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Presented at the 1994 Applied Superconductivity Conference,  
Boston, MA, October 16-19, 1994, and to be published  
in the IEEE Transactions on Applied Superconductivity

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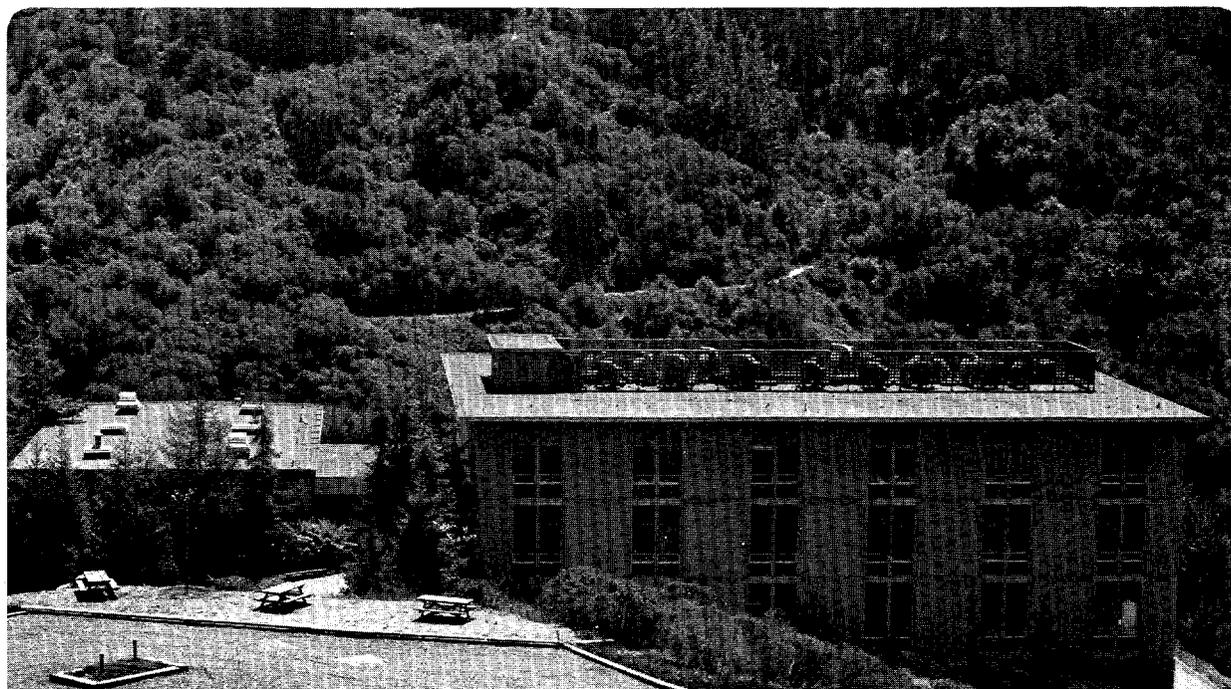
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# Quantum Effects in the Hot Electron Microbolometer

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**Abstract**—The theory of the hot electron microbolometer proposed by Nahum et al. assumed that the photon energy is thermalized in the electrons in the Cu absorber before relaxing to the lattice. Since the photons initially excite individual electrons to  $\hbar\omega \gg k_B T$ , however, direct relaxation of these hot electrons to phonons must also be considered. Theoretical estimates suggest that this extra relaxation channel increases the effective thermal conductance for  $\hbar\omega \gg k_B T$  and influences bolometer noise. Calculations of these effects are presented which predict very useful performance both for ground-based and space-based astronomical photometry at millimeter and submillimeter wavelengths.

