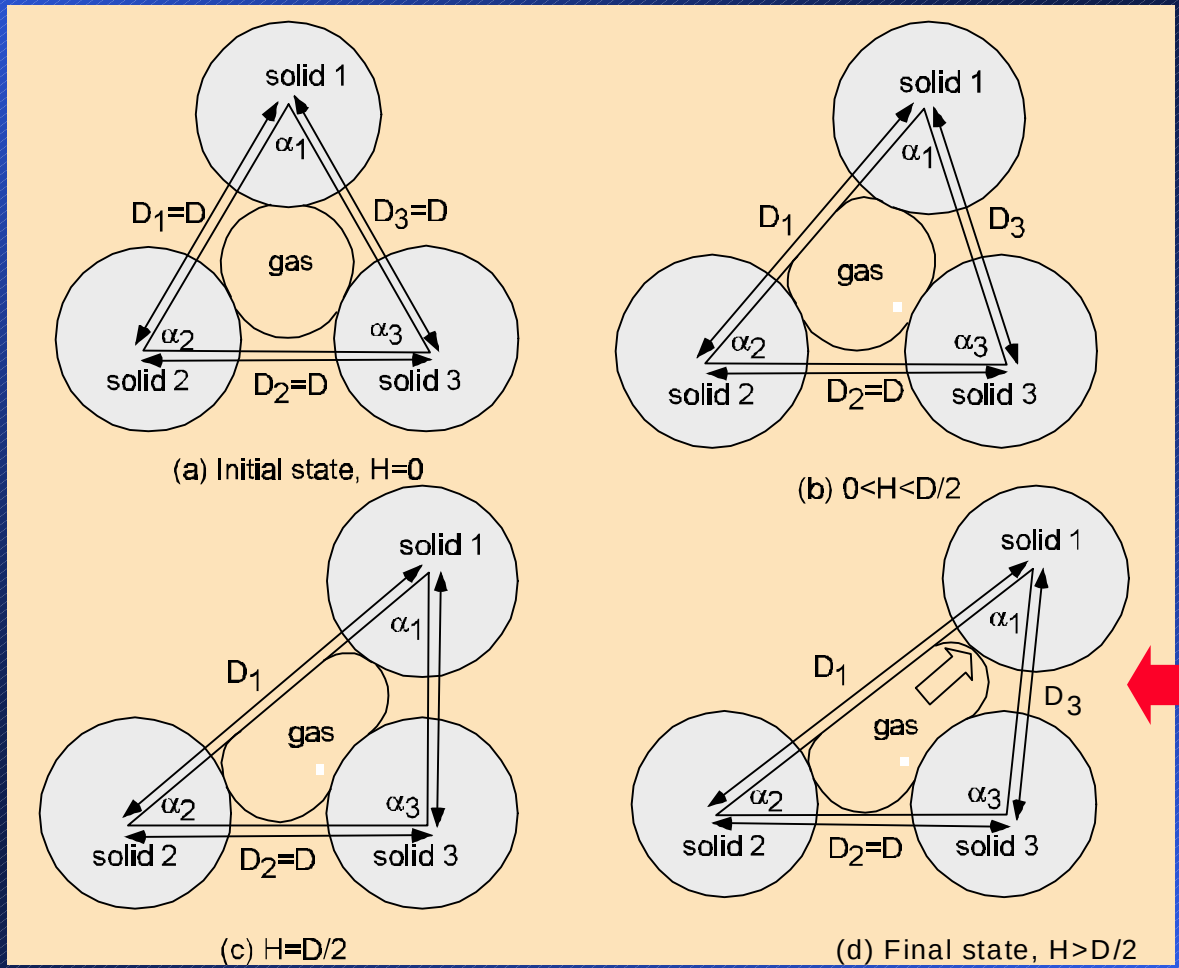


(a) Mode 1 ($r>r_s$) : interface between solids forms lens



