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Herman Feshbach

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

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CONCLUDING REMARKS

Herman FESHBACH

Department of Physics, Massachusetts Institute of Technology
Cambridge, Massachusetts

Introduction

It is not possible in these relatively short concluding remarks to do justice to all the important science we have heard during the past week. Indeed, the program committee in its instructions to me has emphasized that this final presentation is not to be the customary summary. What I have chosen to do is to consider how five major themes have been illuminated in the course of this conference and, within these themes, the problems which need to be resolved in the near and far future. Some of the important results and issues which have occupied us during this conference will not fit into this pattern and consequently they will not be discussed.

The five grand topics about which this contribution will be organized are:

(1) "New" degrees of freedom.
(2) "New" forms of matter.
(3) "New" reaction mechanisms.
(4) "New" aspects of weak interactions inside nuclei.
(5) "New" symmetries of the nuclear Hamiltonian.

The word "new" is put in quotes in order to indicate that the discussion will include both new aspects of well-known examples in each category as well as new examples which have been realized recently.

1. "New" degrees of freedom

(a) Single particle motion in the mean field is the degree of freedom associated with the great discoveries of the fifties, the shell model and the unified model of Bohr and Mottelson. Particular attention is now being paid to the high spin states such as those seen in M(L) contributions to electron scattering as reported by Sick[1]), particle-hole excitations seen particularly in inelastic scattering of high energy protons by Gales[2]), and in the aligned particle orbits in rapidly rotating nuclei discussed by Stevens[3]) and Mottelson[4]). The properties of these orbitals and their interaction with the core nucleus are the interesting aspects of these studies. We have learned already from the study by electron scattering of ^{17}O that the magnetic moment, (M(1)), is not given as originally thought by the moment of the neutron in a d( ) orbit. Core excitation plays a very important role and the resulting wave function must be considerably more complex than the wave function given by the naive independent particle shell model. The residual interaction is thus of great importance.

Arima[5]) has pointed out in his lecture that the problem of the residual interaction remains unresolved. According to Arima, its derivation from nucleon-nucleon forces has not been successfully accomplished. Calculational techniques have improved sufficiently that one must ascribe most of the problem to the starting point -- the description of the nuclear forces. Of particular importance is the tensor force which is weaker in the Paris potential or the Bonn-Julich one.

The size of the tensor force must be determined. Similar remarks can be made with respect to the effective three body force which empirically should be weak. At the same time, one must ensure saturation under these conditions. One should mention the important understood result that the quadrupole-quadrupole force between like nucleons is much smaller than that between neutrons and protons. Still remaining is the problem of relating the p-h interactions used in the discussion of giant resonances and the p-p interaction used in the interacting shell model.

(b) This conference saw a great emphasis on giant resonance (Bertsch[6]), Bertrand[7]), Cardman[8]), Petrovich[9])). In recent years the giant quadrupole resonance and the monopole have been discovered and their systematics determined (Bertrand[7])). But a great deal remains to be done as one can see by looking at the investigations of the properties of the giant dipole resonance since its discovery. In particular, one wants to understand the origin of the width of the giant resonances, as well as the way it decays (Cardman[8])). But determining the width experimentally is not trivial. Background subtraction has been referred to, but there is also the question of interference with other resonances. I am also concerned with the heavy reliance on the DWBA to make the separation between monopole and quadrupole resonances. Fortunately, reaction theory can treat the background and evaluate the correction to the DWBA. One is struck by the extensive use of a number of different experimental methods used to investigate these resonances. In this connection there seems to be a large discrepancy between the strength of the GQR as determined by electron scattering[10]) and by hadron interactions. A second remark has to do with the use of the high energy probes such as the 800 MeV (p,p') (Bertrand[7])) and (α,α') studies (480 MeV) at Saturne which reveal clearly not only these GQR and GMR but other structures as well. These may be the long sought for resonances of higher multipolarity. A theoretical problem I should mention is the extraction of the nuclear compressibility which may turn out to be more subtle than it was first anticipated because of the presence of a surface as well as a volume response.