## I. INTRODUCTION

Bolometric detectors are chosen for astronomical photometry at millimeter and submillimeter wavelengths to achieve high sensitivity. Although optically coupled (composite) bolometers are widely used in practice,<sup>1</sup> it has long been recognized that antenna-coupled microbolometers have potential advantages in heat capacity and optical efficiency. Such bolometers, like the SIS mixer, could use lithographed filters and be coupled to planar antennas or to waveguides and horns.<sup>2,3</sup>

An antenna coupled hot electron microbolometer was proposed by Nahum, Mears and Richards.<sup>4</sup> A diagram of this device is shown in Fig. 1. A thin film of copper with micron dimensions is connected between the antenna terminals. This resistive load is assumed to thermalize the infrared currents. The increase in the electron temperature in the load is measured from the temperature dependence of the current-voltage (I-V) characteristic of an aluminum superconductor-insulator-normal metal (SIN) tunnel

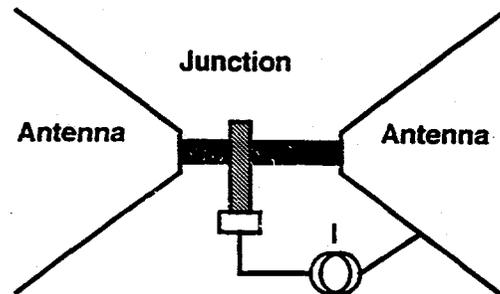


Fig. 1. Schematic of the radiation coupling and temperature readout configuration. The rf current from the superconducting antenna is dissipated in the resistive copper strip (shown in black), the resulting temperature rise of the electrons in the strip are measured as a change in the voltage across the junction which is biased at a constant current  $I$ . The contact to the superconducting electrode is made of a superconductor whose  $T_c$  is much higher than that of the electrode.

junction, which is deposited on the copper strip. The analysis originally presented<sup>4</sup> assumed that the energy of the absorbed photons raises the temperature of the free electrons in the Cu. These hot electrons are trapped by Andreev reflection at the superconducting terminals of the antenna. This energy subsequently relaxes to phonons in the Cu which escape into the substrate. The value of the effective thermal conductance  $G$  computed from the electron phonon coupling in Cu was appropriate for experiments with very low background power  $P_0 \leq 10^{-13}$  W. This model predicts that applications with higher backgrounds would require a much larger volume of Cu.

In this paper we pursue the device consequences of the quantum viewpoint suggested to us by H. Kinder that the energy of the photons is initially absorbed by individual electrons whose energy increases by  $\hbar\omega$ . This energy increase is much larger than  $k_B T$  at millimeter and submillimeter wavelengths for detectors operated with  $T \leq 300$  mK. Theoretical relaxation rates predict that direct relaxation of the energy of these excited electrons to phonons can be faster than relaxation to phonons by way of a hot electron bath. This extra relaxation channel is illustrated in Fig. 2. Because of the energy dependence, the experiments of Nahum and

Manuscript received October 15, 1994.

This work was supported in part by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U. S. Department of Energy under Contract No. DE-AC03-76SF00098.

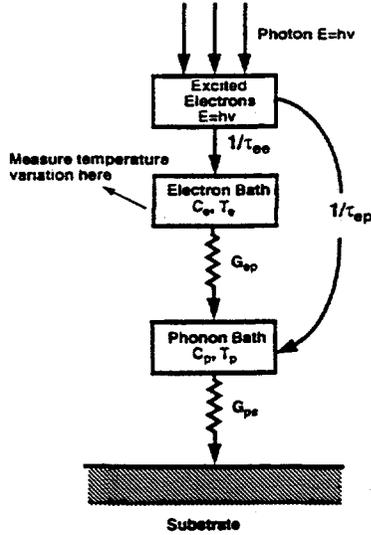


Fig. 2 Energy relaxation channels for the hot electron microbolometer. Individual electrons excited to energy  $\hbar\omega$  can relax to phonons by way of hot electron bath as assumed by Nahum et al., or directly by the emission of a phonon.

Martinis,<sup>5</sup> which used dc excitation, did not probe this important aspect of the device performance at high frequencies.