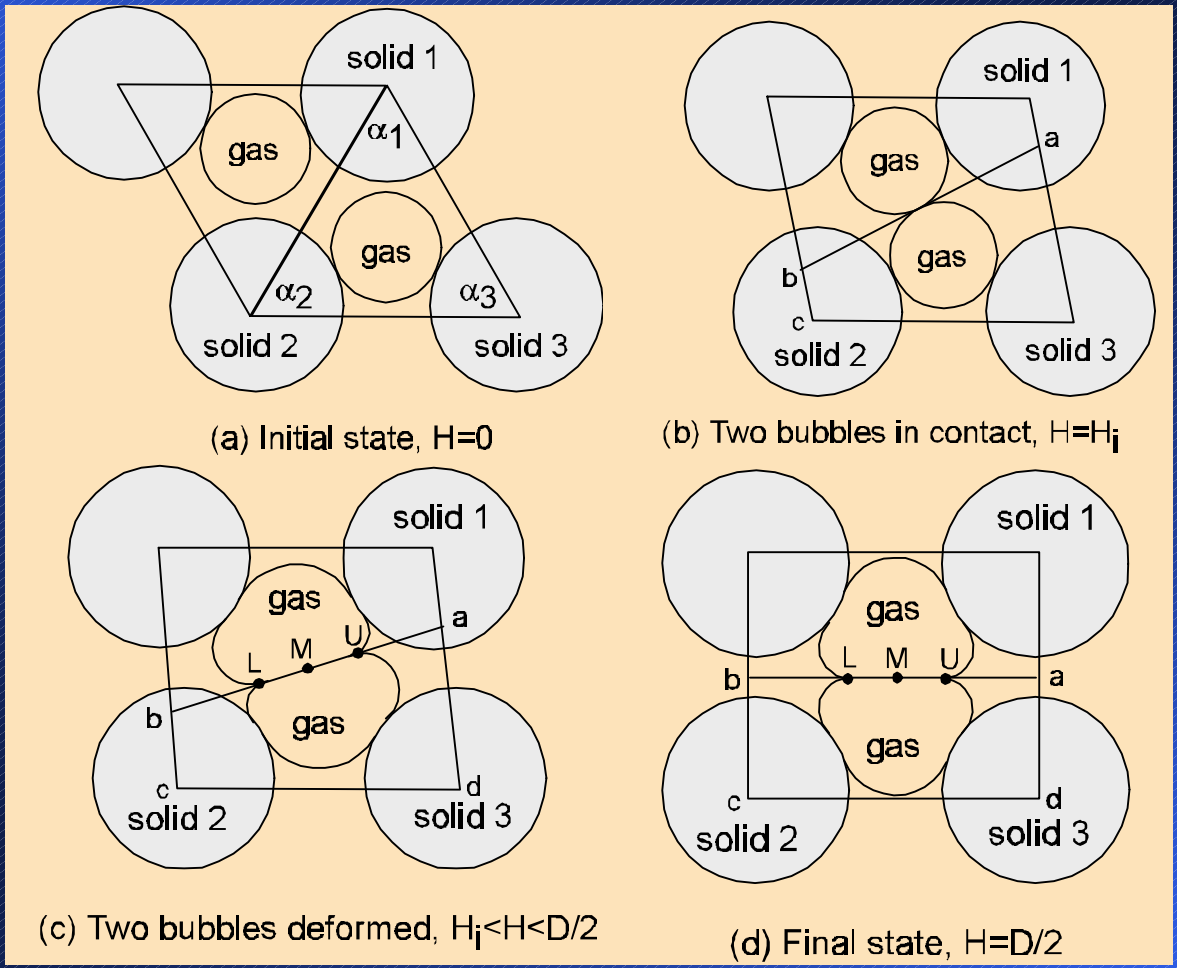
(b) Mode 2 ($r<r_s$) : interface between solids forms lamella and Plateau border

