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Effect of Alpha Particles on Toroidal Alfvén Eigenmodes

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Abstract

An overview is given of the analytic structure for the linear theory of the Toroidal Alfvén Eigenmode (TAE), where multiple gap structures occur. A discussion is given of the alpha particle drive and the various dissipation mechanisms that can stabilize the system. A self-consistent calculation of the TAE mode, for a low-beta high-aspect-ratio plasma, indicates that though the alpha particle drive is comparable to the dissipation mechanisms, overall stability is still achieved for ignited ITER-like plasma. A brief discussion is given of the nonlinear theory for the TAE mode and how nonlinear alpha particle dynamics can be treated by mapping methods.

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I. Introduction

Tokamaks have achieved plasma confinement properties that are close to energy breakeven conditions.[1,2] Hence it is now realistic to assess the quality of high energy particle containment that is to be expected in fusion ignition experiments, such as ITER,[3] and Ignitor.[4] In these experiments a significant fraction of the stored kinetic energy will be in the fusion produced high energy alpha particle component, and to achieve ignition it is essential that the alpha particle energy is available to directly heat the plasma background so that fusion ignition is achieved and the burn sustained.

Recently, there has been considerable theoretical[5-10] and experimental activity[11,12] in trying to understand the role of Alfvén waves in the containment of alpha particles. In a tokamak ignition experiment the alpha birth speed is expected to be comparable, though somewhat larger, than the Alfvén speed. Further, because of the high alpha particle energy, the diamagnetic frequency of the alpha particles, the alpha particle drift frequency $\omega^*_{\alpha 0} \approx mc\, E_{\alpha 0}(\partial \ell n\, n_\alpha / \partial r)/2 e_\alpha B$ ($n_\alpha \equiv$ alpha particle density, $E_{\alpha 0}$ birth energy of alpha particles, $e_\alpha \equiv$ alpha charge, $B \equiv$ modulus of magnetic field, m the poloidal mode number) can readily exceed the Alfvén frequency, $\omega \approx v_A/2qR_0$. (q being the local value of the safety factor, R_0 the major radius and v_A the Alfvén speed.). Under this condition the universal instability's free energy mechanism of the alpha particles is available to excite Alfvén waves and thereby cause enhanced alpha particle diffusion. In the worst case, alpha particles would be rapidly lost to the edge, thereby not allowing their energy to directly heat the background plasma.

A number of theories have been developed to study this problem quantitatively. Several aspects to the theory are needed: (1) a linear analysis, (2) a description of the alpha particle interaction with the Alfvén wave[13] as well as the background dissipation mechanisms,[14-17] (3) a nonlinear theory to quantify the anomalous loss.[18,19] Here we will give an overview of the state of these theories. First we will present a brief background for describing Alfvén waves in large-aspect ratio tokamaks. In particular, we discuss the singular structure of the equations when $\omega^2 = k_{m,n}^2 v_A^2$, where $k_{m,n}^2 = (n - m/q)^2/R_0^2$. Understanding this structure enables one to construct a toroidal coupling theory at large-aspect ratio where a harmonic pair, m and $m+1$ strongly interacting at a local position r where $k_{m,n}^2(r) = k_{m+1,n}^2(r)$. This occurs at $r = r_m$ where $q = (m + 1/2)/n \equiv q_m$, and it excites the mth TAE "couplet." This interaction also causes the m-th harmonic to interact with its other resonant harmonic neighbor $m - 1$ at $q(r) \approx q_{m-1} \equiv (m - 1/2)/n$, while the $m + 1$ mode interacts with its other resonant neighbor $m + 2$ at $q_m(r) \approx q_{m+1} \approx (m + 3/2)/n$. To a good approximation, no other toroidal interactions of the m and $m + 1$ mode is needed. The result is that the eigenmode can be described by a set of amplitudes $C_m^{\pm}$ (which we call fluxes) that determine the structure of the entire eigenmode.

For many cases we need only obtain the set of fluxes $C_m^{\pm}$, to determine the stability properties of the system. For the mth harmonic, the mode amplitude peaks, with a known form, at $q(r) \approx q_m$ and $q(r) \approx q_{m-1}$, from which the dissipation and alpha particle drive can be expressed in terms of the mode amplitudes $C_m^{\pm}$. Using perturbation theory, one can then determine when the wave energy of the eigenmode is driven unstable by the alpha particles in the presence of appropriate dissipation mechanisms

The relatively simple form for the eigenmode in terms of the fluxes also allows for a nonlinear description of alpha particle dynamics in the presence of a TAE wave. A mapping method[18] has been developed to understand the nonlinear particle orbit interaction. An analysis of the map enables one to determine the level of mode saturation[19] and to determine if the alpha particles orbits are stochastic.

II. Alfvén Continuum Resonance

In a cylindrical plasma, the reduced MHD equation at low beta for the mode amplitude $\phi_{m,n}$, takes the form,

$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{v_A^2}-k_{m,n}^2(r)\right)\frac{d\phi_{m,n}}{dr}\right]-r(m^2-1)\left(\frac{\omega^2}{v_A^2}-k_{m,n}^2(r)\right)\phi_{m,n}$$

$$+\ r^2\omega^2\frac{d}{dr}\left(\frac{1}{v_A^2}\right)\phi_{m,n}=0\ . \tag{1}$$

This equation is singular at points where the coefficient of the highest derivative vanishes. From a limited point of view, the MHD equation cannot be inverted through the points $r = r_s$ where $\omega^2 = k_{m,n}^2(r_s)v_A^2(r_s)$; for an increasing $q(r)$ profile there can be no more than two such points for a given ω. However, by introducing causality arguments, these singular points can be interpreted as "dissipation" poles (similar to what arises in conventional Landau damping) that give rise to a predictable damping rate that is independent of the detailed mechanism for damping.[20] Hence if an oscillator is applied at a real frequency ω_0, or an Alfvén wave is excited by alpha particles at a frequency, ω_0, damping from the Alfvén wave continuum occurs at points r_s where $k_{m,n}^2(r_s)v_A^2 = \omega_0^2$. Equation (1) is then integrated with $\mathrm{Im}\,\omega > 0$ but infinitesimally small, and it follows that the flux $C_m \propto r^3(\omega^2/v_A - k_{m,n}^2(r))d\phi_{m,n}/dr$ is slowly varying near $r = r_s$, while $\phi_{m,n}(r)$ exhibits a rapid jump. This case leads to dissipation.