Section II of this paper gives detailed estimates for electron-electron and electron-phonon relaxation rates and sets up the rate equations that govern the operation of the bolometer. Section III describes the parameters which govern the geometry of the Cu absorber. Section IV describes the optimization of the SIN thermometer and calculates the bolometer noise. Finally, Section V summarizes the properties of the hot electron microbolometer as they are now understood.

## II. RELAXATION FROM QUANTUM EXCITATION

Electrons excited by photons to energies  $\hbar\omega \gg k_B T$  relax through electron-phonon (e-p) interactions, electron-electron (e-e) interactions, and probably through surface relaxation mechanisms.

### A. Electron-Phonon Interaction

In thermal equilibrium, the (e-p) interaction is relatively well understood and is in reasonable agreement with experiments of Wellstood et al.<sup>6</sup> on a thin Cu film at millikelvin temperatures. This agreement was obtained by assuming that surface relaxation was negligible. We will therefore neglect surface relaxation in the following. Under certain simplifying assumptions, the electrons radiate energy to phonons at a rate,

$$P = \Sigma V (T_e^5 - T_p^5), \quad (1)$$

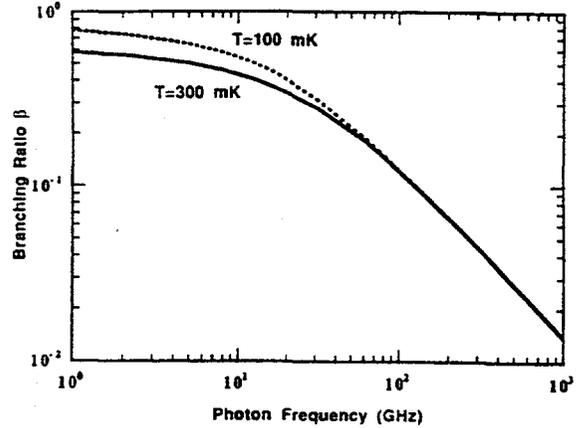


Fig. 3 The theoretical branching ratio, which gives the fraction of the absorbed power  $P_0$  which enters the electron bath, as a function of photon frequency for  $T=100$  and  $300$  mK. The original work of Nahum et al. assumes this ratio to be unity.

where  $\Sigma = \ln W \mu m^{-3} K^{-5}$ ,  $V$  is the volume of the metal, and  $T_p$  is the phonon temperature. The thermal conductance between the electron bath and phonon bath is then  $G_{ep} = \Sigma V T^4$ . From this, we can also estimate the inelastic scattering rate  $\tau_p = C_e / G_{ep}$ , where  $C_e$  is the heat capacity of the electron bath. The calculated value is  $\tau_p = 10.6/T^3$  ns. When the energy of an individual excited electron is much larger than the thermal energy of the other electrons, we must consider the dependence on energy. Allen<sup>7</sup> gives a general expression for the rate of energy relaxation for an electron of energy  $\hbar\omega$  which assumes that the Fermi sea is at the same temperature  $T$  as the phonon bath,

$$1/\tau_{ep}(\omega, T) = 2\pi\lambda^* \omega_D (T/\theta_D)^3 [4\zeta(3) + p(y)], \quad (2)$$

where

$$p(y) = y^{-1} \int_0^\infty dx x^2 [(x-y)(e^{x-y} - 1)^{-1} - (x+y)(e^{x+y} - 1)^{-1}], \quad (3)$$

and  $y = \hbar\omega / k_B T$ . Equation (2) can not be integrated in terms of elementary functions, but the behavior can be found for large or small  $y$ ,

$$1/\tau_{ep}(\omega, T) = [\pi\lambda^* \omega_D / 6(\hbar\omega_D)^3] \cdot [(\hbar\omega_D)^3 + (2\pi k_B T)^2 (\hbar\omega) + 48\zeta(3)(k_B T)^3 + \dots], \quad (4)$$

where  $k_B T \ll \hbar\omega \ll \hbar\omega_D$ , and

$$1/\tau_{ep}(\omega, T) = [24\pi\lambda^* \omega_D / (\hbar\omega_D)^3] \cdot [\zeta(3)(k_B T)^3 + k_B T (\hbar\omega)^2 / 18 + \dots], \quad (5)$$

Table. 1. Design parameters for bolometers used in three infrared astronomical experiments listed as Case 1, 2 and 3. All experiments have 25% bandwidth and optical efficiency of 0.2. A transformer coupled amplifier is assumed.