Case 1

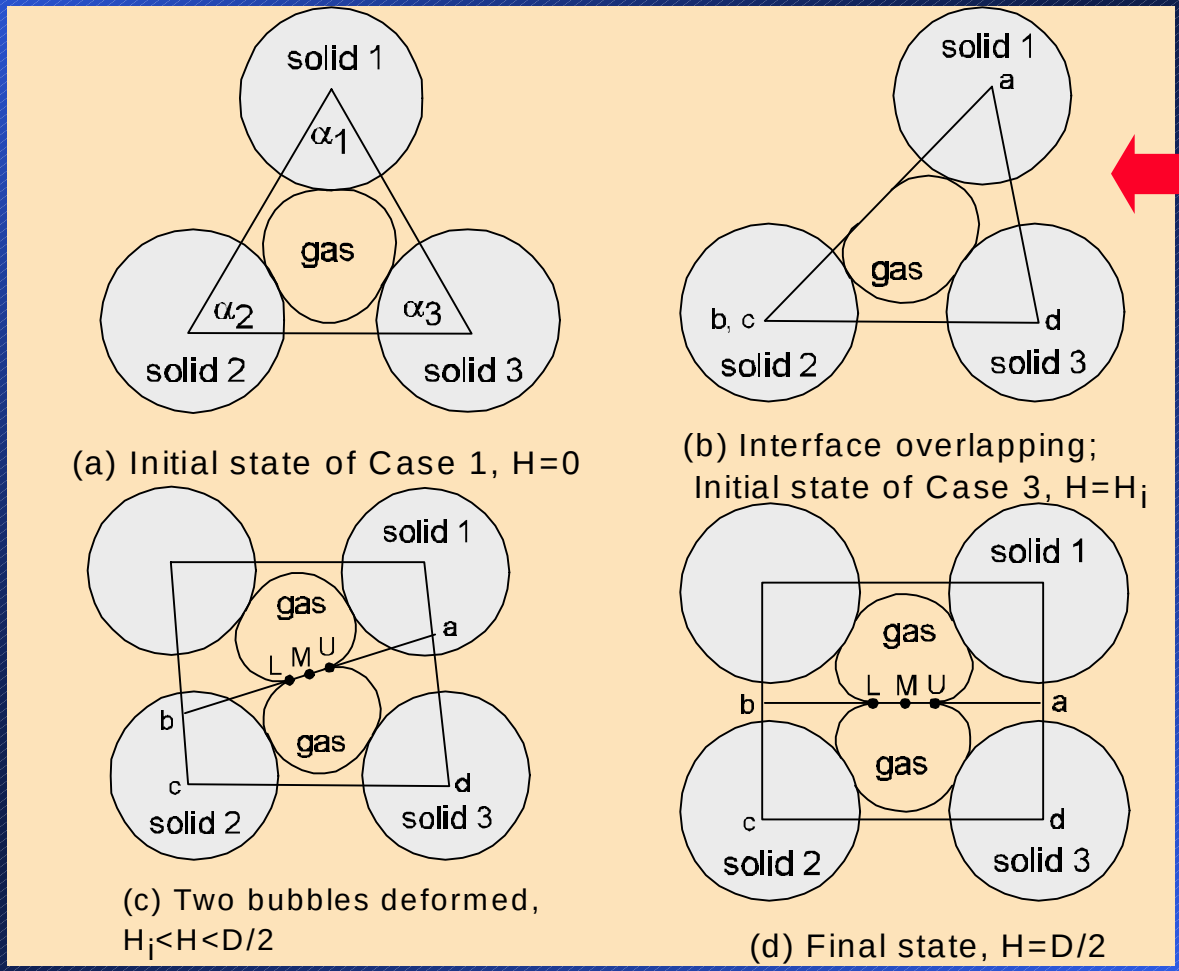


Bubble detaches late

Case 2


solid 1
gas
α1
gas
α2
α3
solid 2
solid 3
(a) Initial state, H=0
solid 1
a
gas
gas
b
c
solid 2
solid 3
(b) Two bubbles in contact, H=Hi
solid 1
gas
a
L
M
U
b
gas
c
d
solid 2
solid 3
(c) Two bubbles deformed, Hi<H<D/2
solid 1
gas
L
M
U
b
a
gas
c
d
solid 2
solid 3
(d) Final state, H=D/2

