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SHELL MODEL TESTS OF THE INTERACTING BOSON MODEL DESCRIPTION OF NUCLEAR COLLECTIVE MOTION*

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The IBM assumes that collective behavior arises from the coupling, through the neutron-proton interaction, of the separate low-lying states systems of valence protons and neutrons defined with respect to a major shell closure. The eigenstates of the proton (neutron) systems are assumed to be constructed purely from combining two-particle "bosons" with L = 0 and L = 2 to form many-particle states. The model is capable of handling nuclear systems which are far beyond the domain of applicability of any reasonably complete shell-model calculation. It is the purpose of this report to present the results of a large shell-model calculation of pseudo-nucleus which displays striking collective behavior suggestive of rotational phenomena. In these calculations, a specific and physically reasonable single-particle structure is given to the wave functions, and an explicit two-body residual interaction is used. An analysis of the shell model wave functions of the eigenstates of the resulting K = 0 rotational bands offers strong support to a primary assumption of the IBM; i.e., wave functions which describe collective behavior can be constructed from many-particle states of valence nucleons which are constructed only from J = 0 and J = 2 two-particle states.

The calculations here were suggested by earlier calculations of Hecht et al.[1] The model space for the calculations reported here includes the $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ proton single-particle orbits, and the $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, and $2s_{1/2}$

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neutron single-particle orbits. The single-particle levels are taken to be degenerate. For a reasonably large number of particles, one-body spin-orbit effects should be minimal. For the residual shell-model Hamiltonian, the surface delta interaction[2] (SDI) is used with equal strengths in the p-p, p-n, and n-n systems. The SDI is known to be a useful approximation to more empirical and/or more realistic interactions. This model space is still a very large one. A truncation scheme is used here as follows. The Hamiltonian can be written in obvious notation, as

$$H = H_{pp} + H_{nn} + H_{pn}.$$

H_{pp} and H_{nn} are diagonalized exactly in the complete proton and neutron model spaces. The basis space of the truncated n-p-model is formed by coupling together a selected set of eigenstates of the neutron and proton systems; i.e.,

$$|\psi^{i,j}_{p,n}\rangle = |\psi^{i}_{p}\rangle \times |\psi^{j}_{n}\rangle.$$

The justification for selecting these particular eigenstates is discussed in some detail in Hecht <u>et al</u>. The reasoning is reviewed briefly here. For the SDI in the n-p basis space used here

$$\langle\psi^{i,j}_{p,n}|H^{SDI}_{p,n}|\psi^{\ell,m}_{p,n}\rangle \approx \sum_{k} \langle\psi^{i}_{p}||Q^{k}_{p}||\psi^{\ell}_{p}\rangle\cdot\langle\psi^{j}_{n}||Q^{k}_{n}||\psi^{m}_{n}\rangle$$

where Q^{k}_{m} is proportional to the spherical harmonic Y^{k}_{m}. It is demonstrated in Hecht <u>et al</u>. that for the set of neutron and proton states listed in Table I, the matrix elements of the surface harmonics, Q^{k}_{m}, between states within the set and states not in the set are very small. Thus, the eigenstates resulting from diagonalizing $H_{p,n}$ in the truncated basis we use are essentially uncoupled to exact eigenstates of H which have significant admixtures of proton or neutron states not included in our model space. In the truncation scheme used here the maximum dimension of any matrix model is 130, as compared to 45,000 in the full space.

Hecht et al. introduce the concept of a "favored pair". A favored pair is an eigenstate of the SDI of a system of two identical particles distributed in all the (degenerate) orbits of a complete oscillator shell with S = 0, L = J, and with a non-zero eigenvalue. There is only one such eigenstate for each even value of L up to the maximum possible L-value for a two-particle state in the given model space. All other two-particle eigenstates are degenerate, with E = 0. Thus, in the (5/2,3/2,1/2) space, there are favored pairs with J = 0, 2, and 4. In the (7/2,5/2,3/2,1/2) space there are favored pairs with J = 0, 2, 4, and 6. Hecht et al. show that the four-particle eigenstates listed in Table I, and included in the truncated model space here, can be constructed essentially completely from coupling of two favored pair states. In the IBM model, the many-particle states are formed from J = 0 and J = 2 identical-particle pairs. In the calculations discussed here, I include J = 4 and J = 6 pairs in addition to the J = 0 and J = 2 pairs. To the extent that the calculations reported here are a good approximation of a realistic shell-model calculation in a many-particle space, the importance of the J = 4 and J = 6 pairs in the many-particle space reflects the adequacy or inadequacy of the restriction to J = 0 and J = 2 pairs of the IBM.