When toroidal coupling is considered, special conditions can be fulfilled that destroy the damping mechanism of the Alfvén wave continuum and allows a nonsingular response from the plasma with the possibility of additional Alfvén wave mode structure. In particular, when one satisfies a degeneracy condition $\omega^2 \approx k_{m,n}^2\,v_A^2 \approx k_{m+1,n}^2\,v_A^2$, dissipation need not occur when there is toroidal coupling. This is illustrated in Fig. 1 where the resonance frequency as a function of position is plotted for a particular q-profile and n-value. The dotted lines are the resonance points neglecting toroidal coupling, whereas the solid lines indicate how the resonance changes with toroidal coupling. We see that "gaps" form in the resonant structure about a frequency $\omega = \omega_m = v_A(r)/2q_m R_0$ as well as at other frequencies. At finite toroidicity, external modes driven at frequencies that thread the gaps will not produce damping near $r \approx r_m$, as they would without toroidal coupling. In addition, self-excited modes become possible for each gap. When many gaps align, these self-excited modes can themselves interact, causing a global eigenstructure with excitations at several gaps; if the frequency goes through resonance at some remote resonant structure, residual damping results.

III. Outline of Model for TAE Mode

The procedure for obtaining the overall perturbed electric field potential Φ, which can be written as $\Phi(r,t) = \Sigma_m \Phi_m(r)\exp[-i(m\theta - n\phi + \omega t)]$, is as follows. Near each Alfvén singularity, the individual poloidal components experience a rapid change in amplitude $\Delta\Phi_m$, whereas the "flux" quantity $C_m(r) \approx r^3(\omega^2/v_A^2 - k_{\|m}^2)(d/dr)(\Phi_m/r)$ is slowly varying spatially. Thus, if the values of $C_m(r)$ are known at the degenerate singular points (call them C_{m+1}^+ and C_m^- at $r = r_m$), the basic form of the eigenfunction is determined within the inner layer. For example, in ideal theory, in the large-n, low-beta limit (with similar results for finite n), the functional form is found to be,

$$\frac{d\Phi_m}{dy} = \frac{(y+g_m)C_m^+ + C_{m+1}^-}{y^2+1-g_m^2} \tag{2}$$

with $y = 4n[q(r) - q_m]/\hat{\varepsilon}_m$, $g_m = (\omega^2/\omega_m^2 - 1)/\hat{\varepsilon}_m$ and $\hat{\varepsilon}_m = 5r_m/2R$. If $g_m < 1$, Eq. (2) is nonsingular for real frequencies, whereas if $g_m^2 > 1$, Eq. (2) can be integrated through the singularity with the use of causality (which formally treats $\mathrm{Im}\,\omega > 0$), to obtain

$$\Delta\Phi_m^+ = \lim_{\delta\gg 1, \hat{\varepsilon}\delta \ll 1} [\Phi_m(r_m + \hat{\varepsilon}\delta) - \Phi_{m+1}(r_m - \hat{\varepsilon}\delta)] = -\pi\left(\alpha_m^+ C_m^+ + \beta_m^+ C_{m+1}^-\right)$$

$$\Delta\Phi_{m+1}^- = \pi\left(\alpha_{m+1}^- C_{m+1}^- + \beta_{m+1}^- C_m^+\right) \tag{3}$$

with $\alpha^-_{m+1} \cong \alpha^+_m = -g_m(1-g_m^2)^{-1/2}$ and $\beta^-_{m+1} \cong \beta^+_m = -(1-g_m^2)^{-1/2}$, where exact equality holds at high n.

These jumps $\Delta\Phi_m$ need to be matched to the solutions of the cylindrical problem away from the singular points. For example, for the mth mode with two singular points at $q \approx q_m$ and $q \approx q_{m+1}$, the solution can be written as

$$\Phi_m(q) = \begin{cases} C^+_m\,\phi^{(b)}_m(q) & q > q_m \\ C^+_m\,\phi^-_m(q) + C^-_m\,\phi^+_m(q)\ , & q_{m-1} < q < q_m \\ C^-_m\,\phi_m\,\phi^{(a)}_m(q) & q < q_{m-1}\ . \end{cases}$$

Here $\phi^{(b)}_m(q)$, $\phi^{\pm}_m(q)$, $\phi^{(a)}_m$ are the solutions to Eq. (1), in the regimes specified, with $\phi^{(b)}_m(q)$ satisfying the appropriate boundary condition at the plasma edge, $\phi^{(0)}_m(q)$ satisfying the regularity condition at the origin, and $\phi^-_m(q)$ and $\phi^+_m(q)$ being regular at $q \simeq q_{m-1}$ and $q = q_m$ respectively. Near the resonances we have

$$\phi^{(b)}_m(q) \longrightarrow \ell n\left[n(q-q_{m-1}) + \pi\Delta^{(b)}_m\right]\ ,\ q \to q_{m-1}$$

$$\phi^{(a)}_m \longrightarrow \ell n\left[n(q-q_m) + \pi\Delta^{(a)}_m\right]\ \ q \to q_m$$

$$\phi^{\pm}_m(q) \longrightarrow \begin{cases} \ell n\left[n(q-q_{m-1/2\mp 1/2})\right] + \pi\Delta^{\mp}_m\ ,\ q \to q_{m-1/2\mp1/2} \\ \pi\tilde{\Delta}^{\pm}_m\ ,\ q \to q_{m-1/2\pm1/2}\ , \end{cases} \tag{4}$$

where the various Δ terms are determined by the properties of Eq. (1). The jumps in Φ_m from the cylindrical solutions can then be matched to the jumps in Φ_m from the toroidal solutions. The result of the matching leads to a 3-term recursion relation[9] for the mode amplitudes C^+_m:

$$C^+_m\left[\overline{\Delta}^+_m + \alpha^+_m - \frac{\beta^+_m\,\beta^-_{m+1}}{\overline{\Delta}^-_{m+1} + \alpha^-_{m+1}} - \frac{\tilde{\Delta}^+_m\,\tilde{\Delta}^-_m}{\overline{\Delta}^-_m + \alpha^-_m}\right]$$
$$+\frac{\beta^+_m\,\tilde{\Delta}^-_{m+1}\,C^+_{m+1}}{\left(\overline{\Delta}_{m+1} + \alpha^-_{m+1}\right)} + \frac{\beta^-_m\,\tilde{\Delta}^+_m\,C^+_{m-1}}{(\overline{\Delta}_m + \alpha^-_m)} = 0 \tag{5}$$

where $\overline{\Delta}^+_m = \Delta^{(b)}_m - \Delta^-_m$ and $\overline{\Delta}^-_m = \Delta^{(a)}_m - \Delta^+_m$, and $C^-_m = \left(\tilde{\Delta}^-_m\,C^+_m - \beta^-_m\,C^+_{m-1}\right)/(\overline{\Delta}^-_m + \alpha^-_m)$.