Case 3



(a) Initial state of Case 1, H=0

(b) Interface overlapping; Initial state of Case 3, $H=H_i$

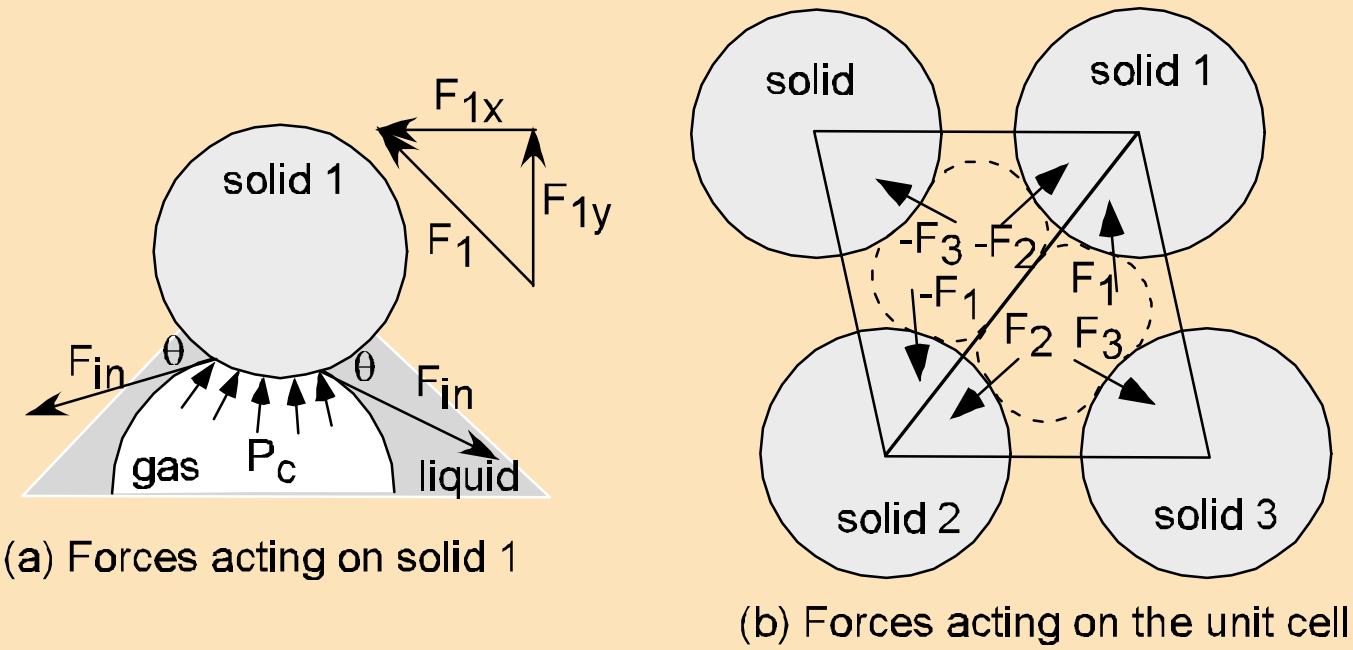
(c) Two bubbles deformed, $H_i<H<D/2$

(d) Final state, H=D/2