In Fig. 1 is shown the calculated spectrum of low-lying states of a system (4 x 6) with six protons and four neutrons in the model space described above. The first column in Fig. 1 shows the spectrum of all states up to 5 MeV excitation and selected high-spin states up to 20 MeV. B(E2) values for transitions between these states have been calculated with total proton and neutron charges of 1.0e, respectively. In succeeding columns in Fig. 1, states are grouped into bands determined by observing which states are connected by strong B(E2)-values.

The last column in each figure shows the states not included in the "bands".

In Fig. 1 we see that a number of states in the (4 x 6)-system can be grouped into three bands. The "ground state" band has the level ordering and spacings very similar to the J(J+1) spacing of a rigid rotor with K = 0. The level energies of the second and third bands strongly resemble those of K = 2 and a K = 0 rotational bands, respectively. In Table II, the relative B(E2)-values for transitions within the two "K = 0^+" bands and the "K = 2^+" bands of the (4 x 6) system are shown. For comparison, the relative B(E2)-values predicted by the usual rigid rotor model for K = 0 and K = 2 bands are shown, i.e.,

$$B(E2) \alpha \left| C^{J_i 2 J_f}_{K\ 0\ K} \right|^2 .$$

There is remarkable agreement between the shell model and the rigid rotor for the K = 0^+_1 and the K = 2^+_1 band for states with J $\leq$ 8. There is a band crossing at the J = 8^+ level between the second K = 0^+_2 band and the K = 2^+_1 band. The J = 6^+ members of the K = 0^+_2 and K = 2^+_1 bands are almost degenerate, and this fact may be reflected in the deviations between shell model and rigid rotor for transitions involving these states. There are similar results in the system with four neutrons and four protons (4 x 4), but the rotational behavior breaks down above the J = 6 state in both bands and the second K = 0^+ band is not present in the (4 x 4) calculation. Thus, from these calculated results, one sees that there is clear evidence of rigid-rotor behavior appearing in the large calculations, considerably more striking than I have seen in previous shell-model calculations. The collective behavior is more distinct when two protons are added to go from the 4 x 4 to the 4 x 6 system. This behavior appears with a neutron-proton interaction (the SDI) which does not obviously enhance the possibility for rotational behavior.

Some analysis of the wave functions of the ground-state band of the 6 x 4 system is shown in Table III. In this table, for each spin, all basis

states admixed with coefficients greater than 0.2 are shown, and the components are listed in decreasing order of magnitude. In all cases these components account for 60% or more of the wave function amplitude. For states up to the $J = 6^+$ state, these large components are dominated by coupling of $J = 0$ and $J = 2$ two-particle bosons, as assumed by the IBM. One might argue that this results from simple perturbation theory arguments; i.e., that the $J = 0^+$ and $J = 2^+$ states lie lowest in energy. However, note that besides the $J = 0_1^+$ and $J = 2_1^+$ states, the other important states are the $J = 2_2$ state with seniority $\nu = 4$, the $J = 4_2$ state with $\nu = 4$, and the $J = 4_3^+$ state with $\nu = 6$. The low-lying $J = 4_1$, $\nu = 2$ (which must be the coupling of a $J = 0$ and a $J = 4$ pair) state plays essentially no role in the important components of the $J = 0$, 2, 4, and 6^+ levels. As shown in Hecht et al., these two $\nu = 4$ and one $\nu = 6$ states are constructed essentially from the coupling of the $J = 0^+$ and $J = 2^+$ two-particle states of the neutron and proton system. The $J = 8$ state has important components which cannot be accounted for by coupling $J = 0$ and $J = 2$ two-particle states, but many of the largest components can be so described. Note that the agreement with the rigid rotor model breaks down at the $J = 8^+$ state for the B(E2)-values. Thus, when the $J = 0^+$ and $J = 2^+$ two-particle states are not so dominant a factor, the rotational pattern is violated.

A similar analysis of the $K = 0_1^+$ band of the 4 x 4 system leads to similar conclusions. There, the "rotational behavior" and the dominance of the $J = 0^+$ and $J = 2^+$ states deteriorates after the $J = 4^+$ state in the 4 x 4 system. These results suggest that when additional two particles are added, the collective behavior extends to two higher units in angular momentum.

The $K = 0_2^+$ band in the (4 x 6) system is dominated by $J = 0$ and $J = 2$ couplings up to the $J = 4^+$ state. The states in the $K = 2$ band in both the (4 x 4) and (4 x 6) systems are much more complicated than are the states in

the $K = 0_1^+$ band. In the $K = 2$ bands, the states are not dominated by the $J = 0$ and $J = 2$ couplings. In particular, there are significant admixtures of the lowest $J = 3^+$ two-particle eigenstates.