The solution of the three-term recursion relation given by Eq. (5) then determines the overall eigenmode structure. A typical example is given in Figs. 2 and 3 for the eigenvalues (obtained for $n = 3$) as well as the relative values of the flux amplitudes $C_m^{\pm}$.

IV. Marginal Stability Calculation

Having determined the mode amplitudes, the growth rate can be determined perturbatively by the relation

$$\gamma = \frac{\mathrm{Re} \int d^3r \, \mathbf{E}^* \cdot \mathbf{j}}{2 \int d^3r |B_\theta|^2/4\pi}$$

where $\int d^3r|B_\theta|^2/4\pi$ is the wave energy of an Alfvén wave for which the field energy and kinetic energy are nearly in balance and in the dominant regions of excitation $B_\theta \gg B_r$. The wave energy, expressed in terms of the C_m's is given by

$$\int d^3r \, \frac{|B_\theta|^2}{4\pi} \propto \sum_n \frac{nr_m^3 \, q'(r_m)}{v_A^2(r_m)\hat{\varepsilon}_m} \frac{(\hat{\Phi}_m^2 + \hat{\Phi}_{m+1}^2)}{(1 - g_m^2)^{1/2}} \tag{6}$$

with

$$\hat{\Phi}_m^2 = |C_m^+|^2 + |C_{m+1}^- + g_m \, C_m^+|^2 \Big/ (1 - g_m^2)$$

$$\hat{\Phi}_{m+1}^2 = |C_{m+1}^-|^2 + |C_m^+ + g_m \, C_{m+1}^-|^2 \Big/ (1 - g_m^2) \; .$$

The power transfer due to kinetic processes from the background plasma and alpha particles is also calculated perturbatively, given the eigenfunction. For the alpha particles, one needs to take into account that their orbit width is appreciable compared to the width of a typical TAE mode $\delta r \sim \frac{\hat{\varepsilon}/4}{nq'(r_m)}$. An evaluation of the alpha particle drive in the large-aspect-ratio limit, with the orbit excursion

$$\Delta_b \equiv q(r_b) \, v_{\|} \Big/ (1 + v_\perp^2/v_\|^2)\omega_{c\alpha} < \frac{1}{nq'(r_m)} \; ,$$

($\omega_{c\alpha} = c_\alpha \, B/m_\alpha \, c$) has been performed with the use of the universal form of the eigenmode given by Eq. (2). The result from one TAE structure (for $\omega_\alpha^* \gg \omega$) for a large-aspect ratio tokamak[18]

$$\mathrm{Re} \int d^3r(\mathbf{E}^* \cdot \mathbf{j}_\alpha) = -\frac{2r_m^2\,\pi^3\, ce_\alpha}{B\omega\, Rq_m^2} \int dr\, d^3v\, \frac{\partial f_\alpha}{\partial r} \sum_{-\infty}^{\infty}(m+\ell)\delta\left(\omega - \frac{v_{\|}(nq-m-\ell)}{Rq}\right)$$

$$|\ell\phi_{m,l} + (\ell-1)\phi_{m+1,\ell-1}|^2$$

where

$$\ell\phi_{m,\ell}(r) = \ell\int_0^\infty \frac{d\theta}{2\pi}\cos\ell\theta\,\Phi_m(r+\Delta_b\cos\theta) \propto \left[C_m^+ + i\left(C_m^+ g_m + C_{m+1}^-\right)\right] z^\ell$$

$$z = -(x+iy) + \left((x+iy)^2 - 1\right)^{1/2}\ , \quad x = (r-r_m)/\Delta_b\ , \quad y = \frac{\hat{\varepsilon}(1-g_m^2)}{4n\, q'(r_m)\Delta_b}$$

and the branch of the square root must be chosen so that $|z| < 1$.

In general, $\int d^3\mathbf{r}\ \mathbf{E}\cdot\mathbf{j}$ should be calculated from the kinetic contribution for each species. The general result has the form,

$$\gamma = -\gamma_c + \int d^3r\,\mathbf{E}\cdot\mathbf{J}\Big/2\int d^3r|B|^2/4\pi = -\gamma_c + \sum_{m,j}\gamma_{m,j}\,\lambda_m\Big/\sum_m \lambda_m\ , \qquad (7)$$

where $-\gamma_c$ is the damping rate of the zeroth-order problem, $\lambda_m = \lambda_m(C_m, C_{m+1})$ is a quadratic form in the wave amplitudes (related to the C_m's) with $\Sigma_m \lambda_m = \int d^3r|B|^2/4\pi$ the wave energy, and $\gamma_m^{(j)}$ is the growth rate that would arise due to the jth instability source if only a single resonance pair of poloidal harmonics are excited. In Eq. (7) only the inner region field structure, given by Eq. (1) with self-consistent values for $C_m^\pm$, is needed. The various contributions to $\gamma_m^{(j)}$ are as follows:

1. The destabilizing contribution, $\gamma_m^{(\alpha+)}$, from the alpha particle spatial profile gradient: For the important case when $\hat{\varepsilon}/4n\,q'(r_m) < \Delta_b < \frac{1}{nq'(r_m)}$, this contribution is given approximately by

$$\frac{\gamma_m^{(\alpha+)}}{\omega_m} = 5q_m^2(1-g_m^2)^{1/2}\,r_m\left(\frac{d\beta_\alpha}{dr}\right)\left(\frac{v_A}{v_{\alpha 0}}\right)\left(1-\frac{v_A^2}{v_{\alpha 0}^2}\right) H\left(1-\frac{v_A}{v_{\alpha 0}}\right)\ . \qquad (8)$$

Note that $\gamma_m^{(\alpha+)}$ is independent of the mode number n. If $v_A < v_{\alpha 0}$ (the alpha birth velocity), there are additional, smaller contributions. [For numerical calculations, a more general formula for $\gamma_m^{(\alpha+)}$ is used, which, however, is restricted to $\Delta_b < 1/nq'(r_m)$.]

