Risk Based Groundwater Remediation Design Using a Tunneling Optimization Algorithm

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The Main Objective

- To develop a **Risk Based** least-cost pump-and-treat remediation design or redesign for contaminated aquifers.

Linkage to Site Needs

- To design or redesign risk-based remediation systems to reflect least cost
- To accommodate uncertainties in hydrogeological parameters
- To penalize, but not eliminate minimal risk designs, i.e designs that are less expensive, but have a small possibility of failure

The Approach

- Formulate and solve an optimization problem that has an objective function that minimizes the total remediation cost and also considers the uncertainty in the hydraulic-conductivity field.

Main Characteristics of the Optimization Problem

- The **constraints** are non-linear (risk-based concentration constraints).
- The **objective function** is non-convex with multiple local minima.

Methodology

- Use **Robust Optimization** theory. A multi-scenario approach (Mulvey, 1995).
- Introduce the **Tunneling Method** (Levy, 1984 and 1985), an efficient method for solving non-convex global optimization problems with multiple local minima.

Why a Multi-Scenario Approach?

- The scenarios chosen to represent the variance in the hydraulic conductivity field are randomly selected using a new method called '**Equal Area Selection**.'
- The multi-scenario optimization approach addresses the issue of **design risk** due to hydraulic conductivity uncertainty.

The Robust Optimization Formulation

Pumping cost + Penalty term

Objective: $$\min \sum_i c_i q_i + \omega \frac{1}{|S|} \sum_{S \in \Omega} \max(0, \xi_S)$$

Subject to:

$$g - \sum_i (h_i^1 - h_i^2) \le 0$$ **Gradient constraint**

$$c - c_j \le 0$$ **Maximum pumping constraint**

$$0 \le q_i \le \max q \; \forall S$$

$c_i \equiv$ cost associated with pumping one unit of water from well i.

$q_i \equiv$ pumping rate at well i.

$\omega \equiv$ scaling term for the penalty term.

$S \equiv$ the number of scenarios.

$\frac{1}{|S|} \equiv$ the individual weighting term on each scenario.

$g \equiv$ lower bound on gradient necessary for capture.

$h^1 - h^2 \equiv$ resultant head gradient.

$\xi_S = g - (h^1 - h^2) \equiv$ the penalty.

The Groundwater Model

The flow equation:

$$S(\mathbf{x})\frac{\partial h(\mathbf{x},\mathbf{t})}{\partial t} - \nabla \cdot \mathbf{K}(\mathbf{x})\, b(\mathbf{x}) \cdot \nabla h(\mathbf{x},t) + q_i \delta(\mathbf{x} - \mathbf{x}_i) = 0$$

$S(\mathbf{x}) \equiv$ storage coefficient.

$h(\mathbf{x},t) \equiv$ hydraulic head.

$\mathbf{K}(\mathbf{x}) \equiv$ hydraulic conductivity of the aquifer.

$b(\mathbf{x}) \equiv$ aquifer thickness

$q_i \equiv$ discharge from well i.

$\delta(\mathbf{x} - \mathbf{x}_i) \equiv$ Dirac delta function that defines the location of the well.

The Groundwater Model

The transport equation:

$$\frac{\partial c(\mathbf{x},t)}{\partial t} + \mathbf{v}(\mathbf{x},t) \cdot \nabla c(\mathbf{x},t) - \nabla \cdot \mathbf{D} \cdot \nabla c(\mathbf{x},t) = 0$$

$c(\mathbf{x}, t) \equiv$ concentration

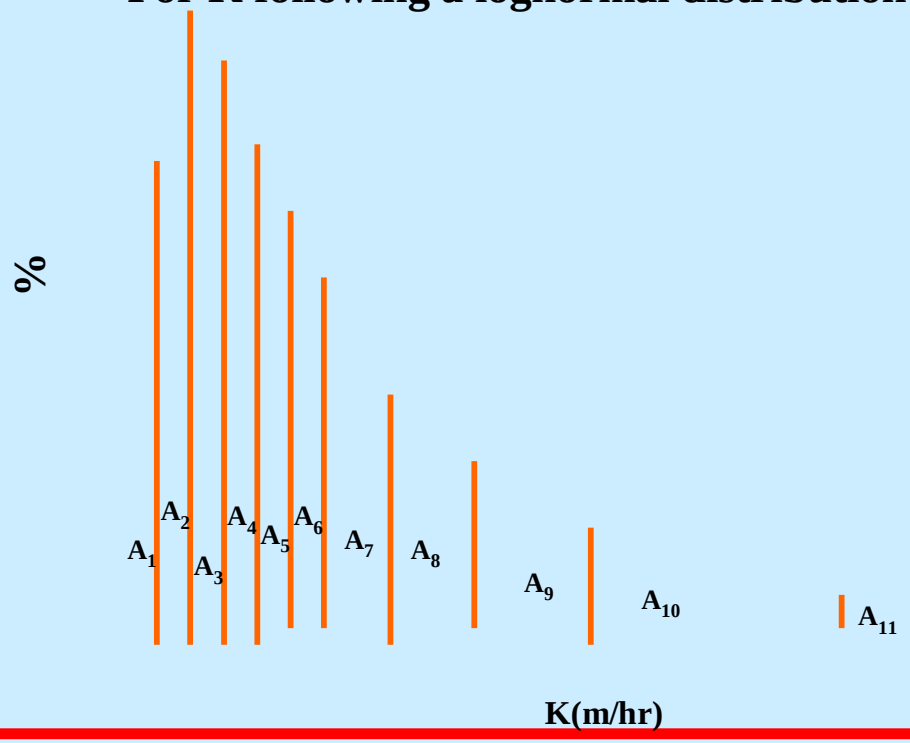
$\mathbf{D}(\mathbf{x}, t) \equiv$ Dispersion coefficient