In summary, the calculations reported here offer considerable supporting evidence for the validity of the assumptions of the Interacting Boson Model. The calculations show:

1. The coupling of key low-lying neutron and proton states of valence particles can lead to collective rotational features which appear to be more distinct and which extend to higher angular momenta as the number of particle increases.

2. Those states which are rotational are dominated by states formed by coupling $J = 0$ and $J = 2$ two-particle states.

References

1. K. T. Hecht, J. B. McGrory, and J. P. Draayer, Nucl. Phys. A197, 369 (1972).

2. I. M. Green and S. A. Moszkowski, Phys. Rev. 39B, 6390 (1965).

Table I. Eigenstates included in truncated calculations. Column headed ν shows the seniority of the state. E is the eigenvalue of the state. Column headed % (2 × 2) shows percentage of given eigenstate which is formed from coupling of J=2 two-particle states.

PROTONS				NEUTRONS			
J	ν	E	% (2 × 2)	J	ν	E	% (2 × 2)
0_1	0	-18.00		0_1	0	-20.00	
0_2	6	-14.16		0_2	4	-12.82	63
2_1	2	-15.43		2_1	2	-16.67	
2_2	4	-13.96	83	2_2	4	-14.08	92
3_1	4	-12.90		2_3	4	-11.48	10
3_2	6	-11.35		3_1	4	-12.31	
4_1	2	-13.43		4_1	2	-13.94	
4_2	4	-12.73	86	4_2	4	-13.30	93
4_3	6	-11.96		4_3	4	-11.70	7
5_1	4	-11.14		4_4	4	-10.12	2
6_1	4	-10.28		5_1	4	-10.86	
6_2	6	- 9.96		5_2	4	-10.41	
				6_1	2	-11.63	
				6_2	4	-10.38	
				6_3	4	- 8.50	

Table II. Relative B(E2)-values in four-neutron, six-proton systems. (The transitions are normalized to one for the first transition in each column. Column headed S.M. gives shell model results. Column headed R.R. gives calculated relative B(E2)'s for rigid rotor model as discussed in the text.)

"K = 0"			"K = 0"			"K = 2"		
$J_i \rightarrow J_f$	S.M.	R.R.	$J_i \rightarrow J_f$	S.M.	R.R.	$J_i \rightarrow J_f$	S.M.	R.R.
2-0	1	1	2-6	1	1	3-2	1	1
4-2	1.48	1.43	4-2	1.34	1.43	4-2	.31	.3
6-4	1.56	1.57	6-4	1.09	1.57	4-3	.72	.75
8-6	1.42	1.65	8-6	0.61	1.65	5-3	.53	.53
10-8	1.06	1.69	10-18	0.55	1.69	5-4	.51	.53
12-10	0.5	1.72	12-10	0.24	1.79	6-4	.49	.66
						6-5	.41	.39
						7-5	.70	.74
						7-6	.34	.30
						8-6	.74	.80
						8-7	.28	.24
						9-7	.60	.83
						9-8	.18	.19
						10-8	.62	.87
						10-9	.18	.16

Table III. Principle components of ground state band wave functions in six-proton, four-neutron system. The number in each column is the observed value of the admixture of the given basis state in the eigenstate. The numbers in parentheses ($J_1 \times J_2$) indicate the basis state ($J_p \times J_n$), where J_p and J_n are given in Table 1.

$J = 0^+$	$J = 2^+$	$J = 4^+$	$J = 6^+$
0.600 ($0_1 \times 0_1$)	0.458 ($2_1 \times 0_1$)	0.454 ($2_1 \times 2_1$)	0.401 ($2_1 \times 4_2$)
0.594 ($2_1 \times 2_1$)	0.424 ($0_1 \times 2_1$)	0.320 ($4_2 \times 0_1$)	0.391 ($4_2 \times 2_1$)
0.229 ($4_2 \times 4_2$)	0.275 ($4_2 \times 2_1$)	0.313 ($2_1 \times 2_2$)	0.328 ($4_2 \times 2_2$
0.218 ($2_2 \times 2_2$)	0.266 ($2_1 \times 2_1$)	0.296 ($0_1 \times 4_2$)	0.296 ($2_1 \times 4_3$)
0.210 ($2_1 \times 2_2$)	0.260 ($2_1 \times 2_2$)	0.229 ($4_2 \times 2_1$)	0.203 ($2_1 \times 4_1$)
	0.255 ($2_2 \times 4_2$)	0.213 ($2_1 \times 4_2$)	0.200 ($4_2 \times 4_2$)
	0.216 ($0_1 \times 2_2$)		

