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Author(s): Bhattacharya, Tanmoy/T8
Chandrasekharan/Duke University
Rajan Gupta/T8
Weonjong Lee/T8
Stephen Sharpe/University of Washington

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Non-perturbative Renormalization Constants using Ward Identities*

Tanmoy Battacharya[a], Shailesh Chandrasekharan[b], Rajan Gupta[a], Weonjong Lee[a], Stephen Sharpe[c]

[a]MS B-285, Los Alamos National Lab, Los Alamos, New Mexico 87545, USA

[b]Department of Physics, Duke University, Durham, North Carolina 27705, USA

[c]Physics Department, University of Washington, Seattle, Washington 98195, USA

We extend the application of vector and axial Ward identities to calculate b_A, b_P and b_T, coefficients that give the mass dependence of the renormalization constants of the corresponding bilinear operators in the quenched theory. The extension relies on using operators with non-degenerate quark masses. It allows a complete determination of the $O(a)$ improvement coefficients for bilinears in the quenched approximation using Ward Identities alone. Only the scale dependent normalization constants Z_P^0 (or Z_S^0) and Z_T are undetermined. We present results of a pilot numerical study using hadronic correlators.

To remove errors of $O(a)$ from physical matrix elements, one must improve both the action and the operators [1]. The former requires the addition of the Sheikholeslami-Wohlert term [2],

$$S_{SW} = -a^5 c_{SW} \sum_x \bar{\psi}(x) \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x) \,. \quad (1)$$

Improvement of flavor off-diagonal bilinears requires [3] both the addition of extra operators,

$$(A_I)_\mu \equiv A_\mu + a c_A \partial_\mu P \quad (2)$$

$$(V_I)_\mu \equiv V_\mu + a c_V \partial_\nu T_{\mu\nu} \quad (3)$$

$$(T_I)_{\mu\nu} \equiv T_{\mu\nu} + a c_T (\partial_\mu V_\nu - \partial_\nu V_\mu) \,, \quad (4)$$

and the introduction of a mass dependence,

$$(X_R)^{(ij)} \equiv Z_X^0 (1 + b_X a m^{ij}) (X_I)^{(ij)} \,. \quad (5)$$

Here $X = A, V, P, S, T$, Z_X^0 are the renormalization constants in the chiral limit, and $m_{ij} \equiv (m_i + m_j)/2$ are the bare quark masses defined using the axial Ward Identity (WI), Eq. (6). The bare unimproved bilinears are $A_\mu^{(ij)} \equiv \bar{\psi}^i \gamma_\mu \gamma_5 \psi^j$, etc. The task is to determine the coefficients c_{SW}, Z_X^0's, c_X's, and b_X's non-perturbatively.

Previous calculations have shown how Z_V^0, Z_A^0 and Z_P^0/Z_S^0 [4], c_{SW}, c_A and b_V [5–7], c_V [8], c_T [9], and $b_P - b_A$ and b_S [10] can be determined non-perturbatively using axial and vector WI. We discuss here an extension that yields b_A, b_P, and b_T. The two remaining constants Z_P^0 (or Z_S^0) and Z_T^0 are scale and scheme dependent, and so cannot be determined using WI. Note that our method does not extend directly to the unquenched theory, which requires additional improvement constants and a more complicated set of conditions [11].

We begin by recalling the ALPHA method for determining c_{SW} and c_A [6]. The bare WI mass

$$2m_{ij} = \frac{\sum_{\vec{x}} \langle \partial_\mu [A_\mu + a c_A \partial_\mu P]^{(ij)} (\vec{x}, t) J^{(ji)}(0) \rangle}{\sum_{\vec{x}} \langle P^{(ij)}(\vec{x}, t) J^{(ji)}(0) \rangle} \quad (6)$$

should be independent of t, up to corrections of $O(a^2)$, since it is proportional to the average renormalized quark mass

$$\frac{m_i^R + m_j^R}{2} = m_{ij} \frac{Z_A^0 (1 + b_A a m_{ij})}{Z_P^0 (1 + b_P a m_{ij})} \,. \quad (7)$$

This is achieved by simultaneously tuning c_{SW} and c_A. Our approach differs from the Schrödinger functional method of Ref. [6] in that we use standard 2-point correlation functions. Consistency of these estimates are checked by varying the initial state using $J = P$ or A_4 and with different types of sources for the quark propagators (Wuppertal smearing, Wall, point). In the following we use the abbreviation $m_i = m_{ii}$, where m_{ii} refers to two degenerate flavors.

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With c_{SW} fixed, Z_V^0 and b_V are obtained using charge conservation. We use the forward matrix elements of $(V_I)_4$ between pseudoscalars,

$$\frac{1}{Z_V^0(1+b_V am_2)} = \frac{\sum_{\vec{x},\vec{y}}\langle P^{(12)}(\vec{x},\tau)(V_I)_4^{(22)}(\vec{y},t)J^{(21)}(0)\rangle}{\langle\sum_{\vec{x}}P^{(12)}(\vec{x},\tau)J^{(21)}(0)\rangle}\,. \quad (8)$$

with $\tau > t > 0$ and $J = P$ or A_4. Note that the c_V term in V_I does not contribute.