$\mathbf{v}(\mathbf{x}, t) \equiv$ Darcy velocity

Darcy's Law $\mathbf{v}(\mathbf{x},t) = -\mathbf{K}(\mathbf{x}) \cdot \nabla h(\mathbf{x},t)$

Equal Area Sampling: Choose K values by examining equal area partitioning of the distribution function.

For K following a lognormal distribution: $A_i = A_j$



$$\frac{i}{n+1} = \int_0^{y_i} f(y)dy$$

10 realizations K (m/hr)

$K_1 = 0.0026$	$K_6 = 0.0112$
$K_2 = 0.0040$	$K_7 = 0.0142$
$K_3 = 0.0055$	$K_8 = 0.0183$
$K_4 = 0.0071$	$K_9 = 0.0248$
$K_5 = 0.0089$	$K_{10} = 0.0380$

Lognormal Distribution Formula

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma_x y} \exp\left(\frac{-(\ln y - \mu_x)^2}{2\sigma_x^2} \right)$$

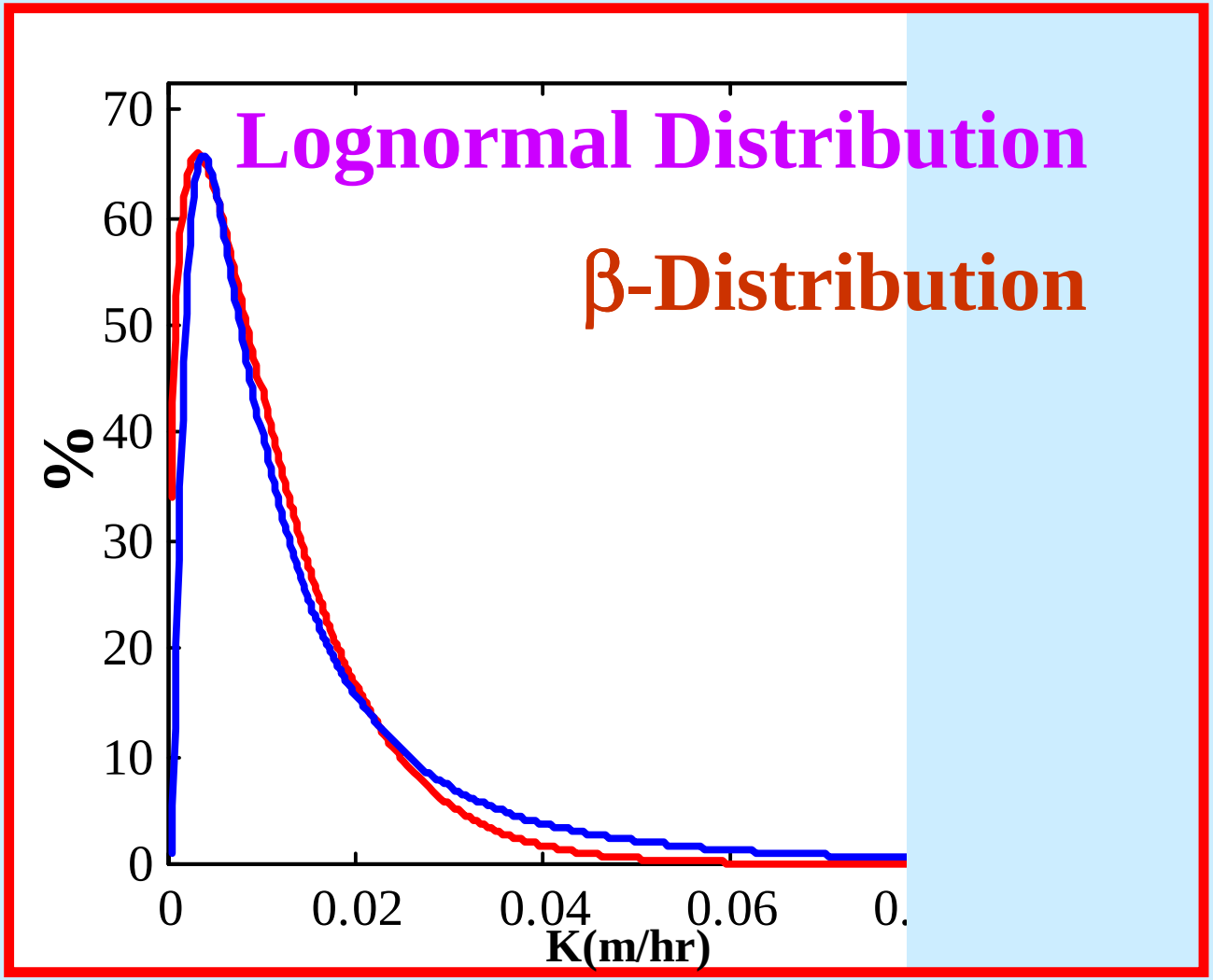
$\sigma_x \equiv$ mean in the associated normal distribution.