$J = 8^+$	$J = 10^+$	$J = 12^+$
0.425 ($4_2 \times 4_2$)	0.476 ($4_2 \times 6_1$)	0.515 ($6_1 \times 6_1$)
0.331 ($4_2 \times 4_3$)	0.464 ($4_2 \times 6_2$)	0.491 ($6_1 \times 6_2$)
0.315 ($2_1 \times 6_2$)	0.425 ($6_2 \times 4_2$)	0.486 ($6_2 \times 6_1$)
0.314 ($2_1 \times 6_1$)	0.373 ($6_2 \times 4_3$)	0.472 ($6_2 \times 6_2$)
0.307 ($6_2 \times 2_1$)	0.209 ($6_2 \times 4_1$)	
0.278 ($6_2 \times 2_2$)		
0.258 ($4_2 \times 4_1$)		

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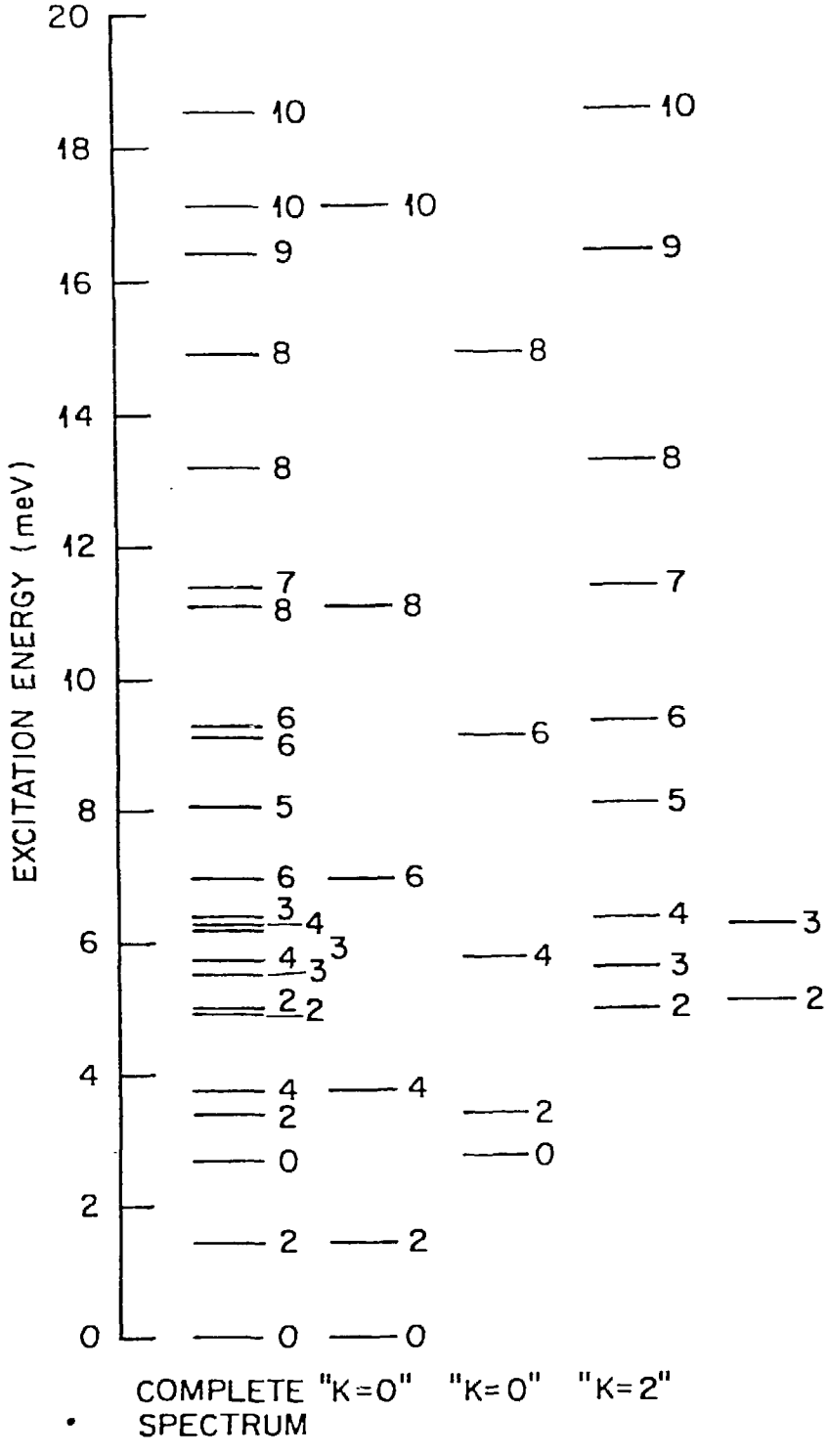

Fig. 1. Calculated Spectrum, states in the four-neutron six-proton system, described in the text.