2. Alpha particle damping, $\gamma_m^{(\alpha-)}$, from the energy gradient of a slowing-down distribution: Roughly, this is given by $\gamma_m^{(\alpha-)} = -\gamma_m^{(\alpha+)}(\omega/\omega_{*\alpha})$ and is small for large-n .

3. Ion Landau damping, $\gamma_m^{(i)}$, arising from the alpha particle interaction with the magnetic curvature at the wave-particle sideband resonance $v_{\|} = v_A/3$: This is given by[14]

$$\frac{\gamma_m^{(i)}}{\omega_m} = -\sum_j \left(\frac{n_j}{n_e}\right) \frac{\sqrt{\pi}}{\beta_j^{3/2}} \frac{m_j}{\langle m_j \rangle} \frac{q_m^2}{3^5} \exp\left(-\frac{m_j}{9\langle m_j \rangle \beta_j}\right) \tag{9}$$

where the j-summation is over deuterium ($j = d$) and tritium ($j = t$) ion species, with the brackets designating a density average, e.g., $\langle m \rangle = (n_d\, m_d + n_t\, m_t)/n_e$.

4. Collisional electron damping due to magnetic curvature, $\gamma_m^{(e,c)}$ and parallel electric field, $\gamma_m^{(e,\|)}$, effects[15,16] (the latter is here newly calculated by Rosenbluth for the TAE mode structure): The combined result for $\gamma_m^{(e,c)} + \gamma_m^{(e,\|)}$ is given by

$$\left[\gamma_m^{(e,c)} + \gamma_m^{(e,\|)}\right]/\omega = -0.5\left\{6.7\beta_e\, q_m^2 + \frac{16m^2\, S_m^2\, \rho_s^2}{\bar{\varepsilon}^2(1-g_m^2)r_m^2}\right\} \frac{\left(\frac{\nu_e}{\omega_m}\right)^{1/2}}{[1+(\nu_e\, r_m/\omega_m\, R)^{3/2}]}$$

$$\times\, \frac{1}{\left[1.4 + .25\ln\left(1+\frac{\omega_m r}{R_0\, \nu_e}\right)\right]^{3/2}} \tag{10}$$

with $\nu_e = 4\pi\, n_e\, e^4 \ln \Lambda/m_e^{1/2}(2T_e)^{3/2}$. Note that $\gamma_m^{(e,\|)} \propto -m^2$, so that this contribution can cut off high-mode-number instabilities.

5. Dissipation due to ideal continuum mode damping: This is also part of γ_c and is discussed in Refs. 8-10.

6. Recently it has been noted that damping from radiation of kinetic Alfvén waves is significant.[21] A formula for the damping rate, that can be derived from a formalism developed by Rosenbluth,[22] gives for the nonideal damping, $\gamma^{(ni)}$,[17]

$$\frac{\gamma^{(ni)}}{\omega_m} = -\frac{\hat{\varepsilon}(1 - g_m^2)^{3/2}}{2} \exp\left(-f(g_m)\tau_m^{-1/2}\right)$$

where

$$f(g) = \int_0^{(1+g_m)^{1/2}} dk \left[1 - (k^2 - g_m)^2\right]^{1/2}$$

and

$$\tau_m = \left(8n^2 S_m^2 q_m^2 / \hat{\varepsilon} r_m^2\right) \left[3\rho_i^2/4 + \rho_s^2\right] ,$$

is the parameter, which when small, allows the ideal MHD theory to be applicable. Here ρ_i is the ion Larmor radius, and $\rho_s = \rho_i(T_e/T_i)^{1/2}$.

Equation (7) was evaluated numerically, with self-consistent eigenvalues and eigenfunctions. A self-consistent profile based on fusion cross-sections with $T_e = T_i$ was used for the alpha particles. Typical results, shown in Fig. 4, indicate, that for reactor-type parameters, our model predicts TAE stability when $T_e < 25\,\text{keV}$. The tendency for strong stabilization at low-plasma beta occurs because v_A becomes larger than $v_{\alpha 0}$ and have the principal resonances are then not expected, whereas the stronger stabilization tendency at higher beta values occurs because ion Landau damping becomes significant. At high temperatures the dissipation due to radiation of kinetic Alfvén waves is important in achieving stabilization. However, it is important to note that the alpha particle drive is competitive with dissipation mechanisms. Hence, modifications of the theory due to geometry, can conceivably alter the present stability result.

V. Nonlinear TAE Behavior

The nonlinear evolution of the alpha particle distribution has been analyzed[19] as a special case of the problem of a distribution function with a weak beam-like source. The latter is generic to many plasma situations and can be considered in the context of the paradigm of the bump-on-tail instability, as well as for the more complicated alpha particle-TAE wave interaction. The answer to the question of whether unstable waves gives rise to spatial diffusion of alpha particles depends on whether stochasticity due to the perturbed fields develops in the particle orbits. Without stochasticity, the wave amplitude saturates at the natural level determined by when the resonant bounce frequency of a particle in the wave field becomes comparable to the linear growth rate. The amplitude of the radial bounce motion, δq, of a particle in the wave field has the dependence $\delta q \propto \delta B_r^{1/2}$, where δB_r is the magnetic field perturbation and $q(r)$, the safety factor, is used as the radial variable; in the limit $\hat{\varepsilon} r_m/4mS_m < \Delta_b < r_m/mS_m$, the explicit result is $\delta q/q_m \cong 4(2S_m R\,\delta B_r/nr_m B)^{1/2}$. For a single TAE mode, the neighboring gap resonances are separated by $\Delta q \approx 1/n$. Hence, stochasticity is achieved if $\delta q > 1/n$. If there is a spectrum of modes, however, the overlap condition can be considerably less: roughly, $\delta q \geq 1/pn$, with p the number of unstable modes that interact with the particle.