$\mu_x \equiv$ standard deviation in the assoc. norm. distribution.

The equal area calculation gives:

$$y_i = \exp\left(\sqrt{2}\sigma_x inverf\left(\frac{2i}{n+1} - 1 \right) + \mu_x \right)$$

β-Distribution vs.Lognormal Distribution



The Hydraulic Conductivity Distribution - A Beta Distribution

$$P(y) = \frac{1}{B(p,q)} \frac{(y-a)^{p-1}(b-y)^{q-1}}{(b-a)^{p+q-1}}$$

B(p,q) = the beta function

a, b = lower and upper bounds of the range of possible hydraulic conductivities.

p, q = parameters that describe the shape of the beta function.

The Tunneling Method

- The Tunneling method consists of two phases:

1. The minimization phase - A **gradient method** to determine a local minimum of the objective function.
2. The tunneling phase - the objective function is transformed into a "**tunneling function**" and a new starting point for Phase 1 is determined.

The Tunneling Function

$$T(\mathbf{q}) = \frac{f(\mathbf{q}) - f^*}{[(\mathbf{q} - \mathbf{q}^*)'[(\mathbf{q} - \mathbf{q}^*)]^n}$$

f(q) = original objective function

f* = the local minimum determined in the minimization phase

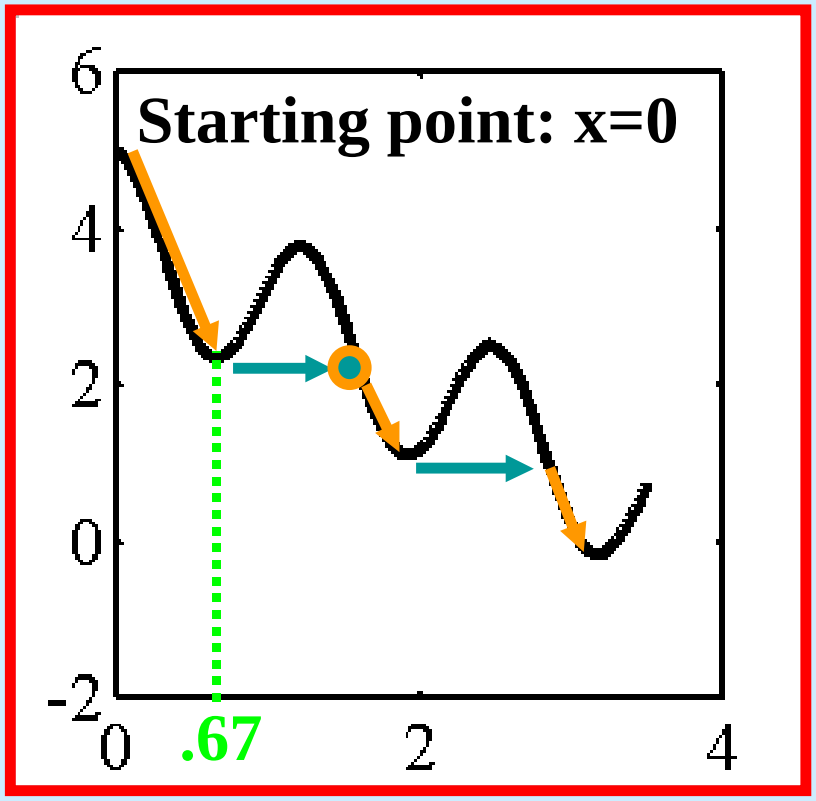
q* = the pumping rate for the local minimum determined in the minimization phase

