MASTER

BNL 28357

CONF-8006127--1

On the Problem of "Phase Transitions" in Nuclear Structure

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

G. Scharff-Goldhaber

Brookhaven National Laboratory, Upton, New York 11973 U.S.A.

and

Cornell University, Ithaca, New York 14850 U.S.A.

Invited Talk Presented at the

International Conference on Interacting Bose-Fermi Systems in Nuclei

Erice, Sicily

June 12-19, 1980

The submitted manuscript has been authored under contract DE-AC02-76CH00016 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

ON THE PROBLEM OF "PHASE TRANSITIONS" IN NUCLEAR STRUCTURE*

G. Scharff-Goldhaber

Brookhaven National Laboratory and Cornell University
Upton, NY 11973 Ithaca, NY 14850

In the setting of Erice, so evocative of Homeric adventures, it may be appropriate to state that the nuclear physicist is forced to steer perpetually between the Scylla of inductive and the Charybdis of deductive reasoning. The first approach consists in attempting to organize the empirical facts in as model independent a fashion as possible with the aim of describing them analytically in order to gain deeper insight. The second approach consists in model building, where the model, although usually based on vastly simplifying assumptions, is expected to yield completeness and accuracy in its predictions. The nuclear structure talks we have heard so far during this workshop have proven very effectively the success of an approach which belongs in the second category, namely that associated with the Interacting Boson Model (IBM), in classifying the complex band structure of even-even nuclei. On the other hand the inductive method yielded the Variable Moment of Inertia (VMI) equations,[1,2] which give a precise description of ground state band energies of non-magic nuclei in terms of the rotational energy expression:

$$E = \frac{C}{2}(\mathcal{J} - \mathcal{J}_o)^2 + \frac{J(J+1)}{2\mathcal{J}} \tag{1}$$

$$\left.\frac{\partial E}{\partial \mathcal{J}}\right|_J = 0 \tag{2}$$

From (1) and (2) follows the "equation of state" for the moment of inertia:

*Research supported by the U. S. Department of Energy under Contract Nos. DE-AC02-76CH00016 and DE-AC02-80ER10576.

$$\mathcal{J}^3 - \mathcal{J}_0\mathcal{J}^2 = \frac{J(J+1)}{2C} \qquad (3)$$

An impressive comparison of the VMI predictions with the predictions resulting from an anharmonic vibrator model (which are identical with those given by the three dynamical IBM symmetries) was published by Das, Dreizler, and Klein[3] (I, Fig. 6) ten years ago.

In a recent talk[2] I have described the evidence for a universal mechanism underlying these bands, as well as the developments preceding this finding. Let me briefly repeat the main points: Fig. 1 depicts the "heroic" stages which immediately followed the classical papers on the shell model: To the right we find the well *known level scheme of the Bohr-Mottelson strongly deformed nucleus* with its rotational ground state, beta and gamma bands. This model contains the assumption that the moment of inertia of the g.s. band as well as the "intrinsic" quadrupole moment are independent of the angular momentum J. To the left we see a near-harmonic level structure displayed by nuclei in the smaller shells, and at the beginning of the rare earths region. This pattern gave rise to the vibrational model as well as to the "gamma unstable" model. An abrupt change was found to exist between near-harmonic nuclei containing $\leq$ 88 neutrons and strongly deformed nuclei with $\geq$ 90 neutrons. Soon afterwards an almost as abrupt transition was found to exist at the beginning of the actinide region, between 86 and 88 protons.[4] The obvious question arose: is there no intermediate stage possible? It was soon answered when a gradual transition was discovered in level schemes of even Os (Z = 76) nuclei from rotational to vibrational. This transition which was shown to continue into the Pt (Z = 78) region appeared to be particularly simple and regular when instead of the level energies the ratios E_J/E_2 were presented. The reason for the increase in simplicity is due to the fact that the energy E_2 can be used as a "scale factor" which includes not only the nuclear size effect ($\propto A^{-2/3}$) but also the influence of residual forces. This procedure soon furnished evidence for the existence of "quasi-gamma bands" and "quasi-beta" bands, in addition to the g.s. bands, in all even-even nuclei.

