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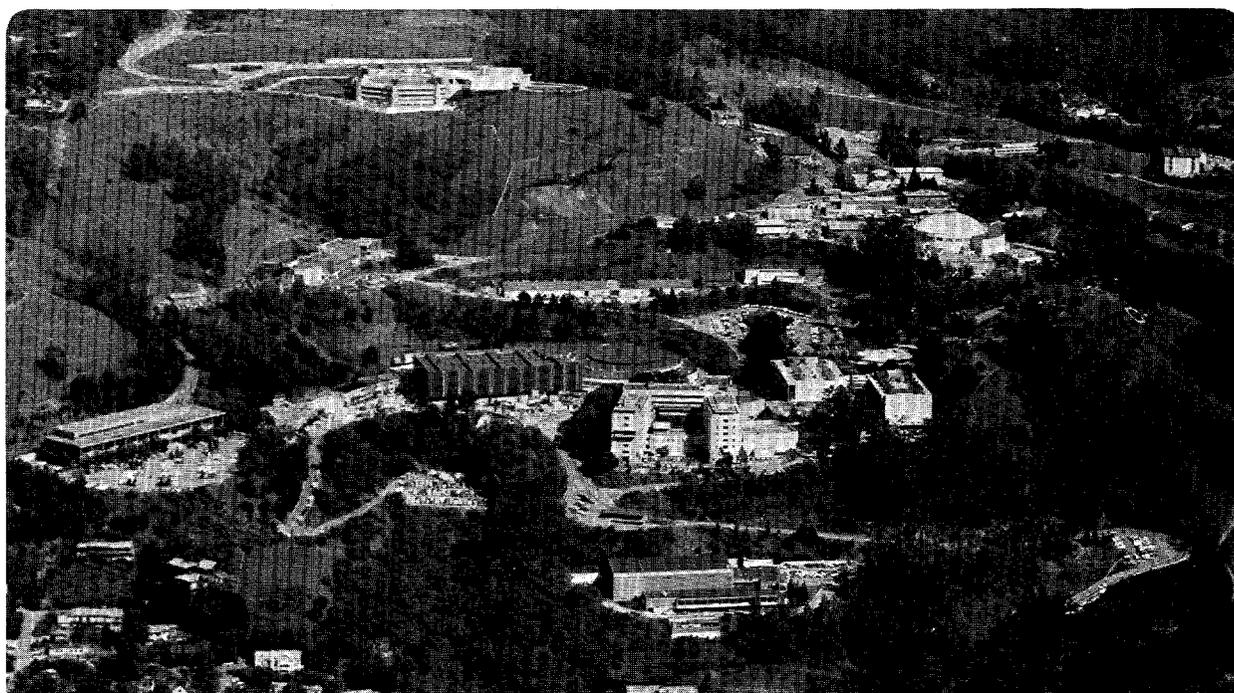
## Engineering Division

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### **Radiation and Gas Conduction Heat Transport Across a Helium Dewar Multilayer Insulation System**

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**RADIATION AND GAS CONDUCTION  
HEAT TRANSPORT ACROSS A HELIUM DEWAR  
MULTILAYER INSULATION SYSTEM**

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**MASTER**

# RADIATION AND GAS CONDUCTION HEAT TRANSPORT ACROSS A HELIUM DEWAR MULTILAYER INSULATION SYSTEM

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This report describes a method for calculating mixed heat transfer through the multilayer insulation used to insulate a 4 K liquid helium cryostat. The method described here permits one to estimate the insulation potential for a multilayer insulation system from first principles. The heat transfer regimes included are: radiation, conduction by free molecule gas conduction, and conduction through continuum gas conduction. Heat transfer in the transition region between the two gas conduction regimes is also included.