Case	1	2	3
$\omega/2\pi(\text{GHz})$	80-100	200-260	750-970
Background $\epsilon T$ (K)	3.0	30	225
Absorbed power $P_0$	$2.1 \times 10^{-13}$	$5.0 \times 10^{-12}$	$1.4 \times 10^{-10}$
Branching ratio $\beta$	$1.35 \times 10^{-1}$	$5.79 \times 10^{-2}$	$1.62 \times 10^{-2}$
Dimension of Cu absorber ( $\mu\text{m}^3$ )	$0.3 \times 4 \times 0.040$	$1 \times 13 \times 0.040$	$4.5 \times 60 \times 0.040$
$T_p - T_s$ (mK)	12	25	33
$T_e - T_s$ (mK)	23	34	36
$G_{\text{eff}} (1/G_{ps} + \beta/G_{ep})^{-1} (\text{W/K})$	$7.5 \times 10^{-12}$	$1.2 \times 10^{-10}$	$3.1 \times 10^{-9}$
Responsivity $S$ (V/W)	$7.0 \times 10^7$	$4.4 \times 10^6$	$1.7 \times 10^5$
Contributions to the square of the noise equivalent absorbed power			
Photon noise ( $\text{W}^2/\text{Hz}$ )	$2.5 \times 10^{-35}$	$1.5 \times 10^{-33}$	$1.6 \times 10^{-31}$
Phonon noise $4kT_p^2 G_{ps}$ ( $\text{W}^2/\text{Hz}$ )	$1.8 \times 10^{-35}$	$1.0 \times 10^{-33}$	$2.2 \times 10^{-32}$
SIN junction noise $V_j^2/S^2$ ( $\text{W}^2/\text{Hz}$ )	$1.2 \times 10^{-37}$	$3.0 \times 10^{-35}$	$2.0 \times 10^{-32}$
Amplifier noise $V_\lambda^2/S^2$ ( $\text{W}^2/\text{Hz}$ )	$2.0 \times 10^{-36}$	$5.2 \times 10^{-34}$	$3.5 \times 10^{-31}$

where  $\hbar\omega \ll k_B T \ll k_B \theta_D$ . These two expressions agree with experiments by Goy and Castaing.<sup>8,9</sup>

When  $\gamma \sim 1$ , we find from (3) and (4) that  $1/\tau_{ep}$  is proportional to  $T^3$ , which agrees with the result derived by Wellstood et al<sup>6</sup> assuming thermal equilibrium.

### B. Electron-Electron Interaction

At infrared frequencies, Lawrence et al.<sup>10,11</sup> have made a detailed calculation of the relaxation time using the Born approximation and Thomas-Fermi screening of the Coulomb interactions,

$$1/\tau_{ee}(\omega, T_e) = (\pi^3 \Gamma \Delta / 12 \hbar E_F) \{ (k_B T_e)^2 + (\hbar\omega / 2\pi)^2 \}. \quad (6)$$

Here  $\Gamma (=0.57$  for Cu) is a constant giving the average over the Fermi surface of the scattering probability,  $\Delta (=0.79$  for Cu) is the fractional umklapp scattering, and  $E_F$  is the free-electron Fermi energy. This formula agrees very well with the experiments by Beach and Christy.<sup>12</sup> For Cu,  $\lambda^* \cong 0.12 \pm 0.02$  and  $\hbar\omega = k_B T_D = 343 k_B$ .

