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Particle Anisotropy

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Destabilization of TAE modes by particle anisotropy

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Abstract

Plasmas heated by ICRF produce energetic particle distribution functions which are sharply peaked in pitch-angle, and we show that at moderate toroidal mode numbers, this anisotropy is a competitive and even dominant instability drive when compared with the universal instability drive due to spatial gradient. The universal drive, acting alone, destabilizes only co-propagating waves (i.e. waves propagating in the same toroidal direction as the diamagnetic flow of the energetic particles), but stabilizes counter-propagating waves (i.e. waves propagating in the opposite toroidal direction as the diamagnetic flow of the energetic particles). Nonetheless, we show that in a tokamak, it is possible that particle anisotropy can produce a larger linear growth rate for counter-propagating waves, and provide a mechanism for preferred destabilization of the counter-propagating TAE modes that are sometimes experimentally observed.

Considerable research in tokamaks has been directed towards studying the spontaneous excitation by energetic particles of low frequency waves such as Alfvén waves^{1,2,3} [in particular the Toroidal Alfvén Eigenmode (TAE)]. It has been understood that the wave-particle resonance interaction, that taps the “universal” instability drive (due to spatial gradients), can destabilize TAE modes whose phase velocity component is in the same toroidal direction as that of the diamagnetic flow of the resonant energetic particles (we call this case co-propagation), while the “universal” interaction is intrinsically stabilizing for waves propagating in the opposite direction (counter-propagation).

In plasmas heated by the Ion Cyclotron Range of Frequencies (ICRF), there is an additional particle anisotropy drive which can also destabilize TAE modes (e.g. see Ref. 4 for a discussion of this effect in toroidal geometry). Particles with turning points in the neighborhood of the cyclotron resonance surface are preferentially heated by ICRF and the particle distribution produced is sharply peaked in pitch angle. This results in an inverted energy population (at constant magnetic moment) for many of the heated particles. Thus particle anisotropy provides another source of free energy in addition to the universal instability drive, and this anisotropy drive can exceed the universal drive for moderate toroidal mode numbers. For co-propagating TAE modes, the two instability drives reinforce each other, and the growth rate is considerably increased by particle anisotropy.

We might expect that counter-propagating TAE modes are not as easily destabilized as co-propagating TAE modes since the universal drive is stabilizing (for monotonically decreasing radial profiles). However, occasionally TAE modes in plasmas heated by ICRF are observed to be unstable with phase velocities opposite to the prevailing ion diamagnetic current.⁵ This of course can arise if a hollow radial profile of energetic particles is formed so that their diamagnetic flow in the wave region is opposite to the prevailing ion diamagnetic current. Here we propose that particle anisotropy provides another mechanism to achieve the strongest destabilization of counter-propagating modes in ICRF heated plasmas. Indeed

we sometimes numerically find, for appropriately chosen parameters, that the most unstable case is for waves whose phase velocity is opposite to the energetic particle diamagnetic flow velocity (i.e. the case which is intrinsically stable if the anisotropy drive is neglected). The reason for the effect is a property of the functional form of the resonance function that is used to calculate the growth rate. The growth rate magnitude is determined by an appropriate phase space integral which involves the 'residues' of the resonance particle interaction at resonance surfaces in the phase space defined by the wave-particle resonance condition. For counter-propagating TAE modes, the 'residues' of the 'Landau poles' become 'singular' when there is a merging of two separate solutions of the wave-particle resonance condition, and as can be inferred from the discussion below, this possibility can only arise for a counter-propagating mode. This means that the strongest interactions are for waves with frequencies ω near a set of critical frequencies ω_{crit} which are such that the wave-particle resonance condition can be satisfied if $|\omega|/|\omega_{\text{crit}}| \leq 1$ but cannot be satisfied if $|\omega|/|\omega_{\text{crit}}| > 1$.