The cryogenic multilayer insulation system is modeled as a stack of flat plates. The following assumptions apply: 1) The spacing between the plates is much smaller than the smallest surface dimension of the plate. As a result the form factor  $F$  for radiation and free molecular heat transfer is one. 2) The spacing between the plates is larger than 0.6 times the predominant wave length of the emitted heat, so there is no radiation tunneling between the plates. At the lowest temperature (say 4 K), the plate spacing must be greater than 0.42 mm. On the room temperature side (300 K), the plate spacing has to be greater than 0.006 mm. 3) The plates shall be metallic or coated with a metal so that they are opaque to the wavelengths of infrared on the plates. At 300 K, mylar with 300 angstroms of aluminum on it is opaque to thermal radiation. At low temperatures, the aluminum thickness has to increase (to 0.03 mm at 4 K). 4) Since the multilayer insulation operates over a temperature range from 4 K on up, the medium between the plates is assumed to be helium gas at pressures that vary from good vacuum ( $<10^{-5}$  Pa) to atmospheric ( $10^5$  Pa). 5) The medium between the plates is non-participating. This means that the gas between the plates does not absorb nor emit thermal radiation. 6) The spacing between the plates is small enough to prevent convection cells from forming during continuum gas conduction.

## RADIATION HEAT TRANSFER BETWEEN PLATES

The radiation heat transfer per unit area  $Q_R$  between plate  $N$  and plate  $N-1$  can be calculated using the following expression:

$$Q_R = E(T_{N,N-1}) \sigma [T_N^4 - T_{N-1}^4] \quad (1)$$

where  $Q_R$  is the radiation heat transfer between plate N and N-1;  $E(T_{N,N-1})$  is form and emissivity factor for the heat transfer;  $\sigma$  is the Stefan-Boltzman constant ( $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ );  $T_N$  is the temperature of plate N; and  $T_{N-1}$  is the temperature of plate N-1. The form and emissivity factor for diffuse reflection is defined as follows:

$$E(T_{N,N-1}) = F \epsilon_{N,N-1} = F \frac{\epsilon_N \epsilon_{N-1}}{\epsilon_N + (1 - \epsilon_N) \epsilon_{N-1}} \quad (1a)$$

where F is the form factor. In our case, we assume  $F = 1$ . As long as F is one, Equation 1a applies for specular reflection as well as diffuse reflection. Another implication of F being one is that radiation heat transfer is independent of plate separation (as long as the plate separation is more than 0.6 times the peak heat wave length).  $\epsilon_N$  is the emissivity of plate N; and  $\epsilon_{N-1}$  is the emissivity of plate N-1. When the plates have the same emissivity  $\epsilon$  and the plate emissivity is small,  $E(T_{N,N-1})$  is approximately  $\epsilon/2$ .

The emissivity of a metal plate is temperature dependent. For metallic plates, the Hagen-Rubens approximation can be used to estimate the plate emissivity as long as the radiation wave length is longer than 5 microns. (Thermal radiation at 300 K has a peak wave length of 9.7 microns; thermal radiation at 4 K has a peak wave length of 725 microns.) When the Hagen-Rubens relationship is integrated over all wave lengths the following temperature dependent form for the emissivity  $\epsilon(T_s)$  can be developed<sup>1</sup>:

$$\epsilon(T_s) = 5.76 [\rho(T_s) T_s]^{0.5} + 12.4 \rho(T_s) T_s \quad (2)$$

where  $\rho(T_s)$  is the temperature dependent electrical resistivity of the metal in the plate that is at a temperature  $T_s$ .

From Equation 2, one can say: 1) The lowest emitters are the best electrical conductors. 2) The emissivity of the plate increases with temperature. 3) Alloying a good reflecting metal increases its emissivity 4) Mechanical polishing that results in work hardening increases emissivity. At 300 K, the theoretical emissivity of 1100 aluminum is 0.0166; at 4 K the emissivity goes down to 0.00034 (provided the oxide layer does not increase the emissivity)<sup>2</sup>. In practical terms, the average emissivity of a stack of multilayer insulation with aluminum should be about 0.02 provided the aluminum layer on each plate is thick enough. If there is a heavy aluminum oxide coating on the aluminum, the emissivity can go up as high as 0.1. Gold coated mylar can have a lower emissivity than aluminized mylar because there is no oxide layer. It should be noted that it does not matter how low the plate emissivity is, if that plate is not opaque to thermal radiation.