These resonances may be considered as manifestations of a vector current type interaction leading to excitation of natural parity 0^+, 2^+, etc. giant resonance states. The excitation of giant resonances by an axial vector type interaction, the (p,n) reaction or a charge exchange reaction more generally the so-called G-T giant resonances (Bertrand[7]) and (Bertsch[6]) is very exciting news. As in the case of the GMR, GDR, GQR, there could be a series of these with differing multipolarities. Another reaction in which the same type of axial vector matrix element occurs is the (γ,π) reaction. It would be interesting to see the strength with which these resonances are excited in this reaction and to compare them with the (p,n) reaction, which can be thought of as a pion stripping reaction.

(c) Another example of a new degree of freedom is provided by the so-called nuclear molecular resonances, which are of the intermediate structure type. These have been most visibly in the $^{12}C + ^{12}C$ and the $^{12}C + ^{16}O$ systems. But there are strong indications that they are present, generally to a lesser degree in many other "light heavy" ion interactions. An example was presented to this meeting[11]). Their presence seems to be the consequence of two circumstances. First, the presence of an angular momentum window in the entrance and exit channel, which selects particular values of J at particular values of the energy for which the systems can approach each other closely. They can then interact and form resonant states. The issue is, of course, under what conditions will this occur? One discussion suggests that this interaction needs to be on the weak side to keep the resonances narrow and located within the angular momentum window. Of course, the possible exit channels need also to be restricted to keep the width small.

(d) Experiments done with beams of what used to be called exotic particles (pions and kaons) have generated an interest in a new class of degrees of freedom in nuclei and in the related subject of other forms of nuclear matter. I do not need to emphasize here the importance of the nucleon-pion interaction and for that matter more generally of the nucleon-boson (ρ,ω,η) interaction for nuclear physics. Indirect confirmation of their presence inside nuclei has been made with the observation of exchange currents with electromagnetic probes (Sick[1])). However, it has been clear in this case, in nucleon-nucleon scattering where interesting structure has been found at several hundred MeV and in 1 GeV proton ^{4}He scattering that the influence of the formation of the Δ-nucleon resonance needs to be taken into account explicitly. This raises the whole issue of the behavior of a particle resonance such as the Δ inside nuclei, a problem which has its counterparts in a wide range of phenomena in physics. How are they modified by their nuclear environment? How can they be used to probe that environment? These questions must be resolved before adequate quantitative treatment of the pion-nucleus interaction in its various aspects can be achieved. Moniz[12]) points out applications to the (e,e'π) reaction and to three body forces in nuclei. One of the regimes in which these questions have been addressed and answered is pion-nucleus scattering in an energy region which includes the free Δ-resonance energy and width. The methods employed were initiated by Kisslinger and Wang in their isobar doorway model and have been carried to their present final form by Lenz and Moniz and their collaborators (Koch, Theis, Hirata, Yazaki). The principal idea behind these investigations is that there exists collective modes of the Δ-hole type which overlap strongly with the incident channel, a phenomenon reminiscent of that which occurs with the particle-hole states, the principal components of giant multipole resonance states. As in that case these modes are not exact eigenfunctions of the Hamiltonian and therefore couple to more complex modes, as well as to absorption modes, giving rise to a spreading width. These effects are taken into account semi-empirically in matching theory with experiment.

The Lenz-Moniz procedure should be used in the calculation of three body forces in nuclei which are produced by the excitation of a Δ by a virtual pion emitted by another nucleon. The existence of such a force has been suggested as a way to resolve problems in the 1 GeV p - ^{4}He elastic scattering.

Beside its direct importance in the understanding of pion-nucleon scattering, the general method can be used for the description of other baryon resonances in nuclei, such as the Y*, formed in this case when a K^- strikes a nucleon.