To begin our detailed discussion of evaluating growth rates, we first consider the resonance function. Let the wave perturbations be of the form $\sim \exp(-i\omega t + in\xi)$, where $\omega \equiv$ mode frequency, $t \equiv$ time, $\xi \equiv$ toroidal azimuthal angle, $n \equiv$ toroidal mode number. The resonance function is

$$\frac{1}{\Omega_\ell(\Gamma)} = \frac{1}{\omega - n\Omega_\xi(\Gamma) + \ell\Omega_b(\Gamma) + i\epsilon} \xrightarrow{\epsilon \rightarrow 0} -i\pi\delta(\Omega_\ell). \quad (1)$$

where $\Omega_\xi(\Gamma) = \frac{v^2 q(r_g)}{2\omega_c r_G R_0} g(\lambda, r_g)$ is the mean toroidal drift frequency of a magnetically trapped particle, $\Omega_b(\Gamma) = \frac{v}{q(r_G)R_0} \left(\frac{r_G}{2R_0}\right)^{1/2} h(\lambda, r_G)$ the mean bounce frequency of a trapped particle, v the particle speed, r_g the particle's mean minor radius, R_0 the major radius, ω_c the cyclotron frequency, $\lambda = \mu B_0(r_g)/H$ the pitch angle (where $B_0(r) = B(r, \theta = \pi/2)$), μ the particle magnetic moment, $H = \frac{1}{2} Mv^2$ the particle energy, M the energetic particle mass, and $q(r)$ the safety factor. The functions g and h are functions of order unity and generally positive (they can be expressed in terms of elliptic functions. For shorthand we use Γ to represent the dependence on phase space variables, in particular the constants of motion in

the unperturbed magnetic field: the energy H , the pitch angle $\lambda = \mu B_0/H$ and the canonical angular momentum P_ξ which is related to the mean minor radius r_G , by

$$P_\xi \equiv MRv_{\parallel} - \frac{e_E}{c} \int_0^r dr \frac{rB}{q(r)} \equiv -\frac{e_E}{c} \int_0^r dr \frac{rB}{q(r)}$$

with e_E the charge of the energetic species, and v_{\parallel} the velocity component parallel to the magnetic field.

In the most straightforward perturbation theory, the growth rate will be proportional to a phase space integral over the delta function in Eq. (1). The variables of integration can be taken to be v, λ, r_g . To integrate in the variable v , we first obtain the solutions v_ℓ^\pm of the resonant particle equation $\Omega_\ell(v_\ell^\pm, r_g, \lambda) = 0$. There are two roots of this equation and they are given by $v_\ell^\pm = v_{n,\ell} \pm v_{n\ell} \left(1 - \frac{\omega}{\omega_{n,\ell}}\right)^{1/2}$, with $v_{n,\ell} = \frac{\omega_c r}{q^2} \left(\frac{r}{2R}\right)^{1/2} \frac{\ell}{n} \frac{h}{g}$, and $\omega_{n\ell} = -\frac{\omega_c r_0^2 \ell^2 h^2}{4q^3 R^2 n g}$. We can therefore write

$$\frac{1}{\Omega_\ell} = -i\pi \sum_{\pm} \frac{\delta(v - v_\ell^\pm)}{|\frac{\partial \Omega}{\partial v}|} = -i\pi \frac{v_{n,\ell} \left(\delta(v - v_\ell^+) + \delta(v - v_\ell^-) \right)}{2(-\omega_{n,\ell}\omega + \omega_{n,\ell}^2)^{1/2}}. \quad (2)$$

and now the v -integration is trivially performed.