Bubble detaches early

Force Calculation

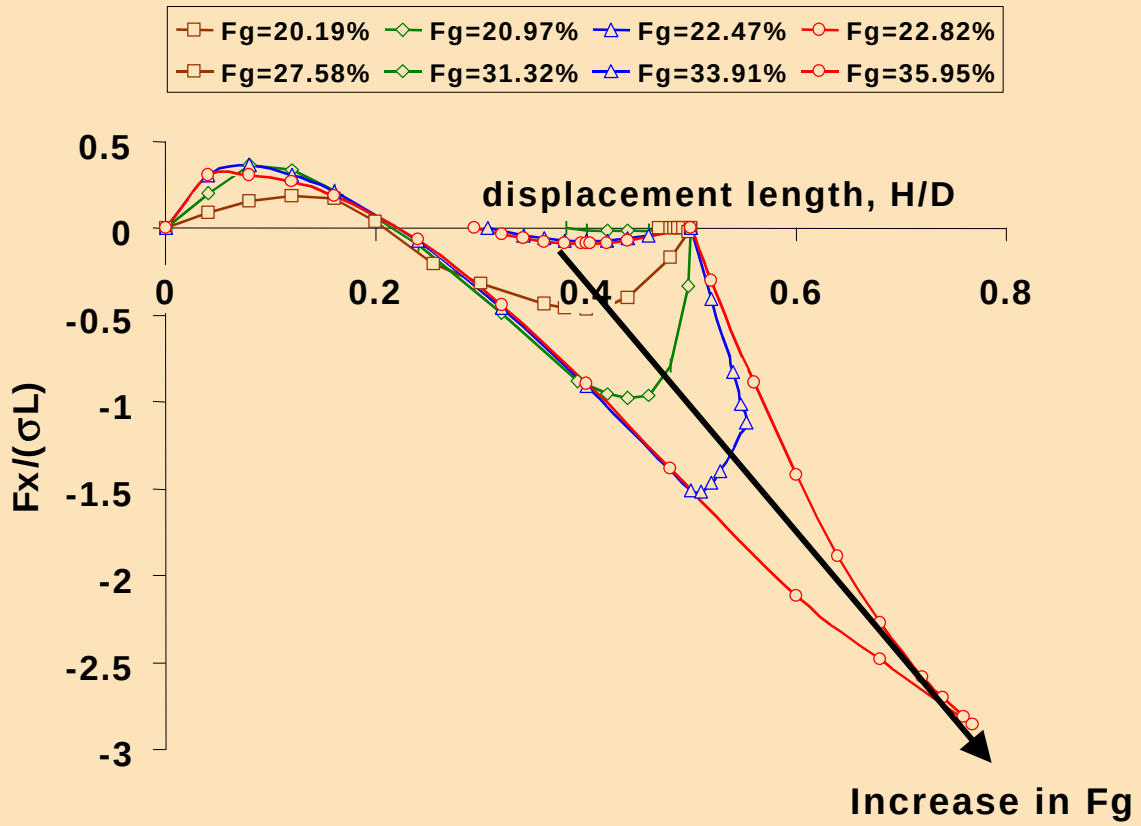
- $F_x = F_{1x} - F_{2x} - F_{3x}$
  - $F_x \equiv$ forces resisting to the horizontal displacement, proportional to yield stress