At the same time that extensive level scheme studies suggested that nuclei in the Os-Pt transition region may not be axially symmetric, Edoardo Mallmann concentrated on the structure of ground state bands alone and discovered that if ratios E_J/E_2 (J = 6,8) are plotted vs. E_4/E_2, the points lie on two "universal curves." This implied that one and the same mechanism is responsible for the ground state bands of all non-magic nuclei (actually Mallmann included also the magic nuclei; the advent of the extended VMI model,[5] however, proved that this inclusion was incorrect). The mechanism, as I stated in the beginning, appears to be rotation in a harmonic vibrator potential. Apart from the strongly deformed, stable

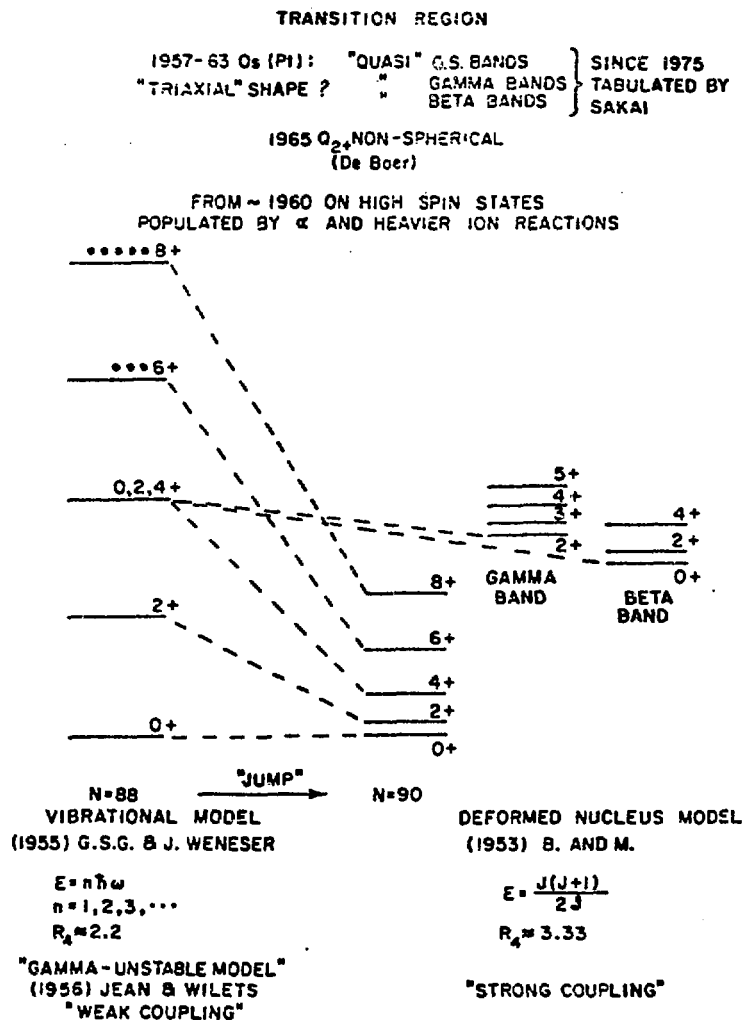

Fig. 1. Schematic graph depicting evolution of models of collective motions in e-e nuclei from the early 1950s on.