Recent experiments[11,12] have observed pulsation behavior in a steady-state, weakly beam-driven plasma. Such pulsations lead to benign oscillations if the wave saturation occurs below the stochasticity threshold, since then global alpha particle diffusion does not occur. However, if the saturation level is above the stochasticity threshold, a phase-space "explosion" is predicted,[18] in which the diffusion process itself allows the release of free energy that pumps the wave amplitude. Ultimately this causes rapid diffusion and either flattening or loss of the alpha particle distribution function, consistent with rapid particle losses observed in the TAE experiments with fast ions simulating alpha particles. These bursts occur periodically, with a relatively long quiescent phase during which a flattened alpha particle distribution builds up with benign pulsations, approximately according to classical theory, to the point where a phase space explosion occurs and the distribution again flattens.

Mapping methods[19] were developed to provide a more quantitative analysis of the nonlinear pulsations. The map to describe wave-particle interaction for one particle transit is based on linear theory, whereas the orbit nonlinearity is described by a map over many transits. Figure 5 shows examples of the transition from orbit integrability to orbit diffusion. The map is now being generalized to incorporate the dynamics of both particles and waves.

VI. Conclusions

We have indicated how a self-consistent model can be established to estimate the effect of alpha particles on Alfvén instability particularly for the TAE-mode. In both linear and nonlinear theories significant advances have occurred. Several areas where improvement is needed, and is being worked on are the following:

1. Introducing nonideal effects into the general structure of the three-term recursion relation. The appropriate generalization has been formulated by Rosenbluth.[22] Such effects are important, as recently shown by Mett and Mahajan.[21] As mentioned already; it gives significant damping by radiation of kinetic Alfvén waves and it also allows kinetic Alfvén waves to form a separate standing wave spectrum.

2. Bringing finite beta effects into the theory. This is an important element, particularly since the Alfvén phase velocity is reduced,[23] and the condition for resonance is altered. The inclusion of this effect is probably necessary to be able to quantitatively explain the experimentally observed thresholds in TFTR and D-IIID.

3. Incorporating elongated plasma cross-section into the theory. This will certainly bring into consideration the ellipticity-induced Alfvén mode (EAE).[14] One also needs to explain the gap structures and resonances in shaped experiments such as D-III-D experiment. There is a challenge to use complicated MHD codes in such a way that the continuum damping can be described as a perturbation to the zeroth-order results. A possible perturbation method has been suggested in Ref. 8.

4. Describing the effect of large orbit excursions when $\Delta_b \gtrsim 1/nq'(r_m)$ where our calculation fails. Fu and Cheng have developed a ballooning theory to treat this regime.[24] Special techniques could be developed from the point of view of our nearest neighbor interaction model. However, the result will probably be more numerically intensive than the present work.

5. More study on the character of the alpha particle response to the kinetic Alfvén waves is needed.

Our present results predict stable fusion regimes for temperatures less than 25 keV. However, as the drives for instability are close to the dissipation rates, the changes that arise from including more realistic geometry effects will need to be correctly assessed in the future. Nonetheless, it is encouraging that stability is predicted in this large-aspect ratio low-beta theory.

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Figure Captions

1. Toroidally coupled shear-Alfvén continuum resonance curves for the normalized frequency $\omega R/v_{A_0}$ as functions of the normalized minor radius r/a, for $n = 5$, $q(0) = 1.0, q(a) = 2.5$, and $a/R = 0.25$. The m values indicate the dominant mode number in the regions where toroidal coupling is negligible. The dotted curves are the resonant curves without toroidal coupling.

2. Continuum resonance curves (only the tips of the curves near the gaps are shown) and normalized TAE eigenfrequencies $\omega R/v_{A_0}$, numerically obtained for a typical density profile, with $n = 3$, $q(0) = 1$, $q(a) = 3$, and $a/R = 0.2$. The respective eigenfrequencies and their associated damping rates (imaginary part) are A: $(0.52, -4.0 \times 10^{-2})$, B: $(0.40, -8.5 \times 10^{-6})$, C: $(0.35, -1.1 \times 10^{-2})$, D: $(0.34, -3.4 \times 10^{-3})$, E: $(0.31, -4.7 \times 10^{-3})$, and F: $(0.27, -4.0 \times 10^{-3})$.

3. Harmonic content $|C_\ell|^2$ of the global TAE mode associated with each of the discrete eigenfrequencies shown in Fig. 2, for each sideband number ℓ.

4. Ratio of the power transferred from all bulk plasma dissipation mechanisms to the α-particle destabilizing power transfer as a function of the plasma central β value, with various plasma temperatures (10 to 25 keV), for an $n = 3$ TAE. Stability occurs because this ratio exceeds unity.

5. Surface of section of seven particle orbits plotting the q-position vs. poloidal angle. The mode amplitude is 1/2 the overlap criterion for the onset of orbit stochasticity. Appreciable stochasticity is apparent, but the diffusion is still not global, and this case will not produce appreciable anomalous particle loss.