Next consider the generic axial WI

$$\left\langle \delta\mathcal{S}^{(12)}\mathcal{O}_R^{(23)}(y)J^{(31)}(z)\right\rangle = \left\langle \delta\mathcal{O}_R^{(13)}(y)\ J^{(31)}(z)\right\rangle \quad (9)$$

where $\mathcal{O}^{23} = \bar{\psi}^{(2)}\Gamma\psi^{(3)}$, $\delta\mathcal{O}^{13} = \bar{\psi}^{(1)}\gamma_5\Gamma\psi^{(3)}$, and

$$\delta\mathcal{S}^{(12)} = \int_{\mathcal{V}}\left[(m_1^R+m_2^R)(P_R)^{(12)} - \partial_\mu(A_R)_\mu^{(12)}\right] \quad (10)$$

This results from a chiral rotation on flavors 1, 2 in the 4-volume $\mathcal{V}$, with $y \in \mathcal{V}$ and $z \notin \mathcal{V}$. By enforcing these identities in the chiral limit one can determine c_V and c_T [3,8,9], as shown below.

Away from the chiral limit, operators P and $\mathcal{O}$ in the product $\int_{\mathcal{V}}(m_1^R+m_2^R)(P_R)^{(12)}(x)\mathcal{O}_R^{(23)}(y)$ need off-shell improvement. This requires the addition of a contact term, of unknown normalization, having the same form as the RHS of (9) [9]. Our new observation is that the contact term is proportional to m_1+m_2 and so can be removed by extrapolating m_1 and m_2 to zero. This leaves the freedom to examine the dependence on m_3, and from this one can determine certain combinations of the b_X. In the following, the extrapolation to $m_1 = m_2 = 0$ is implicit[2].

As a first application of this method we show how to obtain b_A, as well as c_V, using the WI

$$r_1 \equiv \frac{Z_A^0(1+b_A am_3/2)}{Z_A^0\cdot Z_V^0(1+b_V am_3/2)} = \frac{\sum_{\vec{y}}\langle\delta\mathcal{S}^{(12)}\ (V_I)_4^{(23)}(\vec{y},y_4)\ J^{(31)}(0)\rangle}{\sum_{\vec{y}}\langle(A_I)_4^{(13)}(\vec{y},y_4)\ J^{(31)}(0)\rangle} \quad (11)$$

$$= \frac{\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle\delta\mathcal{S}^{(12)}(V_I)_i^{(23)}(\vec{y},y_4)A_i^{(31)}(0)\rangle}{\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle(A_I)_i^{(13)}(\vec{y},y_4)A_i^{(31)}(0)\rangle} \quad (12)$$

[2]In practice, we keep the $\int(m_1+m_2)(P_R)^{(12)}$ term in Eq. (10) prior to extrapolation, since it improves the signal.

where $J = P$ or A_4. Eq. (11) is independent of c_V at $\vec{p} = 0$, and its m_3 dependence gives $b_A - b_V$. The intercept provides a second determination of Z_V^0. Eq. (12) is used to determine c_V.

Given c_V, an alternate determination of $b_A - b_V$ is obtained from

$$r_2 \equiv \frac{Z_V^0(1+b_V am_3/2)}{Z_A^0\cdot Z_A^0(1+b_A am_3/2)} = \frac{\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle\delta\mathcal{S}^{(12)}\ (A_I)_i^{(23)}(\vec{y},y_4)\ V_i^{(31)}(0)\rangle}{\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle(V_I)_i^{(13)}(\vec{y},y_4)\ V_i^{(31)}(0)\rangle} \quad (13)$$

This also yields Z_A^0. The same information can be obtained from the combinations

$$\frac{1}{\sqrt{r_1\cdot r_2}} = Z_A^0\,, \quad (14)$$

$$\sqrt{\frac{r_1}{r_2}} = \frac{Z_A^0}{Z_V^0}(1+(b_A-b_V)am_3/2)\,. \quad (15)$$

Similarly, we determine $b_P - b_S$ and Z_P^0/Z_S^0 using

$$\frac{Z_P^0(1+b_P am_3/2)}{Z_A^0\cdot Z_S^0(1+b_S am_3/2)} = \frac{\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle\delta\mathcal{S}^{(12)}\ S^{(23)}(\vec{y},y_4)\ J^{(31)}(0)\rangle}{\sum_{\vec{y}}e^{i\vec{p}\cdot\vec{y}}\langle P^{(13)}(\vec{y},y_4)\ J^{(31)}(0)\rangle}\,. \quad (16)$$

To get c_T we use the WI with $\mathcal{O} = T_{ij}$

$$1 + ac_T\frac{\sum_{\vec{y}}\langle[-\partial_4 V_k]^{(13)}(\vec{y},y_4)T_{k4}^{(31)}(0)\rangle}{\sum_{\vec{y}}\langle T_{k4}^{(13)}(\vec{y},y_4)T_{k4}^{(31)}(0)\rangle} = Z_A^0\frac{\sum_{\vec{y}}\langle\delta\mathcal{S}^{(12)}\ (T_I)_{ij}^{(23)}(\vec{y},y_4)\ T_{k4}^{(31)}(0)\rangle}{\sum_{\vec{y}}\langle T_{k4}^{(13)}(\vec{y},y_4)\ T_{k4}^{(31)}(0)\rangle}\,. \quad (17)$$

At $\vec{p} = 0$, $(T_I)_{ij}$ has no contribution from the c_T term, so the only c_T dependence is on the LHS.