## GAS CONDUCTION HEAT TRANSFER BETWEEN PLATES

Gas conduction between the plates falls in two general regimes: 1) The free molecule regime is where gas molecules carrying heat travel from plate to plate without colliding with each other. The spacing between the plates is much less than a mean free path for gas molecule collisions 2) The continuum gas conduction regime is where the gas molecules carrying the heat from plate to plate collide with each other in the process of traveling from one plate to the other. The spacing between the plates is much greater than a mean free path for gas molecule collisions There is a transition regime between the free molecular gas conduction regime and the continuum gas conduction regime.

The mean free path for collisions between gas molecules can be determined from the viscosity of the gas. The mean free path is defined as follows<sup>3</sup>:

$$\lambda(T_g) = 1.23 \frac{\mu(T_g)}{P} \left[ \frac{RT}{M} \right]^{0.5} \quad (3)$$

where  $\lambda(T_g)$  is the mean free path for gas molecule collisions for gas at a temperature  $T_g$ ;  $\mu(T_g)$  is the gas viscosity at a temperature  $T_g$ ;  $T_g$  is the average temperature of the gas between plate N and plate N-1; R is the universal gas constant ( $R = 8314 \text{ J K}^{-1} \text{ mole}^{-1}$ ); M is the molecular weight of the gas (for helium  $M = 4 \text{ kg mole}^{-1}$ ); P is the pressure measured by a vacuum gauge looking at a space with a temperature T. Note: the equations here are all given in SI units. (A vacuum of 1 torr is 133.29 Pa.) The viscosity of helium gas can be calculated using the following analytic expression (valid from 5 K to 500 K)<sup>4</sup>:

$$\mu(T_g) = 5.03 \times 10^{-7} T_g^{0.65} \quad (3a)$$

In order to determine the gas heat conduction regime one can define a dimensionless number called the Knudsen number<sup>3</sup> Kn;

$$Kn = \frac{\lambda(T_g)}{S} \quad (4)$$

where S is the spacing between the plate N and plate N-1. The free molecular heat transfer equations can be applied precisely when  $Kn > 10$ . In many cases, the free molecule equation can be used as Kn approaches 1, but the accuracy is reduced. The ordinary conductive heat transfer equation can be applied when  $Kn < 0.003$ . The region from  $0.003 < Kn < 10$  is a transition region. Heat transfer in this region can be looked at from either the free molecular or the continuum gas

point of view. Neither method is completely accurate over the whole transition region, but the accuracy of a given method can be surprisingly good.

### Free Molecule Gas Conduction

When the vacuum pressure is relatively low, heat conduction between plates will be by free molecule gas conduction. Free molecule gas conduction occurs when the  $Kn > 10$ . The upper limit pressure  $P_u$  for free molecular conduction (defined as  $Kn = 10$ ) can be estimated using the following expression:

$$P_u = 0.123 \frac{\mu(T_g)}{S} \left[ \frac{R T}{M} \right]^{0.5} \quad (5)$$

where  $S$  is the spacing between plates.  $\mu$ ,  $R$ ,  $T$ , and  $M$  are previously defined.