(c) Kaons have been used to form a new kind of nuclei using the nearly recoilless (for particular values of kaon momentum) reaction (K,π^-). The nuclei formed are hypernuclei, usually with a neutron replaced by a Λ; although recently some nuclei have been formed with the neutron replaced by the more massive Σ. Dr. Dalitz in his talk discussed the various results and issues connected with these nuclei. We can expect their study to provide information on the Λ and Σ nucleon interaction, information which it would be difficult to obtain in any other way. Indeed one qualitative aspect suggested by Gal and Dover is that the spin-isospin character of the ΣN, ΛN coupling is thought to be responsible for the existence of narrow Σ hypernuclear states. We expect the study of hypernuclei to provide information as well on the weak interaction Λ + n → 2n, which is competitive with the simple decay Λ → n + π only inside hypernuclei. Studies with the heavier nuclei as targets will tell us if a collective state of the Λ- neutron hole type can be formed.

Of course, the properties of hypernuclei are intrinsically very interesting and have been discussed by Dalitz[13]). My remarks will be limited to the possible use of the Λ in hypernuclei as a probe of nuclear structure. The interest in such a probe stems in part from the fact that the Pauli principle doesn't apply, yet the Λ is a baryon so that its interaction with nucleons is reasonably strong and short-ranged. The effect of having a baryon in <u>any</u> orbital, and I emphasize the word "any", may therefore become available for study. The Λ will, for example, produce changes in the macroscopic properties of the core from their value in the absence of the Λ. The core radius will change and with it the Coulomb energy. The moment of inertia will change, as well as the vibrator frequency ω, the superconducting gap Δ, and the various electromagnetic properties. These changes will be reflected by the energy spectrum associated with excitations of the core and the transitions between these levels. The notion that many of the excitations of hypernuclei involve the excitation of the core nucleus only seems to be verified by the recent BNL experiments on ^{13}C, ^{14}N, ^{16}O. Measurement of these changes as they apply seem to be feasible for p-shell hypernuclei. But it is of importance in this context to extend the identification and measurement of hypernuclei states to heavier nuclei which exhibit the macroscopic properties described above clearly.

I don't know whether such determinations will prove feasible but I am encouraged by the recent measurement of the gamma ray spectrum produced by a decaying hyper-nucleus.

(f) Certainly one of the most exciting prospects which we face in nuclear physics is the problem raised by the modern picture of hadron structure as described by the M.I.T. bag and by quantum chromodynamics[14]). What indeed is the quark structure of the nucleus? What is the nature of nuclear forces at short distances where the quark structure dominates? The M.I.T. bag involves a hadron radius of approximately 1 fm. This implies a severe non-locality in nuclear forces and makes such simple models as the one boson-exchange model suspect. Indeed it may lead to a revival of the boundary condition model as that model seems especially appropriate in the context of the bag model. Such a model was used by Low and Jaffe to successfully explain pion-pion and pion-nucleon scattering. The question arises as to what happens to the bag when the nucleons are inside the nuclei. Do the bag surfaces dissolve and the nuclear structure description in terms of nucleons remain valid because of the strong quark-quark forces favoring color singlet clusters or does some remnant of the bag surface remain in place? Kerman raises the question of the effective mass of such a color singlet cluster. Remember that the effective mass of the nucleon in a nucleus is a manifestation of the non-locality of nuclear forces. The nature of the exchange currents is substantially altered since one can have color as well as charge carrying currents. Theoretical exploratory studies are being made of the description of simple nuclei using the quark-gluon picture; for example, the ^{4}He nucleus as a 12 quark system and so on. It will be interesting to see how far one can go with this model and what it implies.

On the other hand, one can ask for direct experimental verification of the quark structure of nuclei. We recall that the quark structure of the hadron was demonstrated by the MIT-SLAC group by observation of the deep inelastic scattering of electrons. We need to consider and plan for the analogous experiments which will reveal the quark structure of the nucleus and how that picture changes into the nucleon picture traditionally employed.

2. New forms of matter

New degrees of freedom imply the possibility of new forms of nuclear or more generally hadronic matter. Historically one can go back to the Lee-Wick "abnormal nuclear matter" for which the new degree of freedom is the σ field. If the new degree of freedom is taken to be the pion field, the implied new form of matter is produced by pion condensation which will occur at densities which are sufficiently large that it is energetically profitable to create pions in macroscopic number. If, on the other hand, the new degree of freedom is represented by the quark, we are dealing with quark matter, whose stability may approach that of ordinary nuclear matter for however unusual values of the strangeness quantum number.