(a) Forces acting on solid 1

(b) Forces acting on the unit cell

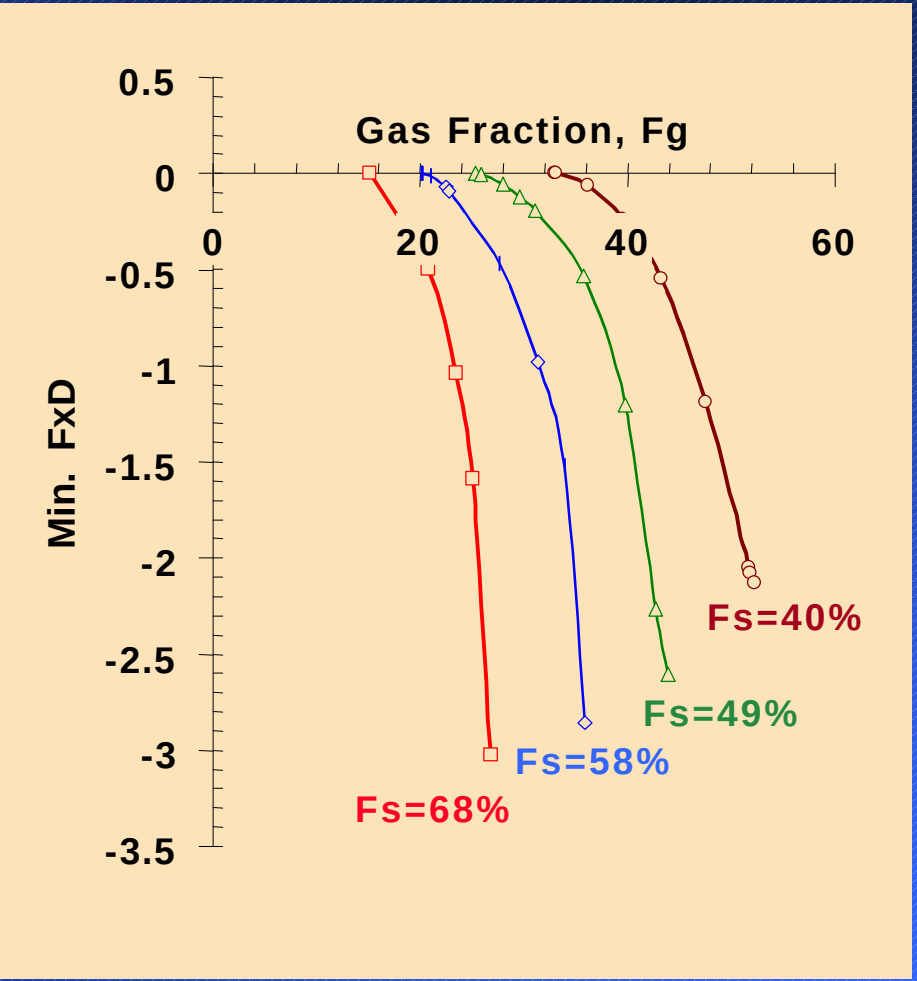
Example of Yield Stress Profile



Fs=49.76%

Yield Stress vs. Phase Fraction

- At given solid fraction, yield stress increases with gas fraction
- At given gas fraction, yield stress increases with solid fraction

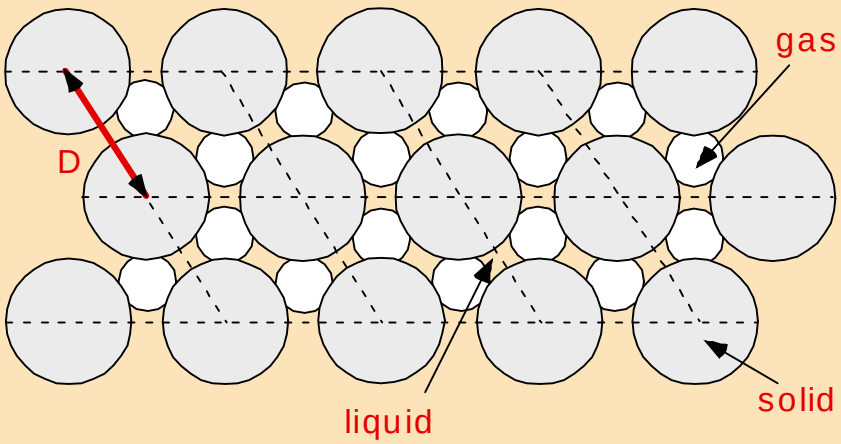


Conclusions

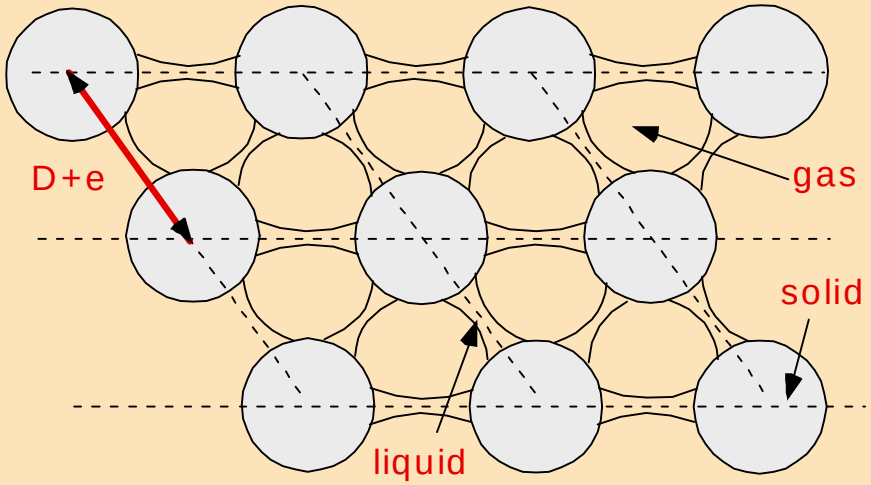
- Origin of the yield stress with gas/liquid/solids mixture has been investigated with 2D model. Forces resisting to the movement of three-phase mixture has been calculated by considering interfacial tension and capillary pressure only (no colloidal forces between solids).
- Maximum yield stress increases with gas fraction at given solid fractions between 39 to 68 %. There is no yield stress at very small gas fraction. Maximum yield stress increases with solid fraction at given gas fraction.