These theoretical expressions predict that for frequencies above 6 GHz the electron-phonon relaxation rate  $1/\tau_{ep}$  is larger than the electron-electron relaxation rate  $1/\tau_{ee}$  for typical temperature of  $T_e = T_p$  of 100 to 300 mK.

### C. Bolometer Thermal Model

From the above discussion, we can see that the (e-e) interaction is not fast enough to thermalize the

electrons when  $\hbar\omega \gg k_B T$ . An improved thermal model is shown in Fig. 2. The rate equations for the electron and phonon baths are,

$$C_p (dT_p / dt) = P_0 \tau_{ee} / (\tau_{ep} + \tau_{ee}) + \sum V (T_e^5 - T_p^5) - \sigma A (T_p^4 - T_s^4), \quad (7)$$

$$C_e (dT_e / dt) = P_0 \tau_{ep} / (\tau_{ep} + \tau_{ee}) + \sum V (T_p^5 - T_e^5). \quad (8)$$

Here  $P_0$  is the input signal power,  $\sigma A (T_p^4 - T_s^4)$  is the energy flow between the phonon bath and the substrate, and  $A$  is the contact area between the sample and the substrate. The power which enters the electron bath is reduced by the factor  $\beta = \tau_{ep} / (\tau_{ep} + \tau_{ee})$ , which is interpreted as a branching ratio. We plot this ratio as a function of  $\omega$  for two values of electron temperature in Fig. 3. Since it is  $\sim 10^{-1}$  at 100 GHz and nearly  $10^{-2}$  at 1 THz, deviations from the thermal equilibrium theory are large. Solving Eqs. (7) and (8) in the small signal approximation, we obtain

$$T_e - T_s = P_0 (1/G_{ps} + \beta/G_{ep}) = P_0 / G_{\text{eff}}. \quad (9)$$

Here we define an effective thermal conductance of the device  $G_{\text{eff}} = (1/G_{ps} + \beta/G_{ep})^{-1}$ .

## III. DESIGN OF THE COPPER ABSORBER

We consider the design of an antenna-coupled microbolometer operated with  $T_S = 300$  mK to be used in two very different measurements. In the Case 1 the detector is operated in a 25% bandwidth at 90 GHz for an experiment to measure the anisotropy of the cosmic microwave background. The background temperature is  $T_b = 3$  K with unit emissivity  $\epsilon = 1$  and the detector optical efficiency is assumed to be 0.2. The parameters shown in Table 1 are appropriate for the balloon-borne Millimeterwave Anisotropy experiment (MAX)<sup>13</sup> but are similar to what would be required on the proposed bolometric space missions SAMBA and FIRE. The Cases 2 and 3 are experiments to explore point sources from a large ground-based telescope in 25% bandwidths centered on the atmospheric windows at 230 and 860 GHz. The emissivity temperature products for the sky are assumed to be  $\epsilon T_b = 30$  and 225 K respectively. The values of absorbed background power  $P_0$  shown in Table 1 for these applications are estimated from the single mode Planck formula, which approaches  $\epsilon \eta k_B T_b \omega / 2\pi$  for  $\hbar\omega \ll k_B T$ . These values vary by nearly  $10^3$ . Because the branching ratio decreases with frequency, however, the power entering the electron bath varies only by a factor  $10^2$ .

When the value of background power is stable and known in advance, the sensitivity of a bolometer can be enhanced by choosing parameters which permit significant temperature rise. Practical considerations, however, often limit the useful temperature use  $T_e - T_s$  to  $\sim 10\%$ . One contribution to this temperature rise comes from the thermal boundary resistance. It varies inversely with the area of the Cu-substrate interface. If we limit this contribution to 10%, the area  $A$  can be written

$$A > 8.6BP_0 / T_s^4, \quad (10)$$

where the parameter  $B$  depends on the densities and sound velocities of the materials and is usually about  $20K^4 \text{cm}^2/\text{W}$ . The other contribution arises from the electron-phonon coupling. It varies inversely with the volume of Cu. If we limit this contribution to 10%, the volume  $V$  can be written

$$V > 1.63\beta P_0 / \Sigma T_p^5. \quad (11)$$

All quantities in Eq. (10) and (11) are in SI units. The thickness of the Cu strip is then  $h = V/A$ .