Experimental issues of importance here are (1) the production and (2) the detection of the production of such new forms of matter. Some have suggested that the appropriate densities will be achieved by heavy ion collisions where the energy is large enough to permit substantial interpenetration. Astrophysics forms an area where one might look for manifestations of new forms of matter. For example, pion condensates speed up the process of cooling of neutron stars by several orders of magnitude. A third possibility is that there are indications, so-called "precursors," which can be detected at ordinary nuclear densities. Such a precursor mechanism has been suggested to explain the inelastic scattering of electrons by ^{13}C with excellent agreement obtained employing such a mechanism. One problem is the relation of this mechanism to configuration mixing. Of course, configuration mixing must be taken into account. But then one must ask: Do these two mechanisms really differ or are they differing ways of representing the same underlying phenomenon?

The appeal of these exotic forms of matter should not obscure the importance of understanding the properties of our "ordinary" nuclear matter spinning very rapidly as discussed by Mottelson[4]) and/or at various excitation energies. The regimes under which thermodynamic concepts, those of equilibrium statistical mechanics, equations of state mentioned by Gyulassy[15]) can be applied and when non-equilibrium statistical mechanics and classical dynamics can be used need to be determined. These were discussed at this conference by K. Dietrich[16]) and evidence that a statistical diffusion type theory is implied by experiment has been presented by Gobbi[17]). The notion some quantities change slowly with the number of interactions and others very rapidly seems to me to be of central importance for these discussions. Heavy ion reactions seem to be the specific experimental area on which such questions can be addressed. However, it is of essential importance that the effect of the prompt (direct) reactions be carefully evaluated in order to delineate the phenomena to which these statistical approaches can be applied. As both Dietrich and Scott[18]) warned, a considerable fraction of the reaction proceeds in this fashion. A great deal of physics remains to be done here. Because even if we have the correct formulation the actual physical input still remains to be delineated. Particle transfer, shape change, excitation of giant resonances and so on, which of these occur and in what order? What is the path of the system in reaction space?

3. New reaction mechanisms

(a) The traditional concepts of direct and compound nuclear reactions need to be generalized. In Fig. 1 we indicate the regimes where these concepts apply and where one must include as well multi-step direct and pre-equilibrium processes. The required generalizations have been made in the course of which an unexpected statistical process was delineated. This can be understood as a statistical way of treating the multi-step direct process when very many exit channels and intermediate states are involved. It would be inappropriate for me to attempt a complete description, but since it was not reported during the conference perhaps a few examples by Bonetti and Mlazza-Colli of its satisfactory application would be permissible. In Fig. 2-4 we show the spectrum at three different angles generated in the ^{120}Sn(p,n) reaction. We see how the total is made up of single, double, and three step contributions; the multi-step contributions achieving greater importance as one goes to larger angles and lower neutron energies. At the very low neutron energies, or for that matter as the proton energy is decreased sufficiently, another process becomes important, that of the multi-step compound. There is no time to discuss it except to say that in the limit it becomes the familiar statistical theory of nuclear reactions as formulated by Weisskopf. This problem does therefore appear to be under control. But the physics remains! What steps are important, and how to identify them are central questions. What experiments should be done? One immediate application, for which the details are perhaps not that important, is the calculation of the background for the giant resonances. The calculation of Shakin and Wang for the giant dipole is an example. The removal of the background effects are required before the position and width of these resonances can be accurately determined.

(b) The time dependent Hartree-Fock method (TDHF) involves the approximate solution of the time dependent many body Schroedinger equation using single Slater determinants with the time dependent single particle wave functions to be determined by a self-consistent method. This procedure has been used to discuss heavy ion induced reactions. These calculations have been very instructive, certainly at a qualitative level. At a quantitative level they do leave much to be desired. The quantitative relation to observables is not straightforward, and although some have begun attempts to look at the next order of approximation, it is clearly a formidable task.