nuclei, it turned out that the nuclear moment of inertia is not constant, i.e. independent of J, but increases gradually from the "ground state moment of inertia" $\mathcal{J}_0$. Already in the middle sixties measurements of the static electric quadrupole moments of 2+ states of "spherical nuclei" had produced a contradiction to the assumption that electric quadrupole moments are not affected by the angular momentum.* Thus also the idea underlying the Nilsson diagrams that each nucleus has a definite deformation parameter β is only correct for the ground state. The change in thinking required by these findings constitutes a "revolution" in Thomas Kuhn's sense, a replacement of the old paradigm by a new one. What, then, is the physical meaning of the basic variable of the VMI model, E_4/E_2? At the rotational limit $E_4/E_2 = 10/3$, since $E \propto (J(J+1)$. As shown in I, Fig. 5, the product $\mathcal{J}_0 E_2$ (where $\mathcal{J}_0$ equals the ground state moment of inertia $\mathcal{J}(0)$) decreases very slowly, close to the point where $\mathcal{J}_0 = 0$ and the "hardness" $h(= 2C\mathcal{J}_0^3)$ vanishes, at $E_4/E_2 = 2.23$. Nuclei with $E_4/E_2 \leq 2.23$ possess $\leq$ two nucleon pairs of at least one type (neutrons or protons) away from a magic number. From this point until the magic number is reached, the nuclear ground state is spherical, i.e. $\mathcal{J}(0) = 0$, but the nucleus resists cranking more and more as the threshold energy $\frac{C}{2}\mathcal{J}_0^2$ increases until $\mathcal{J}_0 = -\infty$ corresponding to $E_4/E_2 = 1.82$. This means magic number nuclei cannot be excited by cranking unless one or two nucleon pairs are first promoted to a higher orbit, thus causing deformation.

The VMI model yields predictions for arbitrarily high angular momenta J, but in reality "phase transitions" due to various mechanisms produce (normally downward) deviations starting at a critical spin J_{cr}, or a critical angular velocity corresponding to 1 to 2 MeV excitation energy. (See Ref. 1, section 8.) However, in the deformed actinides energies of states up to J = 28 (∿ 4.5 MeV) agree with (2-parameter) VMI predictions within 1%.[6]

The correspondences between VMI and IBM are obvious: the "vibrational level scheme" ($\mathcal{J}_0 \sim 0$) corresponds to SU(5), the rotational level scheme to SU(3) and the "transitional"[8] nuclei ^{194,196}Pt nuclei and ^{120}Xe to O(6).

*We know now[7] that the moment of inertia $\mathcal{J}_{02}$ $\left(= \frac{\mathcal{J}(0) + \mathcal{J}(2)}{2}\right)$ is strongly correlated with the transition quadrupole moment Q_{02} (0 → 2). This correlation consists of a linear part for spherical nuclei which have small moments, and a quadratic correlation for transitional and strongly deformed nuclei. The linear part can be interpreted by a closed-shell, non-moving core with two clusters (on the average α-particles) at its perimeter which rotate (the "alpha dumbell model"), whereas the quadratic part is given by a two-fluid model (inertial fluid and superfluid). The rather sharp break corresponds precisely to $\mathcal{J}_0 = 0$.

Dr. Dieperink has given a beautiful proof for the occurrence of shape phase transitions of ground states between SU(5) and SU(3) (first order) and SU(5) and O(6) (second order), using an algorithm derived by Gilmore and Feng. No phase transitions between these three types occur within the framework of VMI (which is based solely on the structure of g.s. bands), but it is highly significant that one or the other of the VMI parameters takes on extreme values for the three special subgroups: SU(5) corresponds to $\mathcal{J}_0 \sim 0$, SU(3) to maxima of $\mathcal{J}_0$ (Fig. 2) and O(6) to minima of the stiffness parameter (or spring constant) C (Fig. 3). The question whether phase transitions at $\mathcal{J}_0 = 0$ and $\mathcal{J}_0 = -\infty$ can be deduced from the VMI equations has recently been explored.[9] For this purpose it is instructive to discuss the VMI "equation of state" (3) in comparison with the cubic equations of state characterizing some systems of condensed matter. The systems we chose are a) the van der Waals equation

$$(p + \frac{a}{v^2})\ (v - b) = RT, \tag{4}$$

and b) the equation of state of a ferroelectric (perovskite) under the influence of an external electric field E

$$P^3 + \gamma(T - T_0)P - \alpha E = 0, \quad P = \text{polarization}; \quad \gamma \text{ and } \alpha \text{ are constants.} \tag{5}$$