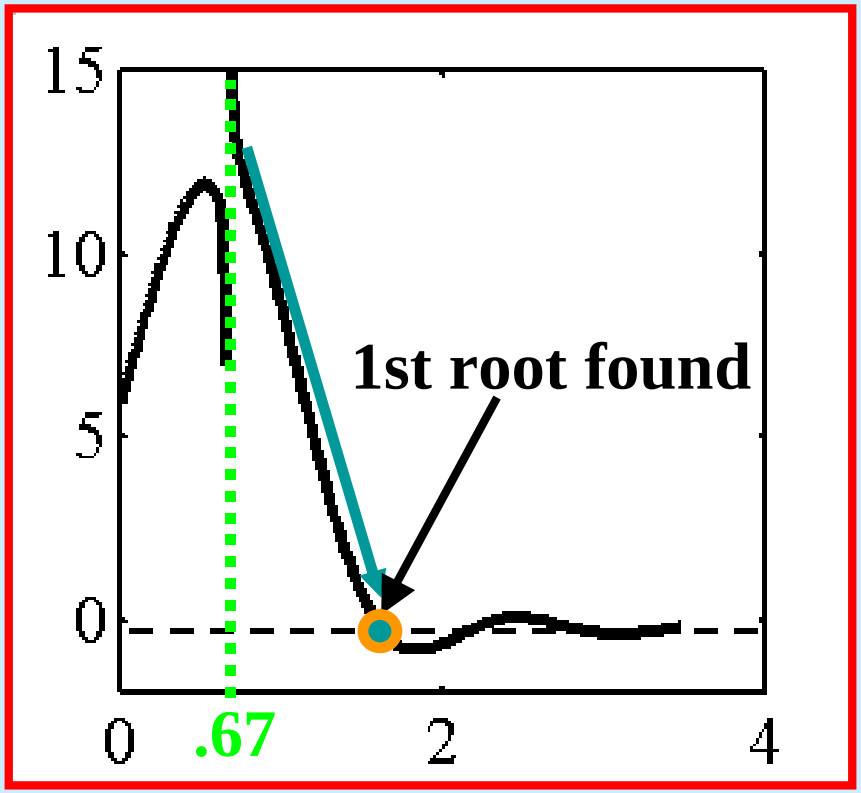
n = the pole strength

Tunneling Function Illustrated