I would like to bring to your attention another procedure which has recently been formulated by Levit, and applied together with Negele, Petrel and Koonin. Recall that it is always possible to formally linearize equations by introducing new dependent variables. In the method of Levit, this is the first step, namely the linearization of the many body Hamiltonian by the introduction of an auxiliary classical field. The second step is the evaluation of the appropriate physical quantities, which can always be expressed as matrix elements of the time-propagation which in turn involves this Hamiltonian, by the method of stationary phase. This procedure yields semi-classical results but has the advantage that it is not perturbative, that it is not limited by the strength of the interaction. Application to the theory of fission has been made and others are in progress.

(c) A great deal of interest has been expressed in recent years in the interaction of relativistic heavy ions with nuclei. In this conference, see the discussions by Scott[15]) and Gyulassy[16]). The hope expressed by some is that such an interaction would result in the production of regions of high density and temperature which might make possible the production of the new forms of matter described earlier or of the super-heavy elements. This would require mechanisms which would deexcite the system without substantially changing the density. From the investigation of collisions of relativistic protons (and more generally hadrons) with nuclei it is apparent that these hadrons deposit a fixed amount of energy in the nucleus and that the space-time character of the hadron-nucleon interactions plays a fundamental role. Further experiments with single hadron projectiles remain to be done to completely characterize this point. Once understood the question of interest is to what extent this is modified when the incident projectile is a relativistic heavy ion.

A major need is the development of useful descriptions of the many particle final states. Correlation type experiments are essential -- but one also needs to know which will provide the most incisive information.

At these energies one can expect the production of many hadron types within nuclei: pions, kaons, Λ's and so on. Some of the interesting consequences might include the formation of ΛΛ type states, the dibaryon states of the bag model (or of double Λ hypernuclei). Here correlation type experiments would be essential.

4. New aspects of the weak interactions in nuclei

Weak interactions were originally discovered in the decay of naturally radioactive nuclei. Present day experiments use the nucleus in testing the consequence of recent developments in the unified gauge theories. We recall nuclear tests of CVC, PCAC, and second class currents. In addition, these studies also provide new tools with which to study nuclei.

(a) By choosing specific nuclei and specific transitions the spatial and symmetry structure of the recently discovered weak neutral currents predicted by the gauge theories can be studied through the scattering of electrons by nuclei.

(b) The recent measurement of the violation of parity conservation in the scattering of protons by protons[13]) will stimulate many experiments; first to delineate the spatial and symmetry structure of the weak nucleon-nucleon interaction; and secondly, once that interaction is understood, to use this interaction which is weak and short-ranged to probe nuclear structure.

5. New symmetries

Nuclear systems form an immensely rich source of phenomena. This is an advantage but there can also be an "embarrassment of riches". For example, one sometimes hears that if we only knew the residual potential the interacting shell model provides the eigenfunctions and by diagonalizing in a sufficiently large space the nuclear structure problem would be solved. Unfortunately, this isn't true. Talmi has provided an example. His point is that as the number of valence orbits and valence particles increase, the number of states which can be formed grows astronomically. In a typical nucleus like $^{154}_{62}Sm_{92}$ the number of valence neutrons is 10 and the appropriate single particle orbits are $^1h_{9/2}$, $^2f_{7/2}$, $^2f_{5/2}$, $^2p_{3/2}$, $^3p_{1/2}$, $i_{13/2}$; the number of valence protons is 12 and the orbits are $^1g_{7/2}$, $^2d_{5/2}$, $^2d_{3/2}$, $^3s_{1/2}$, $^1h_{11/2}$. The number of $J = 0$ states is 4.1×10^{13}, $J = 2$ states is 3.5×10^{14}, $J = 4$ is 5.3×10^{14}. Obviously solving for all these states would be a ridiculous program. We therefore must face the important questions: which of these 10^{14} states are interesting? How can they be selected and studied both theoretically and experimentally? Nature has been helpful in providing some answers -- Nature has selected out the low lying levels of nuclei, which are relatively sparse. These levels are low because in some fashion they take advantage of symmetries of the nuclear Hamiltonian. The symmetries manifest themselves in that one can find families of levels which bear a relatively simple relation to each other. The rotational levels form a simple example. Their energies, electromagnetic properties and transition rates are simply related. When such families of levels can be identified, a symmetry property of the nucleus has been uncovered. In the case of the rotational system the group is known as U(5). Another case in point is the isospin symmetry. Are there other symmetries? If there are they can be used to understand nuclear properties and conversely they reflect upon the nature of the nuclear Hamiltonian.