Two additional conditions constrain the dimensions of the Cu strip shown in Table 1. The RF resistance of the strip must be convenient for matching to a waveguide or a lithographed antenna. Also, the length must be less than the diffusion length of the hot electrons so that they can reach the SIN junction before they relax into phonons. The one dimensional diffusion length of the quasiparticles in the Cu film strip can be estimated from

$$L = \sqrt{v_F \tau_p \lambda}. \quad (12)$$

Here  $v_F$  is the Fermi velocity of copper,  $\tau_p$  is the inelastic scattering rate described in section II,  $\lambda$  is the mean free path of the electrons in the copper film at 0.3 K, which can be calculated from the residual resistance ratio. We assume this ratio to be of order unity for our Cu film and get  $\lambda \approx 300$  Å. Then  $L = 2 \times 10^{-3} T^{-3/2} = 100$  μm.

The dimensions chosen for the Cu strips in Table 1 reveal a natural trend. For low frequencies and low backgrounds it is possible to choose parameters such that  $T_e - T_p \gg T_p - T_s$ . At high frequencies and high backgrounds, however, the natural choice is for  $T_p - T_s \gg T_e - T_p$ . In this limit we have a boundary resistance microbolometer<sup>2</sup> not a hot electron microbolometer.

#### IV. NOISE ANALYSIS AND OPTIMIZATION OF THE SIN JUNCTION

The noise equivalent absorbed power of the detector can be written<sup>1</sup> in the form

$$(\text{NEP})^2 = (\text{NEP})_{\text{photon}}^2 + (\text{NEP})_G^2 + (V_J^2 + V_A^2) / S^2 \quad (13)$$

Here the first term is the photon noise of the absorbed photon power  $P_0$ . The second term is the energy fluctuation or  $G$  noise, which arises from the passage of quantized carriers of energy through the thermal conductances, and the third term is due to the voltage noise of the SIN junction and the equivalent input noise of the preamplifier. These terms are divided by the square of the voltage responsivity of the bolometer, referred to the absorbed power.

For a conventional bolometer with a heat capacity  $C$  connected to a heat sink by a thermal conductance  $G$ , the  $G$ -noise term is simply  $(\text{NEP})_G^2 = 4KT^2G$ . Since the thermal circuit of our bolometer is much more complicated, we must generalize this result. The variance of  $T_e$  arises from the energy fluctuation in the electron and phonon baths.

$$\begin{aligned} \langle (\Delta T_e)^2 \rangle &= \langle [\Delta(T_e - T_p)]^2 \rangle + \langle [\Delta(T_p - T_s)]^2 \rangle \\ &= kT_e^2 / C_e + kT_p^2 / C_p, \end{aligned} \quad (14)$$

where  $C_e$  and  $C_p$  are the heat capacities of the electron and phonon systems.

This fluctuation can be written as an integral over a power spectral intensity  $S_p$ .

$$\begin{aligned} \langle (\Delta T_e)^2 \rangle &= \int_0^{\infty} \left[ \frac{S_p \beta^2}{G_{ep}^2 + \omega^2 C_e^2} + \frac{S_p}{G_{ps}^2 + \omega^2 C_p^2} \right] \frac{d\omega}{2\pi} \\ &= S_p \left[ \frac{\beta^2}{4G_{ep}C_e} + \frac{1}{4G_{ps}C_p} \right]. \end{aligned} \quad (15)$$