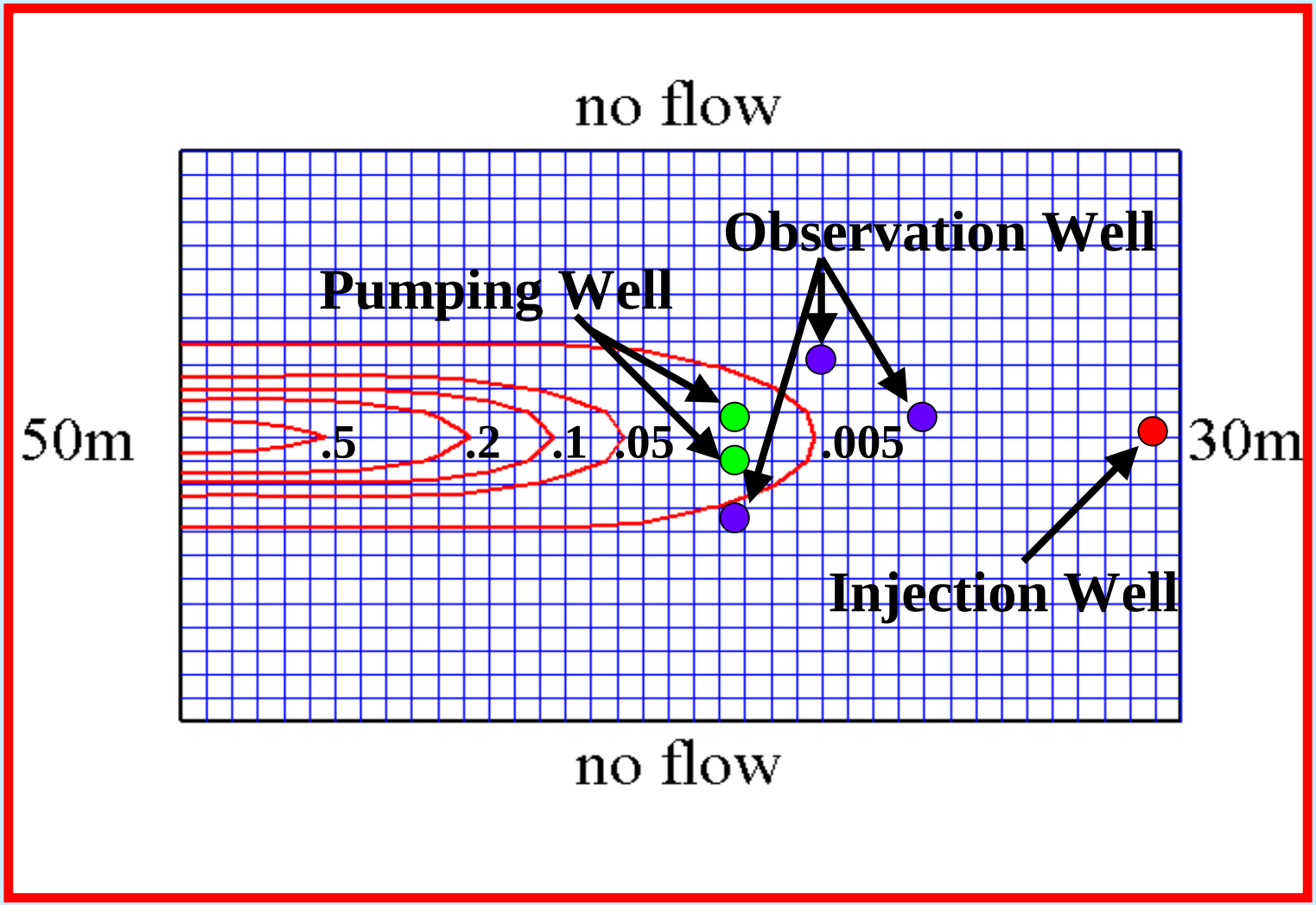
$$f(x) = -x + \cos(5x) + 4$$

$$T(x) = \frac{f(x) - f(.67)}{(x - .67)^2}$$