Compressibility of Three-phase Mixture

- Three phases are present in unit cell.
- Whole system expands/shrinks by fraction e.
- Surfactant stabilizes liquid films between bubbles.



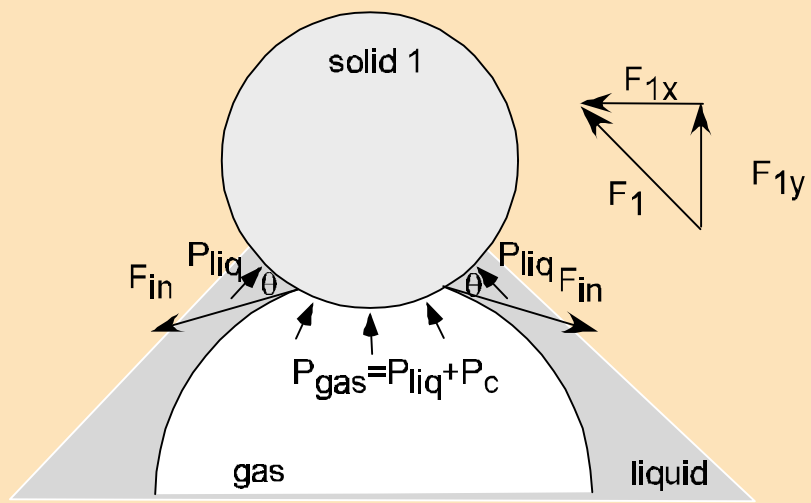
(a) Structure of foamy sands at rest (all phases are evenly distributed in the unit cell)



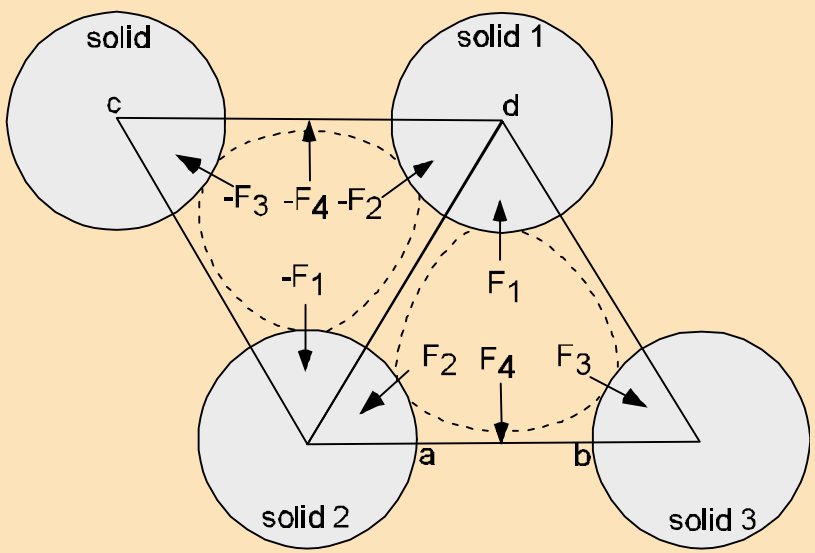
(b) Structure of foamy sands with elongation length e in diagonal direction; all phases are still distributed in the unit cell

Force Calculation

- $P = (F_{1y} - F_{2y} - F_{3y} - F_{4y})/(DL)$
  - $P \equiv$ external pressure applied to this system