During this conference we heard a discussion of groups and nuclear structure by Moshinsky[20] and in particular the role of the interacting boson picture. The use of such a picture is, as you all saw and heard, still very controversial. I will not attempt to describe the nature of that controversy; but with the distinguished theorists on both sides involved, I expect that it will be resolved soon. However, it is a fact that the IBM-IBA developments have had a great impact -- and it is no exaggeration to say that it has led to a renaissance in nuclear spectroscopy of low lying levels by providing a relatively simple theoretical framework.

There is one aspect upon which I would like to comment; namely, the method used by Iachello, Arima and Talmi provides a method which permits a systematic search for symmetries. Their method is quite analogous to the method used by the high energy physicist which, for example, led to the discovery of the SU(3) symmetry among the lighter hadrons of both the baryonic and mesonic types, eventually to be explained by the underlying quark structure. To see how this method is used for nuclear structure let me briefly review the application of IBM. The assumption is made that the elementary excitations of a nucleus like ^{154}Sm are taken to be $s(j = 0^+)$ and $d(j = 2^+)$ bosons. Assuming that the Hamiltonian is invariant in the space formed by the possible multi-boson states implies a group structure; in this case U(6). The properties of the group are reflected in the properties of families of levels of this Hamiltonian. Without going into the controversial question of how well this model fits the data, one sees that if it does fit, a symmetry of the nuclear Hamiltonian would have been discovered. One can of course immediately think of other possibilities such as the larger group formed from the 0^+, 2^+, 4^+ bosons or for other types of nuclei perhaps the 1^-, 3^- bosons might be the most appropriate.

Of course, all these dynamical symmetries are approximate. And the relationship to the actual nuclear Hamiltonian is the important issue for the long run. One must ask and show how an observed symmetry is related to properties of the Hamiltonian for that observation to be ultimately meaningful. In the case of the IBM several theoretical developments which permit such comparisons demonstrate the general techniques which need to be used and thus the feasibility of such an approach.

In conclusion, let me point to the broad generality of the results and problems which have been discussed this week. They are important not just for nuclear physics but in fact for all of physics and in some cases for chemistry. One of the major contributions of nuclear physics has been the discovery of methods which can be used to treat and discuss strong interactions. It is no accident that many of these techniques are now being used by particle physicists. The behavior of resonances inside matter is of interest to solid state and atomic physicists. The results of the study of reaction mechanisms are applicable wherever reactions are used to study systems. The statistical mechanics (equilibrium and non-equilibrium) of relatively small systems is of fundamental importance. The nature of weak interactions has been clarified in the past by studying their impact on nuclear dynamics. This continues to be a fertile source of information regarding these basic forces.

Each sub-field of physics contributes in its own special way to the resolution of these fundamental questions, increasing thereby the ability of the physical sciences to understand, predict and produce natural phenomena. This conference demonstrates how nuclear physics contributes importantly to this effort, an effort that has constructed an intellectual edifice, that is one of the greatest achievements of the human mind.