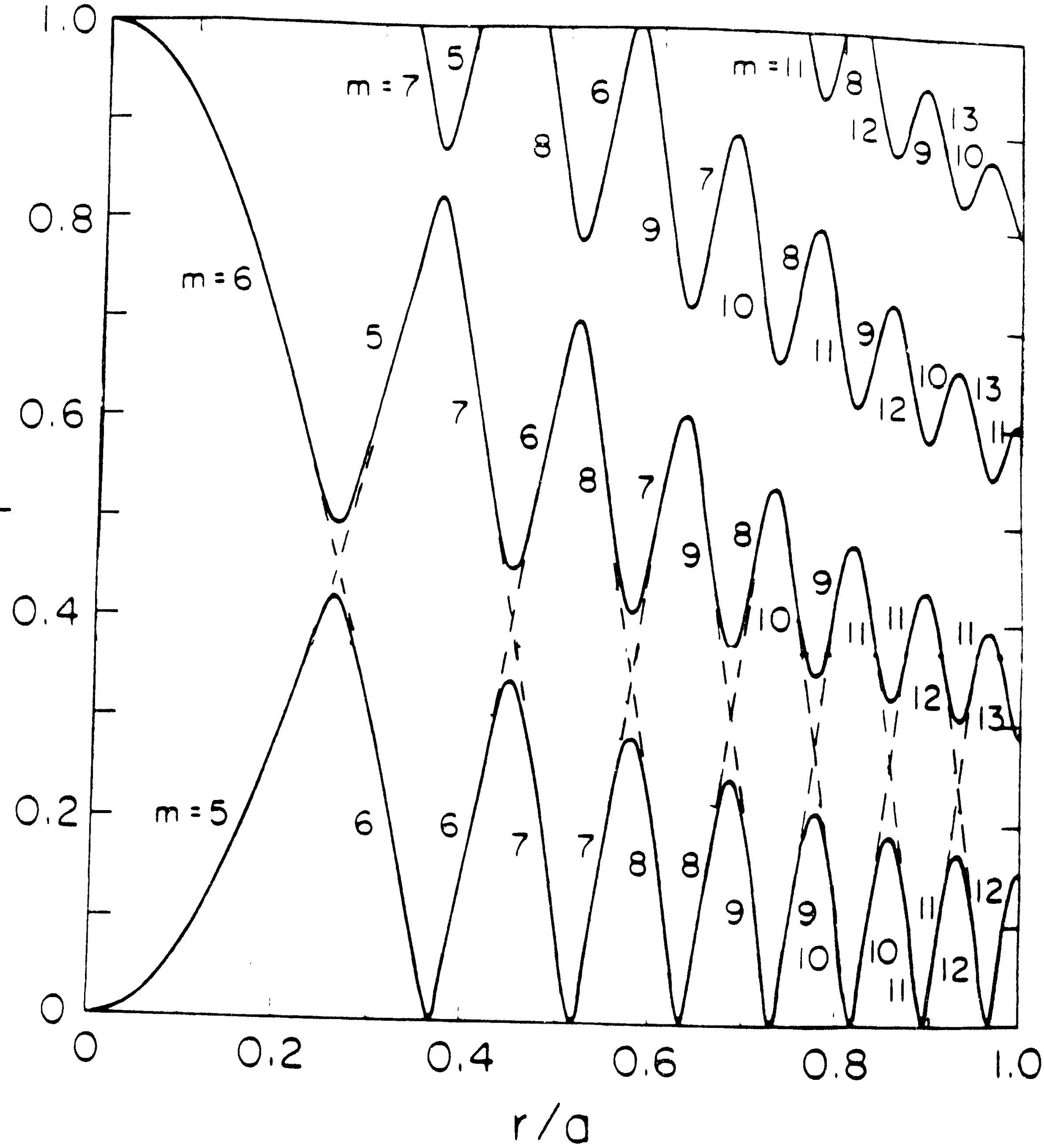


Fig. 1

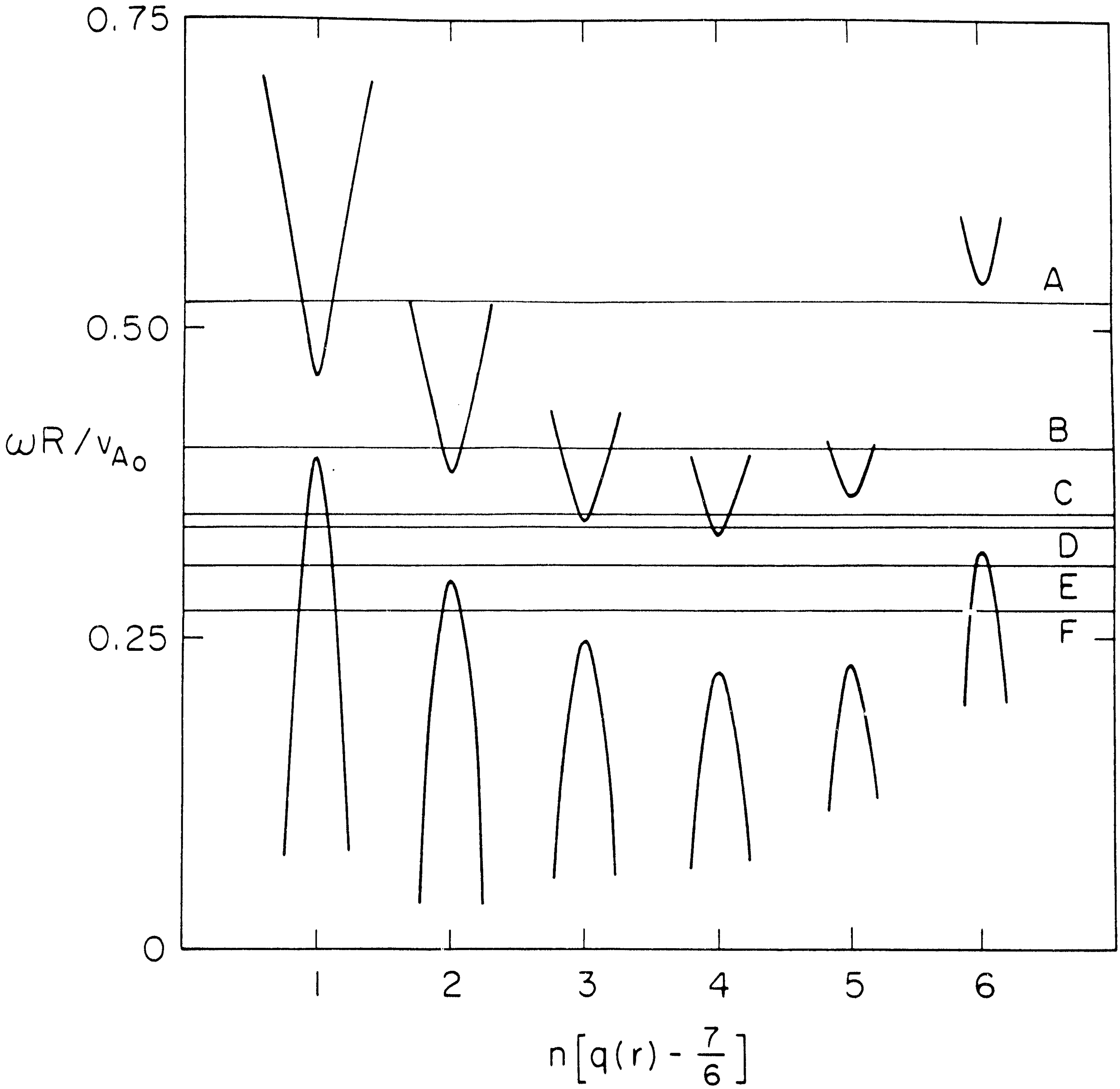