In the free molecular conduction regime, conduction heat transfer is a function of pressure (or number density). Heat flow per unit area  $Q_{N,N-1}$  by free molecular conduction between plate  $N$  and plate  $N-1$  can be calculated using the following expression:

$$Q_{N,N-1} = D(T_{N,N-1},P) [T_N - T_{N-1}] \quad (6)$$

where the transfer function  $D(T_{N,N-1},P)$  takes the form:

$$D(T_{N,N-1},P) = F \frac{\alpha_{N-1} \alpha_N}{\alpha_N + \alpha_{N-1}(1-\alpha_N)} \frac{k+1}{k-1} \left[ \frac{R}{2 \pi M T} \right]^{0.5} P \quad (6a)$$

where  $F$  is the form factor ( $F = 1$ );  $\alpha_N$  is the accommodation coefficient of plate  $N$ ;  $\alpha_{N-1}$  is the accommodation coefficient of plate  $N-1$ ,  $k$  is the ratio of specific heats for the gas (Use  $k = 1.67$  for helium.);  $R$  is the universal gas constant ;  $M$  is the molecular weight of the gas;  $P$  is the pressure. When the accommodation coefficient  $\alpha$  is the same for both plates and it is relatively small,

$$D(T_{N,N-1}) = F \frac{\alpha}{2} \frac{k+1}{k-1} \left[ \frac{R}{2 \pi M T} \right]^{0.5} P \quad (6b)$$

where  $F$ ,  $k$ ,  $R$ ,  $M$ ,  $P$  and  $T$  have been previously defined.

At low temperatures (less than 60 K), the accommodation coefficient is temperature dependent; the helium between the plates collides with molecules of air gasses that stick to the plates. The accommodation coefficient approaches one at 4 K. As the temperature rises, the accommodation coefficient decreases until it drops to about 0.1 between 60 and 120 K. Then the accommodation coefficient

increases to 0.25 at 300 K. The following approximate expression can be used to estimate the temperature dependent accommodation coefficient  $\alpha(T)$  of helium to an aluminum plate:

$$\alpha(T) = 1.23 e^{-T/20} + 8.34 \times 10^{-4} T \quad (7)$$

where  $T$  is the temperature of the plate. Equation 7b applies over a range of temperatures from 5 K to 500 K. The average accommodation coefficient between 4 K and 300 K is between 0.13 and 0.17 (the higher accommodation coefficient applies when radiation heat transfer dominates); the average accommodation coefficient between 4 K and 80 K is around 0.1.

### Ordinary Continuum Gas Conduction

Heat is transferred by ordinary continuum gas conduction when  $Kn < 0.01$ . Gas conduction heat transfer per unit area  $Q_{N,N-1}$  from plate  $N$  to plate  $N-1$  can be estimated using the following expression:

$$Q_{N,N-1} = k(T) \frac{dT}{dx} \quad (8)$$

$$Q_{N,N-1} = k(T) \frac{T_N - T_{N-1}}{S_N} \quad (8a)$$

where  $k(T)$  is the thermal conductivity of the gas at temperature  $T$ ;  $S_N$  is the plate separation distance between plate  $N$  and plate  $N-1$ .

The distance between the high temperature wall and the low temperature wall  $S_N$  is the governing factor for continuum conductive heat transfer. The role of the plates in continuum conduction is to keep convection cells from forming.

The temperature dependent thermal conductivity  $k(T)$  for helium gas can be estimated using the following analytical expression<sup>4</sup>:

$$k(T) = 3.83 \times 10^{-3} T^{0.65} \quad (9)$$

Equation 9 is useful between 5 K and 500 K, from 100 Pa to 10 MPa. For a vacuum space filled with helium gas above 100 Pa, continuum conductive heat transfer per unit area  $Q_{N,N-1}$  from plate  $N$  to plate  $N-1$  can be calculated using the following equation that comes from integrating Equation 9 from  $T_{N-1}$  to  $T_N$ :

$$Q_{N,N-1} = 2.32 \times 10^{-3} \frac{[T_N^{1.65} - T_{N-1}^{1.65}]}{S_N} \quad (10)$$

where  $S_N$  is the distance from the plate N at a temperature  $T_N$  to plate N-1 at a temperature  $T_{N-1}$ .