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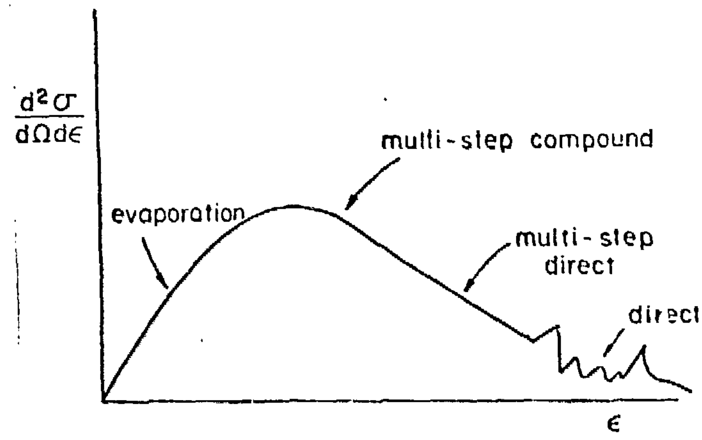

Fig. 1

A typical energy spectrum of particles produced at a given angle by a nuclear reaction. The particle is ε.

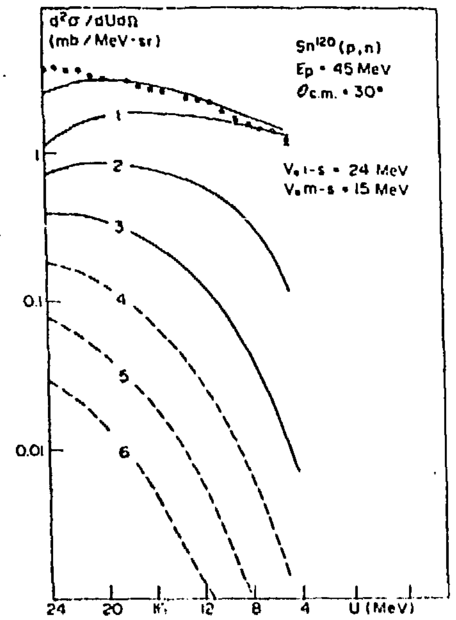

Fig. 2

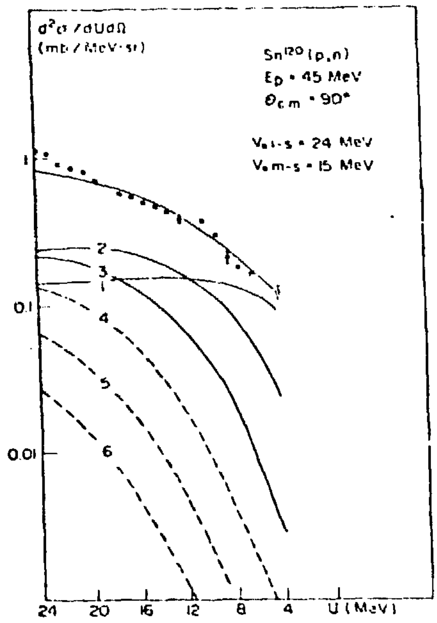

Fig. 3

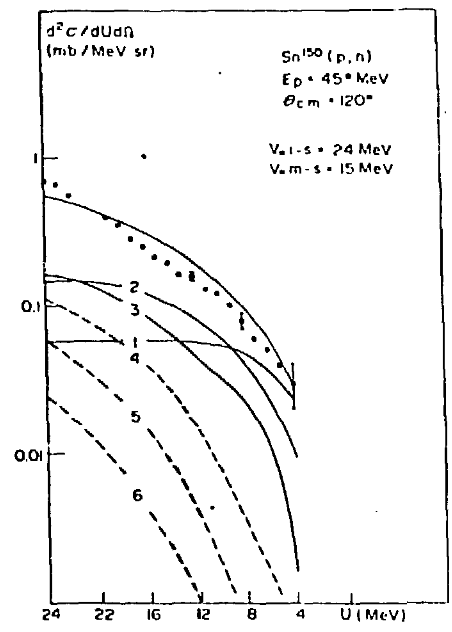

Fig. 4

Figs. 2, 3, and 4. The double differential cross-section for neutron production in the $^{120}Sn(p,n)$ reaction at the indicated angles. U is the excitation energy in the residual nucleus. The numerals labelling each curve indicate the number of steps involved. The solid points are the experimental points.