We showed that to $\mathcal{J}_0 = 0$ corresponds the critical temperature T_c in the van der Waals equation, where

$$RT_c = \frac{8}{27}\frac{a}{b}, \tag{6}$$

and T_0 in the ferroelectric case. For the latter system it was found empirically that the dielectric constant ε_0 obeys the equation $\varepsilon_0 = \frac{a}{\gamma(T - T_0)}$, where a is a constant. In these two many body systems two phases can exist below T_c, resp. T_0 since the cubic equation has there three real roots, two minima and one (unphysical) maximum. However, in the VMI system only one phase exists for $\mathcal{J}_0 < 0$, since, in order to be physically meaningful, the moment of inertia $\mathcal{J}$ has to be positive, corresponding to only one of the three real roots of (3). Hence, it follows that while the van der Waals gas undergoes a first order phase transition at constant pressure from gas to liquid below T_c, and the ferroelectric undergoes a phase transition as ε_0 passes through infinity (which is second order for E = 0, first order for E ≠ 0), no phase transition appears to occur at $\mathcal{J}_0 = 0$.

Yet, as $\mathcal{J}_0 \to -\infty$, a first order phase transition does occur. As mentioned before, at this point an external torque will not produce rotation: excitation of the nucleus can only take place due to a

GROUND STATE MOMENTS OF INERTIA OF EVEN-EVEN NUCLEI

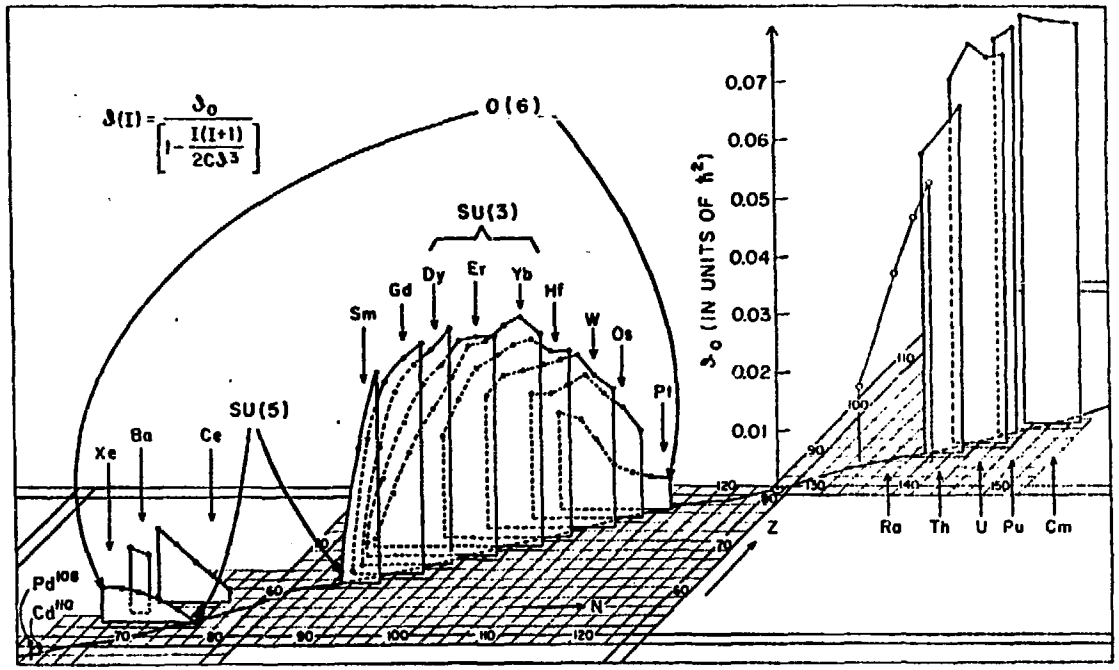

Fig. 2. Three dimensional diagram of $\mathcal{J}_0$ (computed from Eqs. 1 and 3) vs. N and Z. For nuclei corresponding to the group SU(5), $\mathcal{J}_0$ has minima, whereas maximum values occur for nuclei corresponding to SU(3). Small, but finite values characterize ^{120}Xe and $^{194,196}Pt$, the representatives of O(6).