### Gas Conduction in the Transition Region

Heat conduction through gas in the transition region may be obtained by extrapolation from either the continuum region or free molecular region. Lees' linearized four moment Maxwell molecule model<sup>5</sup> approaches the problem from the free molecular end ( $Kn > 10$ ). Experimental measurements taken by Teagan and Springer were found to agree favorably with the Lees' four moment model<sup>6</sup>. The ratio of transition region heat flow to free molecular heat flow  $Q_T/Q_{FM}$  using the Lees' four moment model can be stated as follows:

$$\frac{Q_T}{Q_{FM}} = \frac{\Xi Kn (2/\alpha - 1)}{1 + \Xi Kn (2/\alpha - 1)} \quad (11)$$

where  $Q_T$  is the heat flow per unit area between the plates in the transition region;  $Q_{FM}$  is the heat transfer per unit area between the plates if the heat flow is assumed to be in the free molecule region;  $\Xi$  is a fitting parameter. Lees in his paper proposed that  $\Xi = 3.166$ , but  $\Xi$  may have another value that makes the solution fit better at both ends. (In our case  $\Xi = 1.8$  makes the fit much better.) One can make a strong case for  $\Xi$  being temperature dependent.  $Kn$  is the Knudsen number; and  $\alpha$  is the average accommodation coefficient between the plates. It is interesting to note that if the fitting parameter  $\Xi$  is properly chosen,  $Q_T$  will equal  $Q_{N,N-1}$  given by Equation 10 when the Knudsen number is small. Regardless of the value of  $\Xi$ , the value of  $Q_T$  calculated using Equation 11 will equal  $Q_{N,N-1}$  calculated using Equation 6 when the Knudsen number is large.

## COMBINED RADIATION AND CONDUCTION HEAT TRANSFER

### Combined Heat Transfer in the Free Molecule and Transition Regimes

The heat flow through a stack of multilayer insulation can be approximated using a radiation heat transfer term plus a free molecular gas heat transfer term provided  $Kn > 10$ . Heat transfer through a stack of multilayer insulation can be approximated as follows:

$$Q_{RF} = \frac{H(T_m) - H(T_o)}{m} \quad (12)$$

where  $Q_{RF}$  is the combined heat transfer through multilayer insulation that has an upper stack temperature of  $T_m$  and a lower stack temperature of  $T_o$ . There are  $m$

layers of multilayer insulation in the stack. The values of  $H(T_m)$  and  $H(T_o)$  are defined as follows:

$$H(T_m) = E(T_m) T_m^4 + G(T_m, P) T_m \quad (13a)$$

and

$$H(T_o) = E(T_o) T_o^4 + G(T_o, P) T_o \quad (13b)$$

where  $E$  is the radiation heat transfer function and  $G$  is the free molecular heat transfer function.

One can make the simplifying assumption for  $E$  and  $G$  that neither is very temperature dependent. If one assumes that  $E$  and  $G$  are temperature independent, the following expressions for  $E$  and  $G$  result:

$$E = E(T_m) = E(T_o) = \frac{\epsilon \sigma}{2} \quad (14)$$

where  $\epsilon$  is the emissivity of the multilayer insulation (use  $\epsilon = 0.02$ ); and  $\sigma$  is the Stefan-Boltzman constant ( $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ ).

$$G(P) = G(T_m, P) = G(T_o, P) = \frac{\alpha k+1}{2 k-1} \left[ \frac{R}{2\pi M T} \right]^{0.5} P \frac{Q_T}{Q_{FM}} \quad (15)$$

where  $\alpha$  is an overall accommodation coefficient (use  $\alpha = 0.14$ );  $k$  is the ratio of specific heats for the gas;  $R$  is the universal gas constant;  $M$  is the molecular weight of the gas;  $P$  is the pressure measured by a vacuum gauge looking at a plate at a temperature  $T$ . The ratio  $Q_T/Q_{FM}$  is defined by Equation 11. (The fitting parameter  $\Xi$  in Equation 11 should be about 1.8.)