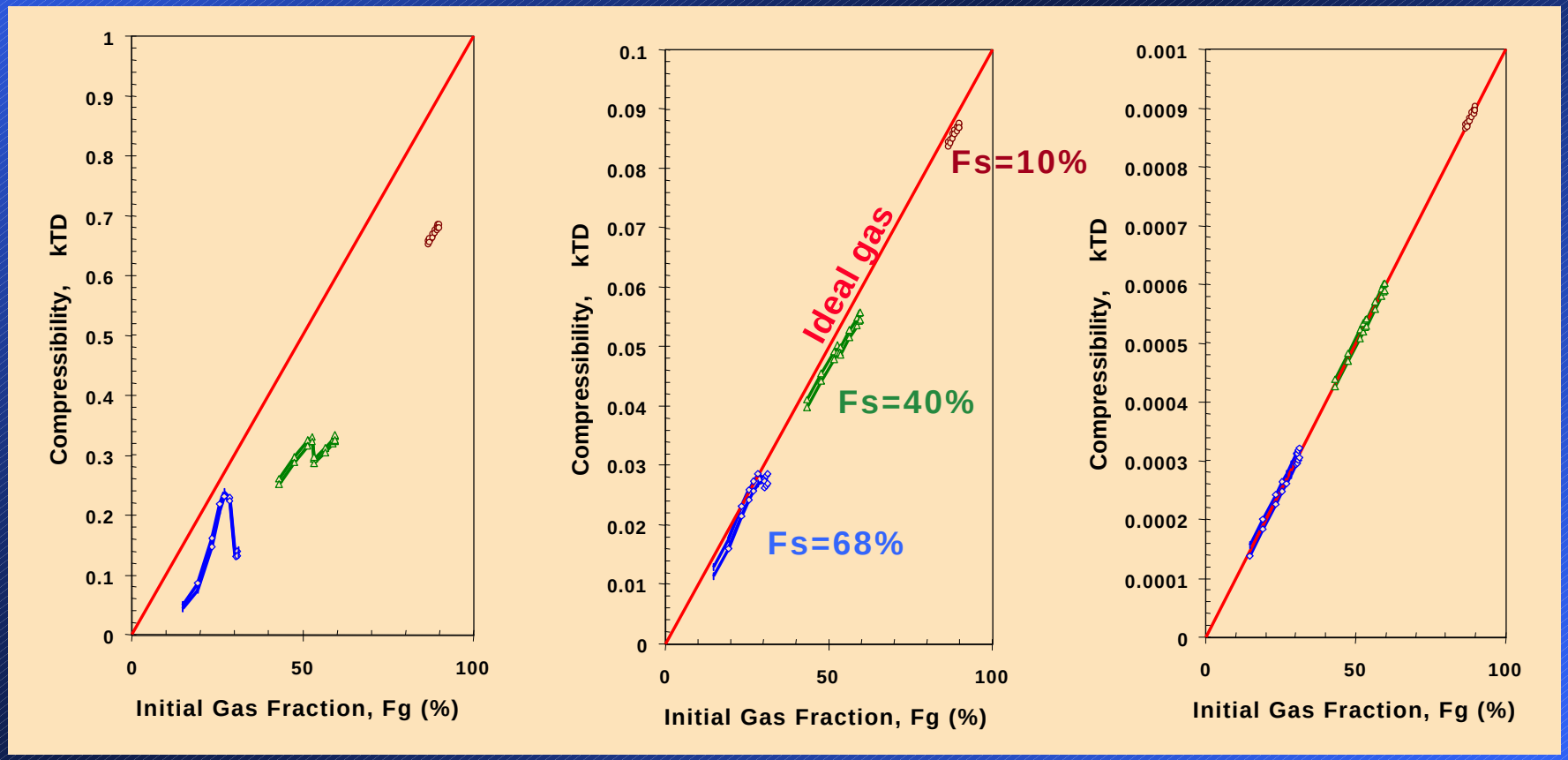
(a) Forces acting on solid 1



(b) Forces acting on the representative unit cell

Results

Smaller Capillary Effect $P_D[\equiv P/(\sigma/R)]$ =1, 10, and 1000 (all at θ=0)



Results (cont'd)

- Compressibility of three-phase mixture deviates more from that of given fraction of ideal gas as P_D decreases.
  - Small P_D represents small particle size
  - for example, P=1 atm, σ=30mN/m, and R=0.3 micron, then P_D=1; substantial capillary effects
- Effect of contact angle is not significant.
- There is discontinuity between two modes:
  - 2nd order phase transition

Conclusions

- Compressibility of gas/liquid/solids mixture has been calculated by 2D analysis. We assume that three-phases are symmetrically distributed such that triangular unit cell represents whole system.
- Compressibility of the mixture deviates from that of ideal gas mixed with solids and liquid. This deviation in compressibility gets insignificant as P_D increases.
- There exists discontinuity between two modes, similar to the second-order phase transition. Compressibility of the mixture is affected by the this mode, more pronounced when P_D is small.