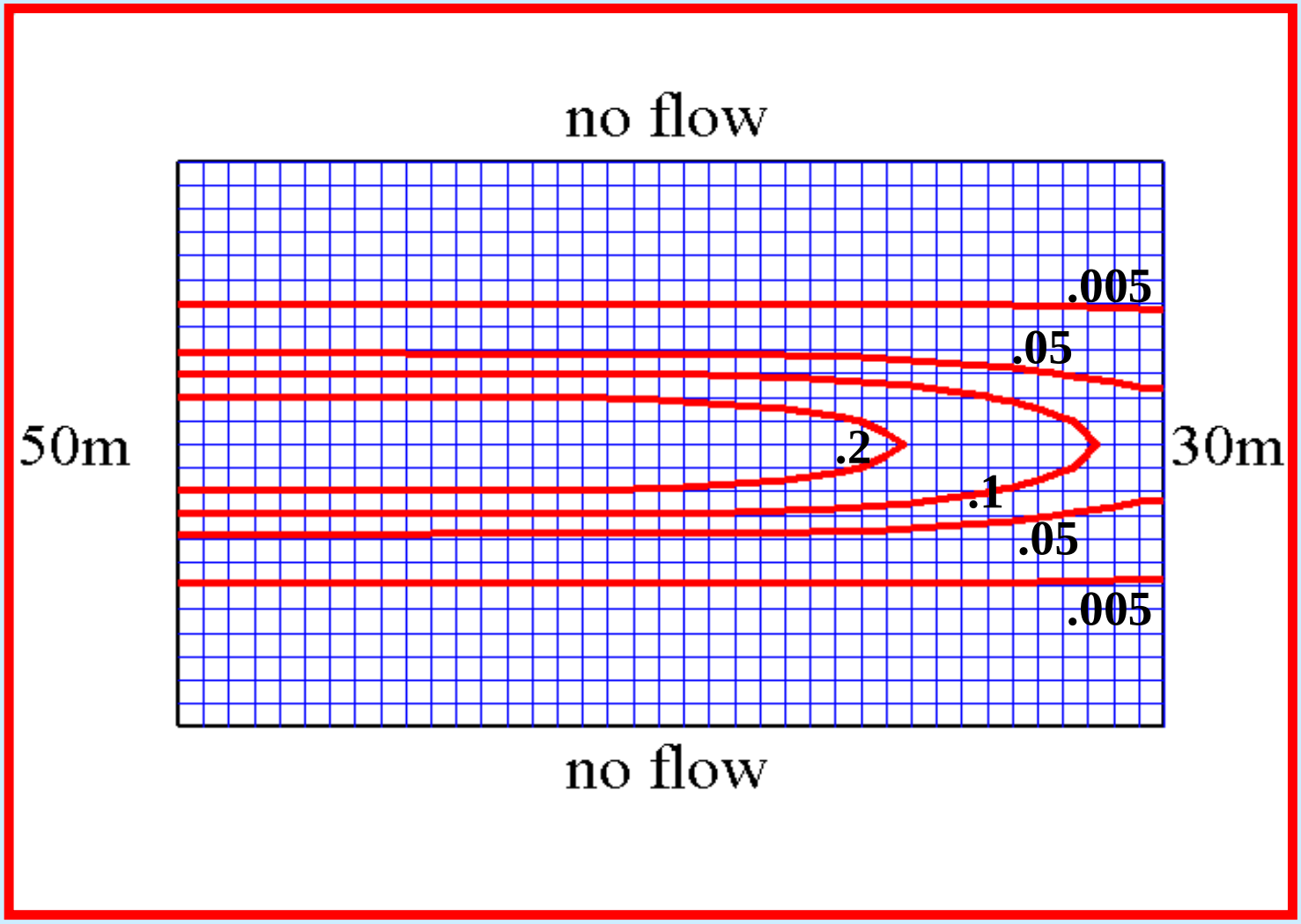




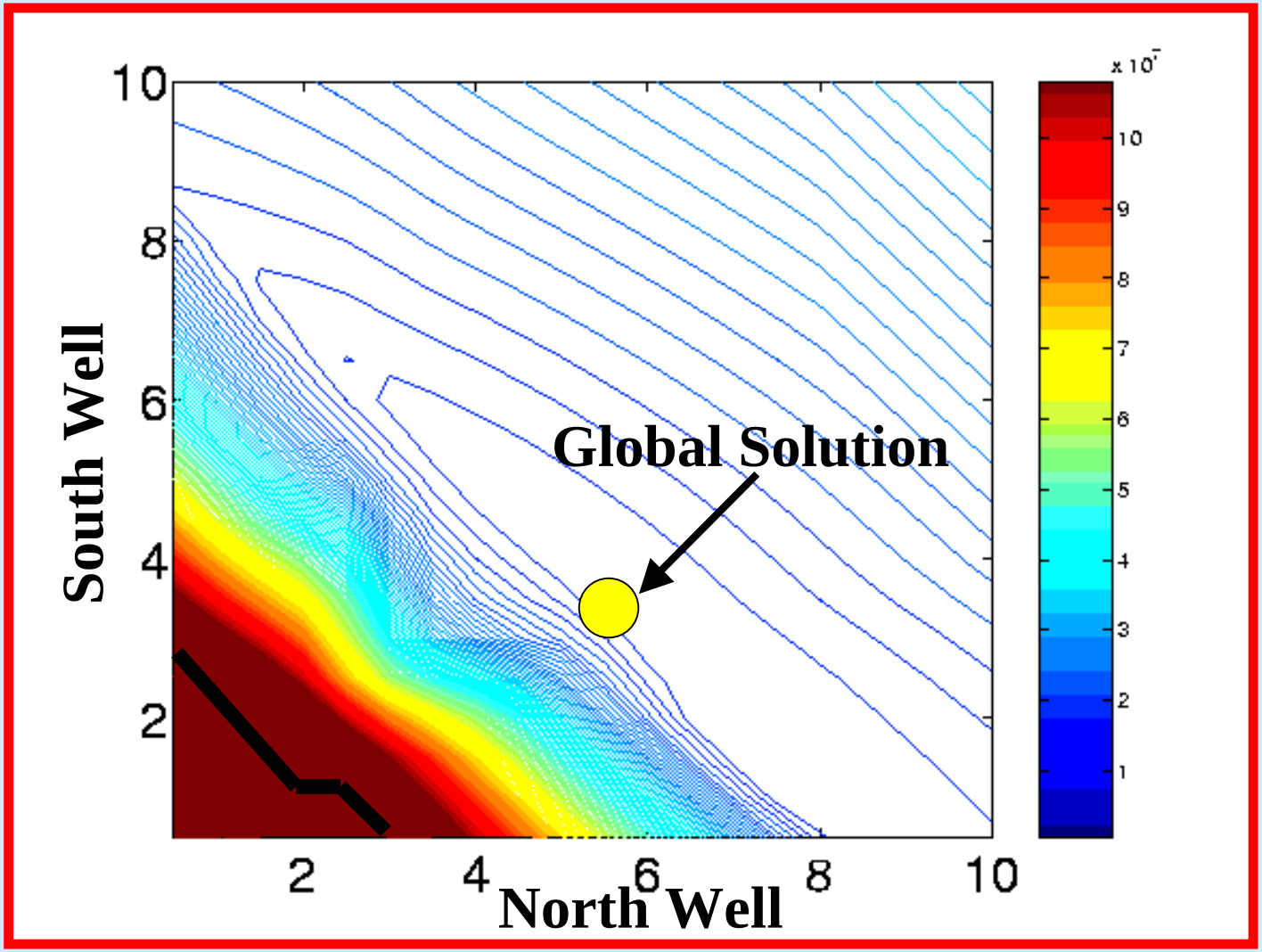
Hypothetical Example Initial State (I)