We adopt the convention that the toroidal mode number n is positive, and hence for g positive (the typical case), $\omega_{n\ell} < 0$ is negative. In the case of ω positive (co-propagation), $(1 - \omega/\omega_{n\ell}) > 1$ and only one root is relevant: $v = v_\ell^+$ if $\ell > 0$, $v = v_\ell^-$ if $\ell < 0$, and $v = \left(\frac{2\omega_c r_G R_0 \omega}{n g g}\right)^{1/2}$ if $\ell = 0$. In the case of ω negative (counter-propagation), there are two relevant roots $v = v_\ell^\pm$, where we require $\ell > 0$ and $(1 - \omega/\omega_{n\ell}) > 0$. Note that the factor $\frac{1}{(\omega - \omega_{n,\ell})^{1/2}}$ blows up when the two roots nearly coincide, and as a result a much enhanced response is expected for $\omega \doteq \omega_{n,\ell}$. In actual fact, we will see that the anisotropy drive gives an even larger enhancement than described here (as we will see, the drive is proportional to $\frac{1}{\Omega_\ell^2} \doteq i\pi \partial \delta(\Omega_\ell) / \partial \omega$). This extra enhancement allows the anisotropy drive to overcome the stabilizing influence of the universal drive for counter-propagating modes, and large growth rates can occur if the frequency of a natural mode of the background plasma is close

to the critical frequency $\omega \sim \omega_{n,\ell}$. However, when $\omega = \omega_{n,\ell}$, a more accurate evaluation is needed to obtain the growth rate. Below we discuss an analytic estimate as well as numerical evaluations of the growth rate.

To be specific we take for the equilibrium particle distribution $F = F(H, P_\xi, \lambda)$,

$$F = \alpha \bar{\beta} \left(r_G(P_\xi) \right) \exp \left(-\frac{H}{T} \right) \exp \left(-(\lambda - 1)^2 / 2\sigma^2 \right), \quad (3)$$

with $\sigma \ll (r/R)^{1/2}$, and $\alpha = \frac{1.68 B_0^2 (r_G/R)^{1/2}}{32\pi^3 \sigma^{1/2} T^{5/6} M^{3/2}}$, so that F is normalized to give $\bar{\beta} = 8\pi/B_0^2 \int_{-\pi}^{\pi} d\theta/2\pi \int d^3p H F$. The distribution function is peaked in the pitch angle variable about $\lambda = 1$.

We consider TAE eigenmodes, which are Alfvén waves whose perturbed electric and magnetic field amplitudes, \mathbf{E}_1 and \mathbf{B}_1 can be taken as,

$$\mathbf{E}_1 = -\nabla_{\perp} \left(\Phi e^{in\xi} \right) e^{-i\omega t}, \quad \mathbf{B}_1 = -ic/\omega e^{-i\omega t} \nabla \times \mathbf{b} \left(\mathbf{b} \cdot \nabla \Phi e^{in\xi} \right),$$

where the potential Φ satisfies

$$\Phi = \sum_m \Phi_m(r) \exp(im\theta).$$

We choose a functional form for $\Phi(r)$ derived in the low shear limit [Ref. 6] ($r_0/R \ll s^2 \ll 1$),

$$\Phi(r) = K_0 \left((x^2 + \hat{\epsilon}^2)^{1/2} \right) \exp \left[in\xi - im\theta \right] (1 + e^{i\theta}) \quad (4)$$

where $x = m(r - r_0)/r_0$, $\hat{\epsilon} = (5\pi/16)r_0/R_0$ and K_0 is the MacDonald function. The TAE mode is localized near $r = r_0$ (the radial position of mode localization). To lowest order, the mode frequency $\omega = \omega_0$ (neglecting the energetic particle corrections) is determined by $\omega_0 = \pm v_A(r_0)/2q(r_0)R_0$, where $q(r_0) = (m + 1/2)/n$ and v_A is the Alfvén speed.

Standard perturbation theory then gives for the frequency shift,

$$\omega - \omega_0 = \frac{-1}{2\{WE\}} \int d\Gamma \sum_{\ell} \frac{|\widehat{H}_{\ell}|^2}{\Omega_{\ell}} \left(\frac{\partial F}{\partial H} \Big|_{\mu, P_{\xi}} + \frac{n}{\omega_0} \frac{\partial F}{\partial P_{\xi}} \Big|_{\mu, H} \right) \quad (5)$$

where, for Alfvén waves, $\{WE\} = \int \frac{d^3r |B_1|^2}{4\pi}$ and $|\widehat{H}_\ell|$ is

$$|\widehat{H}_\ell| = \left| \frac{eE}{\tau_b} \int_0^{\tau_b} d\tau \exp[i(\ell\Omega_b - in\Omega_c)\tau] \{v_D(\tau) \cdot \nabla \Phi(\mathbf{r}(\tau)) e^{in\ell(\tau)}\} \right| \quad (6)$$

The variable τ represents the time dependence of the unperturbed motion, $\tau_b = 2\pi/\Omega_b$, and v_D is the guiding center drift velocity.