From Eq. (14), (15), we get

$$S_p = \left( \frac{kT_e^2}{C_e} + \frac{kT_p^2}{C_p} \right) / \left( \frac{\beta^2}{4G_{ep}C_e} + \frac{1}{4G_{ps}C_p} \right). \quad (16)$$

We now neglect the term  $kT_e^2 / C_e$  and consider two cases. If  $\beta^2 / 4G_{ep}C_e \gg 1 / 4G_{ps}C_p$ , then

$$(NEP)_G^2 = S_p = 4kT_p^2 G_{ep} C_e / C_p \beta^2. \quad (17)$$

When  $\beta^2 / 4G_{ep}C_e \ll 1 / 4G_{ps}C_p$ , which is the case for all the three cases we discussed above, then

$$(NEP)_G^2 = S_p = 4kT_p^2 G_{ps}. \quad (18)$$

The temperature dependence of the tunneling current in an SIN junction typically has a simple thermal activation form for values at the bias voltage parameter  $x = eV/kT$  in the range  $x_{\min} < x < x_g$ .<sup>14</sup> Below  $x_{\min}$  leakage current reduces the temperature dependence. Close to the energy gap  $x_g = \Delta/kT$ , the temperature dependence is more complicated. The nominal value of the gap parameter  $\Delta = 170 \mu V$  in Al. In this range, the voltage responsivity to the absorbed power  $P_0$  can be written as

$$S = -k_B (x_g - x + 1/2) / (eG_{\text{eff}}) \quad (19)$$

Over the same temperature range, the voltage noise in the SIN junction can be written<sup>15</sup>

$$V_J^2 = R_n (2kT)^{3/2} (\pi\Delta)^{-1/2} e^{x_g - x} \quad (20)$$

where  $R_n$  is the normal resistance of the junction. Equations (19) and (20) can be combined to evaluate the noise contribution  $(NEP)_R^2$  of the junction and the amplifier.

$$\begin{aligned} (NEP)_R^2 &= \left[ V_A^2 + R_n (2kT)^{3/2} (\pi\Delta)^{-1/2} e^{x_g - x} \right] \\ &\quad \cdot e^2 G_{\text{eff}}^2 / k^2 (x_g - x + 1/2)^2 \end{aligned} \quad (21)$$

To minimize this contribution to the noise  $V_A^2$  should be small. Transformer coupled amplifiers can give  $V_A = 0.1 \text{ nVHz}^{-1/2}$  for a source resistance  $R_S < 50 \Omega$ . Bipolar transistors give  $V_A = 1.4 \text{ nVHz}^{-1/2}$  for  $R_S < 1 \text{ k}\Omega$ . Also, the noise is reduced if  $R_n$  is small. This unusual behavior arises from the fact that the responsivity in Eq.(19) is independent of  $R_n$ . In most other bolometers the responsivity is proportional to the square root of the resistance at the optimum bias

point, so the resistance divides out of the NEP. The  $(NEP)_R^2$  from Eq. (21) is a minimum when the junction is biased at  $x=x_0$ , where

$$x_0 = x_g - 2V_A^2 / R_n (2kT)^{3/2} (\pi\Delta)^{-1/2} - 3/2. \quad (22)$$

This is the appropriate bias point if  $x_{\min} < x_0 < x_g$  for the smallest achievable  $V_A$  and  $R_n$ . If  $x_0 < x_{\min}$ , then the bolometer should be biased at  $x_{\min}$ .

## V. CONCLUSION

This paper presents a careful analysis of the energy relaxation process in the hot electron microbolometer. Practical guides for design optimization are also given. Calculated values of all contributions to the noise in the microbolometer are shown in Table 1 for Cases 1-3 which represent a wide range of applications. Cases 1 and 2 achieve ideal photon noise limited performance. Case 3 is amplifier noise limited by a small margin. Thanks are due to H. Kinder for suggesting this work and to W. Holmes, S. Grannan, and J. X. He for helpful discussions.

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