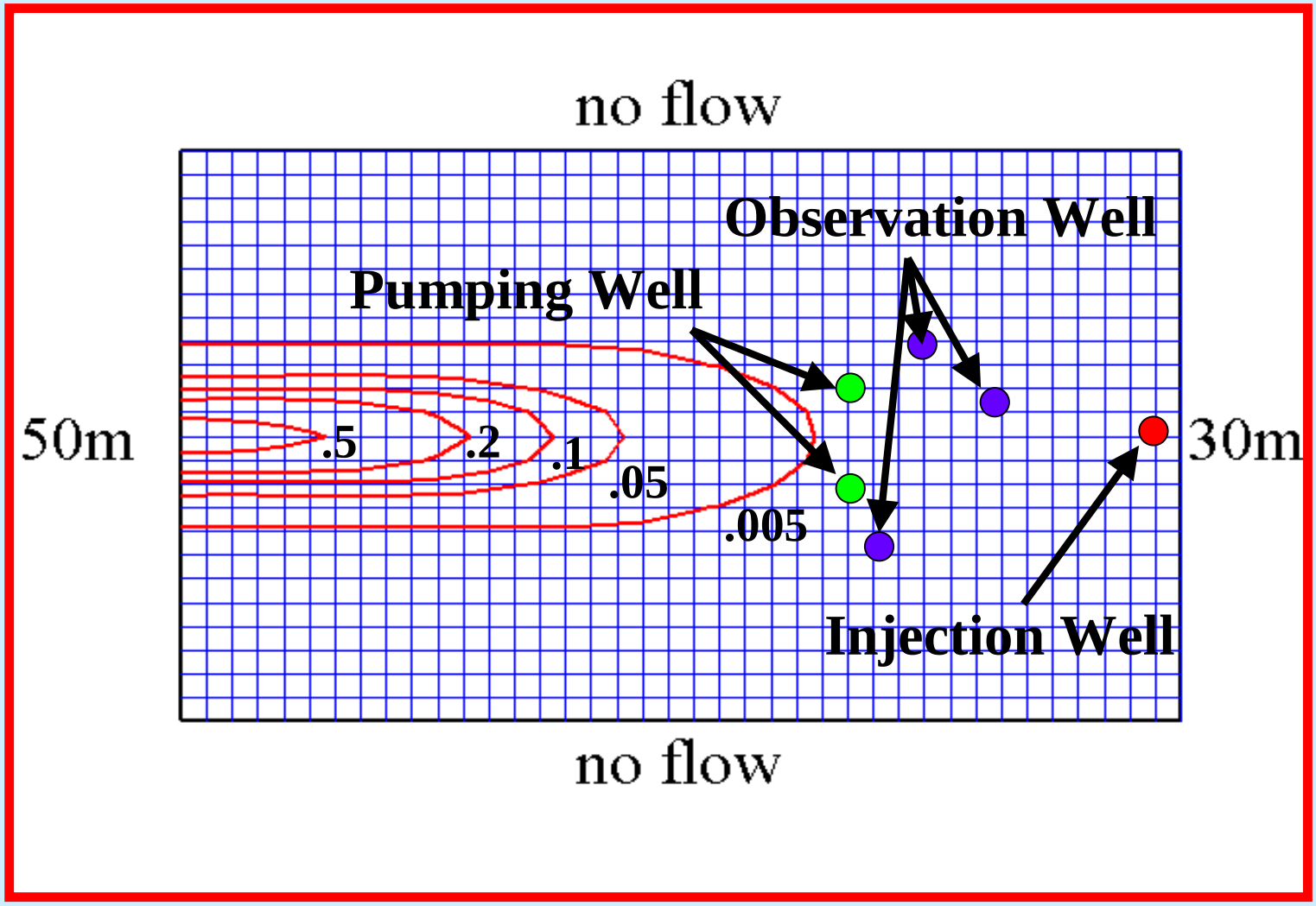
Final State: No Remediation



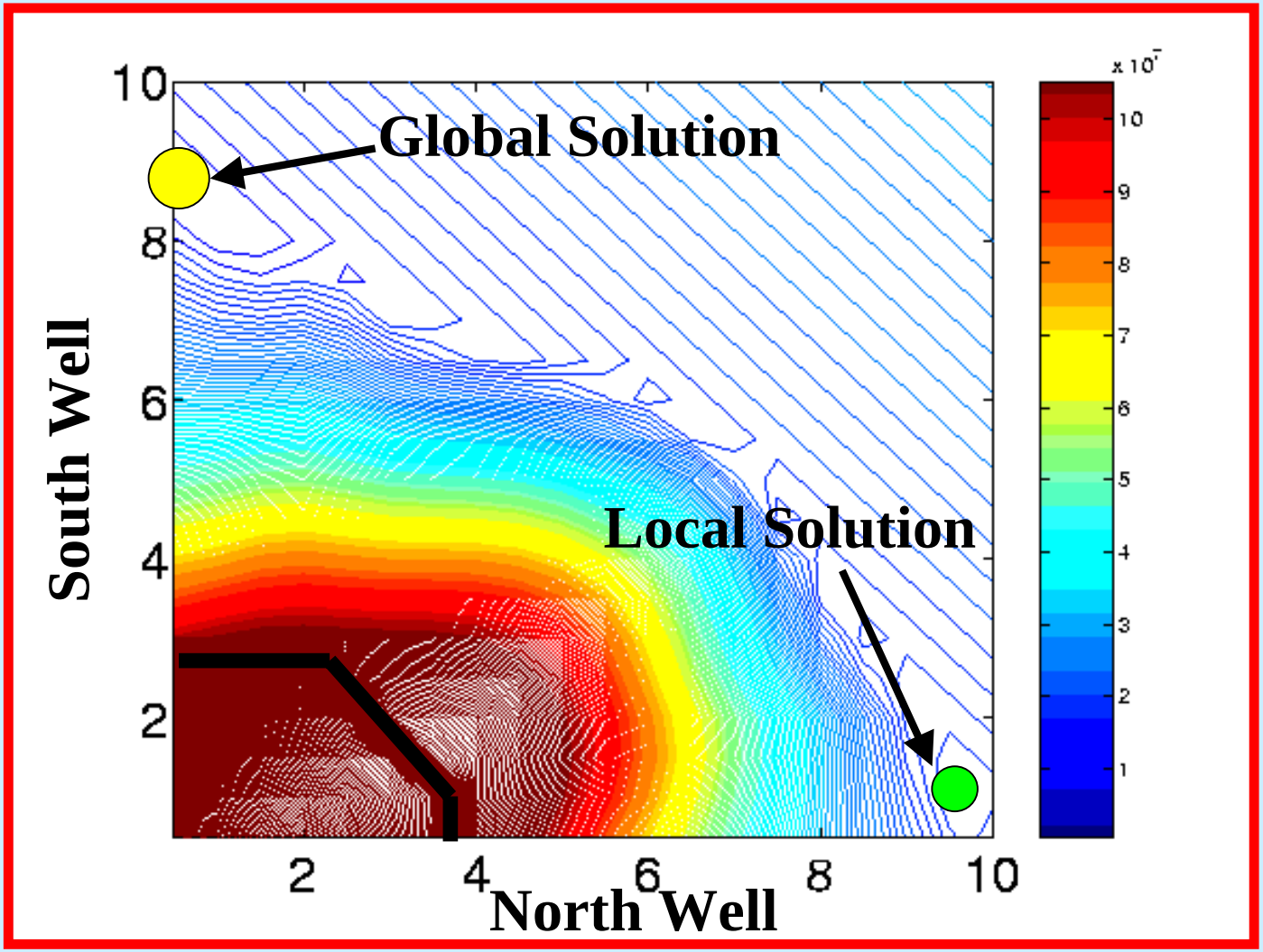
Interior Global Minimum (I)