Fig. 2

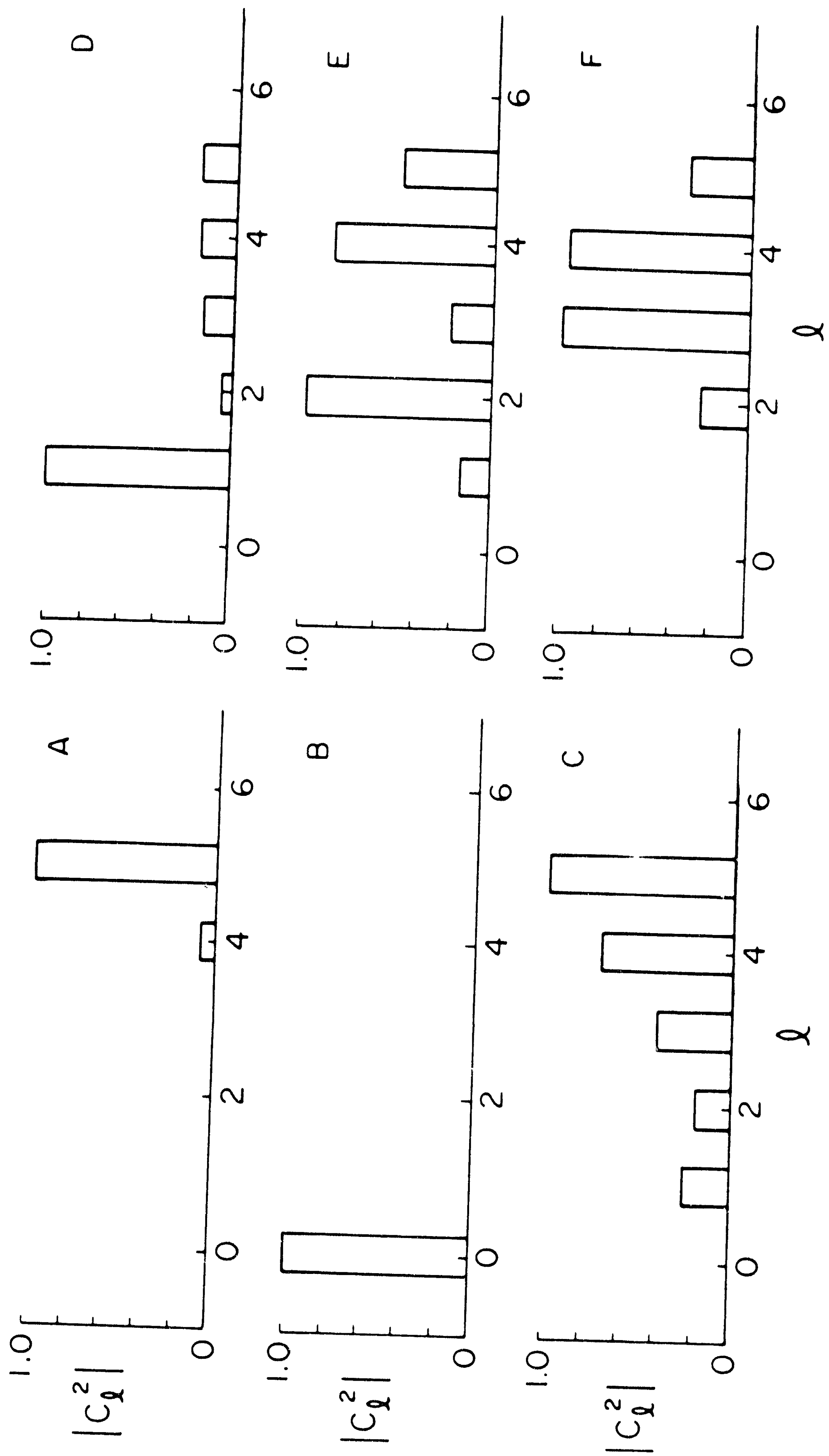
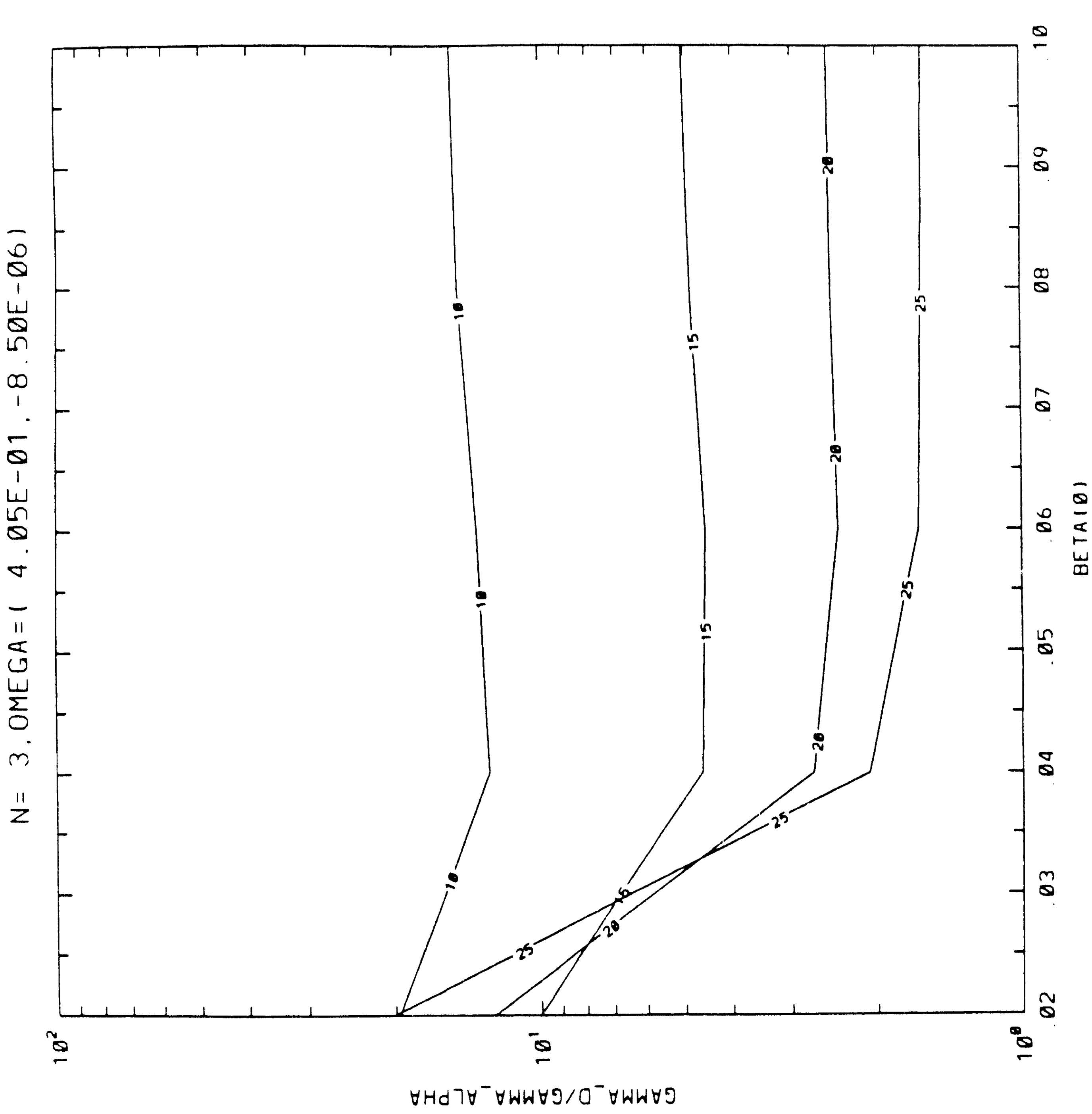