The previous method fails for b_T since both sides of (9) have the same dependence on b_T if $m_1 = m_2 = 0$. The cure is to consider three non-degenerate masses. The contact term required to improve the LHS of (9) is proportional to $m_1 + m_2$, while the relative dependence of the two sides on b_T is proportional to $m_1 - m_2$. Thus the two terms can be separated and b_T determined. More details will be given in [11]. We have not yet implemented this proposal.

Thus far we do not have a separate determination of b_P or b_S. This can be accomplished with

the method of Ref. [10], which uses 2-point correlation functions for non-degenerate masses. In addition to using that method we present a variant which avoids the need to study quantities as a function of the underlying hopping parameter κ. We first note that if we use Eq. (7) and enforce $(2m_1)^R + (2m_2)^R = 2(m_1+m_2)^R$, we find

$$b_P - b_A = -\frac{4m_{12} - 2[m_{11}+m_{22}]}{a[m_{11}-m_{22}]^2}\,. \tag{18}$$

We next make use of the vector two-point WI

$$\begin{aligned} \Delta m_{12} &\equiv \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \partial_\mu V_{I_\mu}^{(12)}(\vec{x},t) J^{(21)}(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle S^{(12)}(\vec{x},t) J^{(21)}(0)\rangle} \\ m_1^R - m_2^R &= \frac{Z_V^0[1+b_V a(m_1+m_2)/2]}{Z_S^0[1+b_S a(m_1+m_2)/2]}\Delta m_{12} \end{aligned}$$

where the source J is either $J^{(21)} = S^{(21)}$ or $\sum_{\vec{z}} P^{(23)}(\vec{z},z_4)P^{(31)}(0)$ for $0 < t < z_4$. Enforcing $2(m_1^R - m_2^R) = (2m_1)^R - (2m_2)^R$, we find

$$\begin{aligned} \frac{b_S - b_V}{2} &= \frac{\Delta m_{12} - R_Z[m_{11}-m_{22}]}{aR_Z[m_{11}^2 - m_{22}^2]} \\ &\quad + (b_P - b_A)\,, \end{aligned} \tag{19}$$

where $R_Z \equiv Z_S^0 Z_A^0/(Z_P^0 Z_V^0)$. Since b_A and b_V are already known, b_P and b_S are given by Eqs. (18) and (19).

The above discussion shows that *in principle* one can determine all the constants, except Z_P^0 (or Z_S^0) and Z_T^0, in the quenched theory using Ward identities. The results of an exploratory study are summarized in Table 1. These were obtained on 83 lattices of size $16^3 \times 48$ at $\beta = 6.0$. Since the action is only tree-level tadpole improved ($c_{SW} = 1.4755$), the results do not represent full $O(a)$ improvement, but they indicate the efficacy of the method. More details will appear in [11].

We draw two preliminary conclusions. First, the determination of c_V has a large uncertainty, which accounts for a substantial fraction of the errors in Z_A^0, Z_P^0/Z_S^0, and c_T. The Schrödinger functional method used in Ref. [8] seems to determine the constants, in particular c_A and c_V, with much smaller errors. Second, even though there are channels in which the errors on the determination of $b_A - b_V$ and $b_P - b_S$ are fairly small, we find the magnitude of these differences are still comparable to their error. This rough equality between all the b_X is consistent with perturbative results [12].

Eq.#	observable	intercept
(6)	c_A	−0.016(11)
(8)	Z_V^0	+0.746(1)
(8)	b_V	+1.55(2)
(11)	Z_V^0	+0.752(7)
(11)	$b_A - b_V$	+0.34(21)
(12)	c_V	+0.46(29)
(14)	$Z_V^0/(Z_A^0)^2$	+1.32(12)
(14)	$b_A - b_V$	+1.8(1.1)
(14)	Z_A^0	+0.78(2)
(15)	Z_A^0/Z_V^0	+1.00(5)
(15)	$b_A - b_V$	+1.2(8)
(16)	$Z_A^0 Z_S^0/Z_P^0$	+0.96(1)
(16)	$b_P - b_S$	−0.08(9)
(17)	c_T	−0.14(7)
[10]	$Z_A^0 Z_S^0/Z_P^0$	+0.96(1)
[10]	$b_A - b_P + b_S/2$	+0.49(1)
(18)	$b_P - b_A$	+0.1(4)
(19)	$b_S - b_V - 2(b_P - b_A)$	+1.7(5)

Table 1
Constants extracted from the different WI.

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