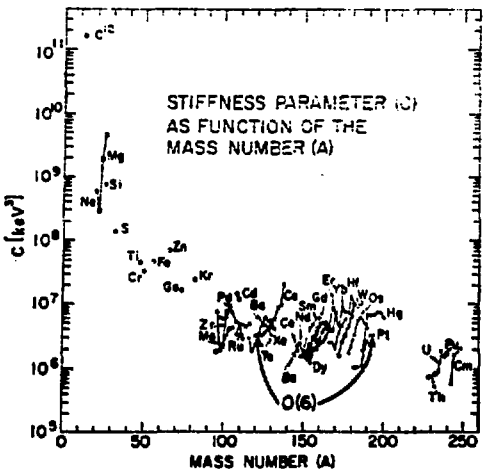

Fig. 3. Stiffness parameter C as a function of A. Points for isotopes of the same element are connected by solid lines, which peak at the most stable nuclei. For nuclei corresponding to O(6), C reaches a minimum.

rearrangement of nuclear matter. Similarly, in a ferroelectric, as $\varepsilon_0 \to 0$, no externally imposed field can produce an internal field in the solid. In the van der Waals case, if the attractive force vanishes, i.e. $a = 0$, the critical temperature $T_c = 0$. Hence it would take infinite pressure to liquefy the gas.

A discussion of phase transitions within ground state bands ("backbending") in comparison with phenomena in condensed matter systems has to be omitted because of lack of space. In all three systems the physical phenomena result from the competition between two opposing tendencies, which are exactly in balance at the critical point: In the van der Waals case these are the repulsive and attractive regions of the intermolecular forces, while the pressure acts as an external influence on the system. In the ferroelectric case these tendencies refer to the polarization which tends to pull the crystal apart, and the elastic forces tending to restore the original shape. In the VMI model the tendency to deform the nuclear shape vies with the tendency to preserve the spherical shape. Since the latter is caused by the analytically rather intractable Pauli principle, no microscopic derivation of the VMI model has yet succeeded.[10]

REFERENCES

1. G. Scharff-Goldhaber, C. Dover, and A. L. Goodman, Ann. Rev. Nucl. Sci. 26:239 (1976) and references therein.
2. G. Scharff-Goldhaber, Int. Conf. on Band Structure and Nuclear Dynamics, Nucl. Phys. A347:31 (1980). This reference will be referred to as I.
3. T. K. Das, R. M. Dreizler, and A. Klein, Phys. Rev. C 2:632 (1970). The deviations of g.s. band energies according to IBM from VMI increase with increasing J. See G. Scharff-Goldhaber, J. Phys. G: Nucl. Phys 5:L207 (1979), Fig. 1b.
4. G. Scharff-Goldhaber, Phys. Rev. 103:837 (1956).
5. G. Scharff-Goldhaber and A. S. Goldhaber, Phys. Rev. Lett. 24:1349 (1970).
6. L. K. Peker et al., private communication (BNL-NCS 28090).
7. A. S. Goldhaber and G. Scharff-Goldhaber, Phys. Rev. C 17:1171 (1978).
8. See Ref. 5, Fig. 1
9. G. Scharff-Goldhaber and M. Dresden, Transactions of the N. Y. Academy of Sciences, Ser. II, 40:166 (1980).
10. B. Buck, L. C. Biedenharn, and R. Y. Cusson, Nucl. Phys. A317:205 (1979).