Fig. 3



N= 3, OMEGA=(4.05E-01,-8.50E-06)
GAMMA_D/GAMMA_ALPHA
BETA(0)
.02
.03
.04
.05
.06
.07
.08
.09
.10
10^2
10^1
10^0
10
15
20
25

Fig. 4

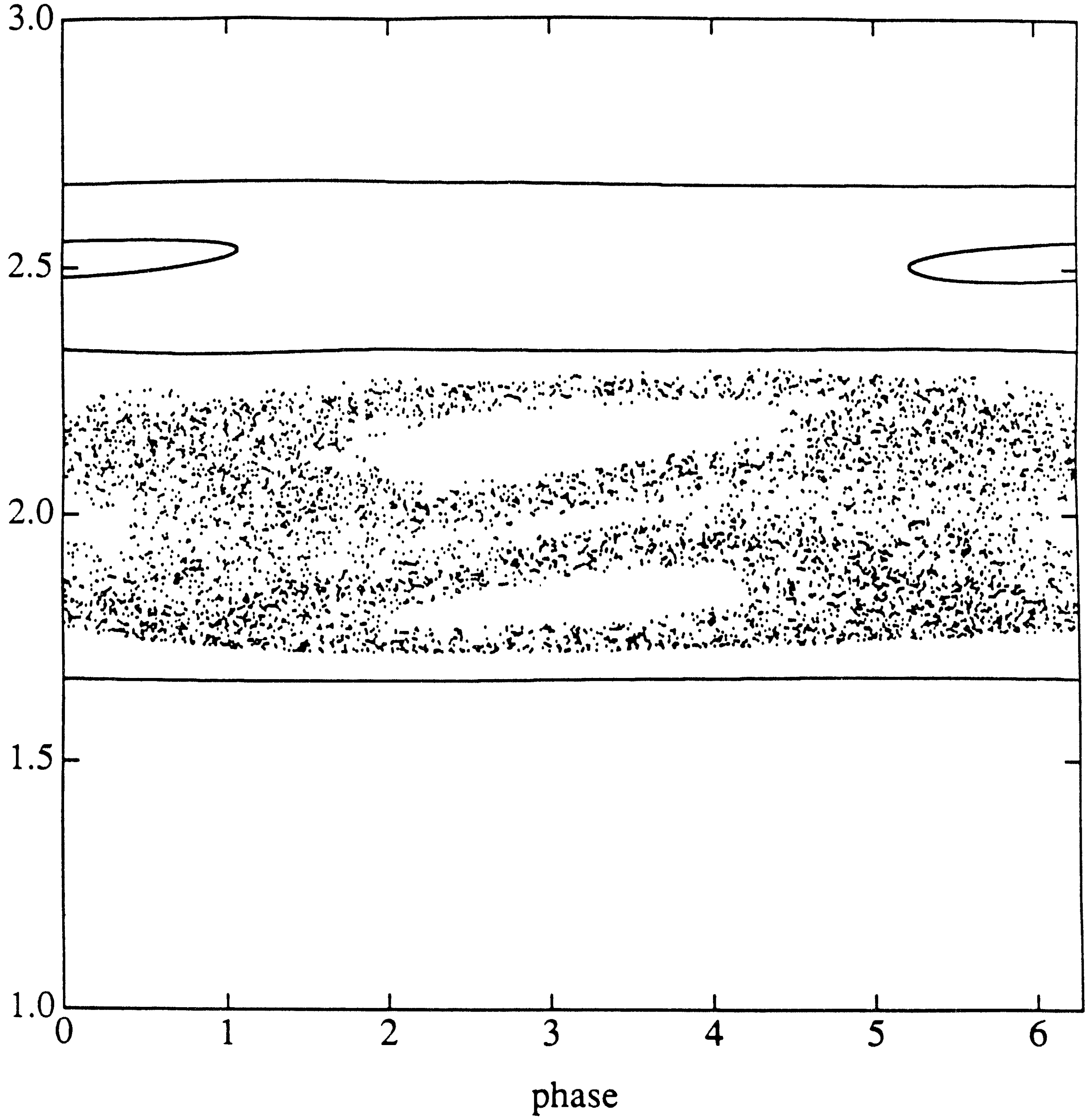


Fig. 5

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