When the high temperature plate is at 300 K (outside an 80 K shield), the radiation heat transfer term and the free molecular gas conduction term are of the same order when the pressure is about 0.055 Pa ( $4 \times 10^{-4}$  torr). When the high temperature plate is at 80 K and the low temperature is at 4 K (inside an 80 K shield), the radiation heat transfer term and the free molecular gas conduction term are of the same order when the pressure is about  $5.5 \times 10^{-4}$  Pa ( $4 \times 10^{-6}$  torr). Since the free molecule heat flow and radiation heat flow equal each other at a relatively low pressure inside an 80 K shield, it is clear that it is very useful to put several layers of multilayer insulation between the shield and the helium vessel to reduce the heat leak in the event that there is a small helium leak into the vacuum space between the plates.

## Combined Heat Transfer in the Continuum Regime

The heat flow through a stack of multilayer insulation can be approximated using a radiation heat transfer term plus a continuum gas conduction heat transfer term provided  $Kn < 0.01$ . Heat transfer through a stack of multilayer insulation with helium in the vacuum space can be approximated as follows:

$$Q_{RC} = \frac{E(T_m) T_m^4 - E(T_o) T_o^4}{m} + 2.32 \times 10^{-3} \frac{[T_m^{1.65} - T_o^{1.65}]}{S_{mo}} \quad (16)$$

where  $Q_{RC}$  is the combined heat transfer through a stack of multilayer insulation that has an upper stack temperature of  $T_m$  and a lower stack temperature of  $T_o$ . There are  $m$  layers of multilayer insulation in the stack.  $S_{mo}$  is the distance between plate at temperature  $T_o$  and the plate at temperature  $T_m$ . The values of  $E(T_m)$  and  $E(T_o)$  are defined by Equation 14.

The heat transfer through a stack of plates in the continuum regime is dominated strongly by the continuum gas conduction heat transfer unless the number of plates is small, the plate emissivity is large, and/or  $T_m$  is substantially above room temperature (300 K). In practical multilayer insulation systems with  $Kn < 0.01$ , the radiation term can be neglected.

### A CALCULATION OF COMBINED HEAT FLOW THROUGH A STACK OF ALUMINUM PLATES

Figure 1 shows the calculated heat flow from 300 K to 4 K through 25 aluminum plates with a spacing of 1 millimeter. The heat flow calculations were made over a range of helium gas pressures from  $10^{-3}$  Pa ( $7.5 \times 10^{-6}$  torr) to  $10^5$  Pa (0.987 atm). The following constants were used to make the heat flow calculations that are shown in Figure 1:  $\epsilon = 0.02$ ;  $\Xi = 1.8$ ;  $T_g = 160$  K; and  $\alpha = 0.14$ .

From Figure 1, one can see that radiation heat transfer dominates the heat flow through the plates at pressures less than 0.01 Pa. From 0.3 Pa to 100 Pa, heat flow through the stack of plates is dominated by free molecular gas conduction. In this region the heat flow through the stack of plates is proportional to the gas pressure between the plates. Above 3000 Pa, ordinary continuum gas conduction dominates. Continuum gas conduction is independent of the pressure between the plates. The four moment transition region solution smoothly connects the free molecular gas conduction region and the continuum gas conduction region. The accuracy of the heat transfer calculation is probably at its worst in the transition region between free molecule gas conduction and continuum gas conduction (at pressures between 100 and 3000 Pa).

$N = 25$ ;  $S = 1$  mm;  $\epsilon = 0.02$ ;  $\alpha = 0.14$ ;  $\Xi = 1.8$ ; and  $T_g = 160$  K.