Hypothetical Example Initial State (II)



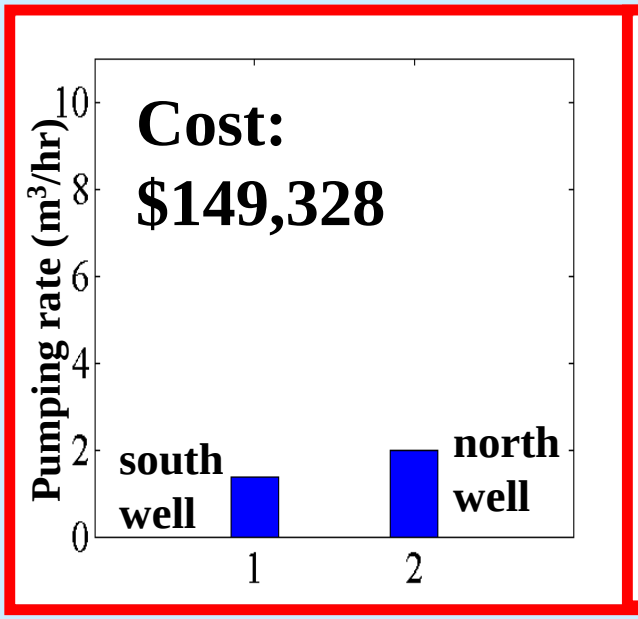
Two Local Minima (II)



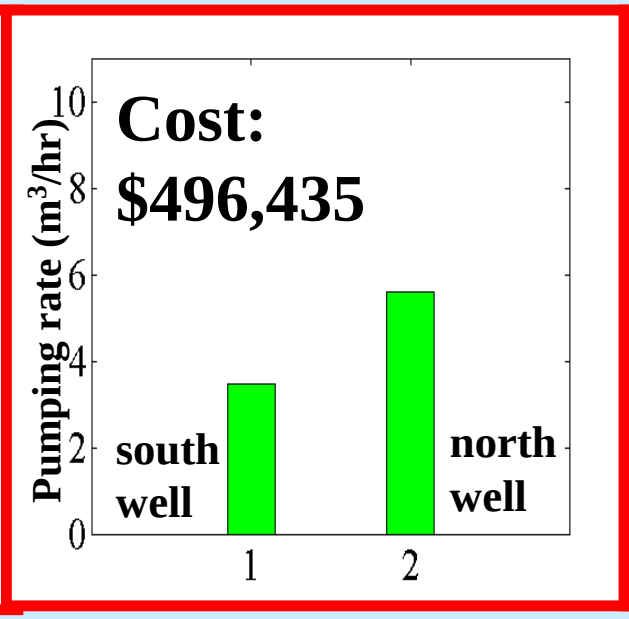
Results

Case I (Interior Global Minimum)

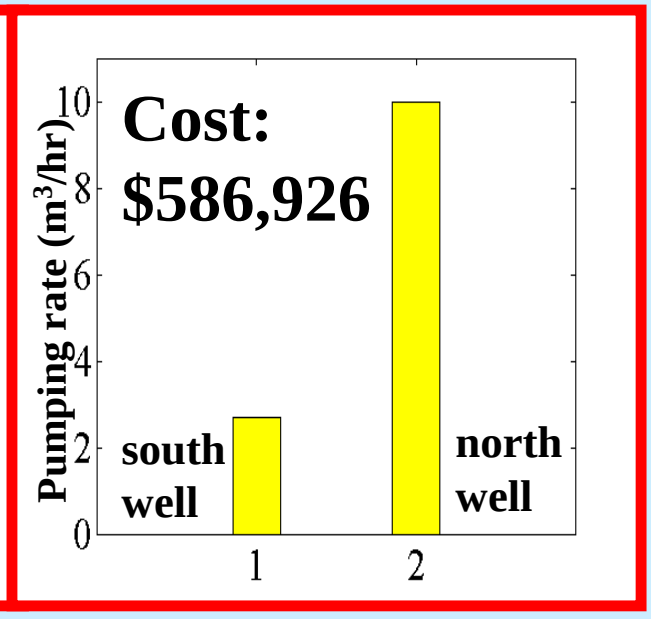
Weight = 0



Weight = 100 mil.



Weight = 500 mil.



Weight indicates measure of risk aversion to system failure

Conclusions

- Risk-based least-cost design can be formulated using a Robust-Optimization approach.
- The Tunneling Method is a viable approach for determining the global optimum for a Robust-Optimization problem where the objective function is non-convex and has multiple local minima.