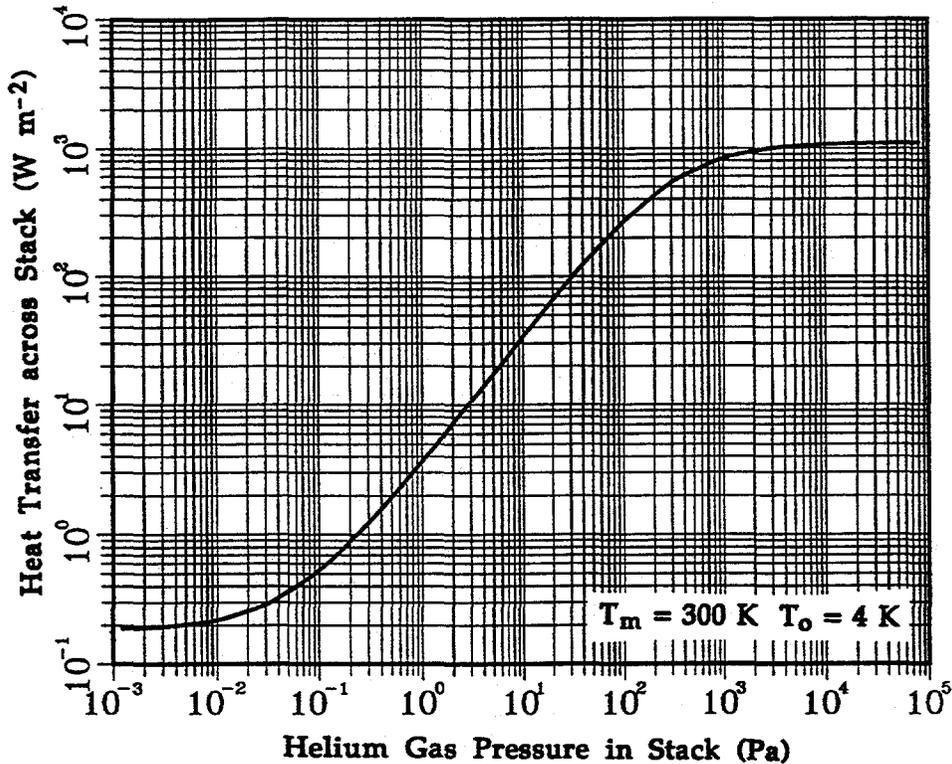


Fig. 1 The Heat Transfer Rate Across a Stack of 25 plates 1 mm Apart as a Function of the Gas Pressure Between the Plates

### CONCLUSION

Heat transfer through a stack of plates in a vacuum is combination of radiation and gas conduction heat transfer. Radiation heat transfer across the stack of plates is proportional to the plate emissivity and inversely proportional to the number of plates in the stack. At pressures below 100 Pa, the gas conduction through the stack of plates is by free molecular gas conduction. Free molecular gas conduction is proportional to the gas pressure and the accommodation coefficient of the gas to the plate material. Like radiation heat transfer, the gas conductive heat transfer through the stack of plates in the free molecular regime is inversely proportional to the number of plates. In both the radiation and free molecular gas conduction regimes, more plates in the stack mean less heat is transferred. In the transition and continuum regimes, the number of plates in the stack is of less concern. In these regimes, conduction heat transfer is controlled by the distance

between the highest temperature plate and the lowest temperature plate. A smooth transition from the free molecular region and the continuum region can occur if the fitting parameter  $\Xi$  is correctly chosen.

### ACKNOWLEDGMENTS

This work is dedicated to Klaus Halbach, a mentor, a colleague, and a friend. The problem that is presented here is one that is usually solved the wrong way. This is the kind of problem that Klaus Halbach would take great delight in solving correctly. Much of the thinking and the methodology that has gone into this work comes from the way that Klaus Halbach would approach a problem of this nature. It is fitting that this paper be dedicated to Klaus Halbach on the occasion of his birthday. Happy Birthday Klaus and may there be many more opportunities for this author to learn from you.

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