Note that with $\text{Im } \Omega_\ell^{-1} = -i\pi\delta(\Omega_\ell)$, the growth rate is proportional to phase space gradients of the distribution function F :

$$\gamma \propto \left(-n \frac{\partial F}{\partial P_\xi} + \frac{\partial F}{\partial H} \right) \delta(\Omega_\ell) = \left(\frac{1}{T} \left(\frac{n\omega^*}{\omega} - 1 \right) F_0 - \frac{\lambda}{H} \frac{\partial F}{\partial \lambda} \right) \delta(\Omega_\ell)$$

where $\omega^* = -\left(Tq/Mr\omega_c\bar{\beta}\right) \frac{\partial \bar{\beta}}{\partial r}$. If there is no particle anisotropy ($\partial F/\partial \lambda = 0$), instability ($\gamma > 0$) requires $n\omega^*/\omega > 1$ so that the universal drive due to spatial gradients can overcome stabilization arising from $\partial F/\partial H$ being negative. However, with sharply peaked anisotropy (where $\partial F/\partial H|_{\mu, P_\xi} > 0$ for a significant fraction of particles), instability can occur if $n\omega^*/\omega < 1$ or even if $n\omega^*/\omega$ is negative.

To analytically perform the integrals appearing in Eq. (5), we model the unperturbed particle orbits as a superposition of its mean drift motion and a relatively simple oscillatory motion. In the numerical evaluation, the orbits are treated accurately, but for the purpose of exhibiting the growth rate scaling with plasma parameters, the models orbits are adequate.

We model the orbits as follows:

$$r = r_G + \frac{v^2 \kappa(\lambda)}{\omega_c R_0 \Omega_b} \sin \Omega_b \tau; \quad v_D \cdot \hat{\mathbf{r}} = \frac{v^2 \kappa(\lambda)}{\omega_c R_0} \cos \Omega_b \tau$$

$$\theta = \theta_T \cos \Omega_b \tau$$

$$\xi = \xi_0 + \Omega_\xi \tau + q(\tau_0) \theta_T \cos \Omega_b \tau$$

$$\Omega_b = \frac{1}{2\pi} \int \frac{d\ell}{v_{\parallel}} = \frac{R_0 q}{\pi} \int_{-\theta_T}^{\theta_T} \frac{d\theta}{\sqrt{\frac{2H}{M} (1 - \lambda(1 - r/R_0 \cos \theta))}}$$

$$\kappa(\lambda) = \left(\frac{R_0}{2r_0}\right)^{1/2} (1 - \lambda(1 - r_0/R_0))^{1/2}$$

We analytically evaluate the integrals in the limit $\hat{\epsilon} r_0/m \ll \Delta_b \ll \{\frac{r_0}{m}, (n(\frac{dq}{dr})^{-1})\}$, where $\Delta_b = \frac{v^2 \kappa(\lambda)}{\omega_c R \Omega_b}$ the orbit width. Then in the region $x \sim m \Delta_b / r_0 \sim \hat{\epsilon}$,

$$\frac{m\Phi_m}{r} \ll \frac{\partial\Phi_m}{\partial r} = -\frac{(r - r_0)}{(r - r_0)^2 + r_0^2 \hat{\epsilon}^2 / m^2}$$

Using our model orbits, we perform several integrals in Eq. (5), and we reduce the expression for the growth rate to a single integral over the particle speed.

$$\begin{aligned} \frac{\omega - \omega_0}{\omega_0} = & -\frac{32}{3\pi(2\pi)^{1/2}} \left(\frac{r_0}{R}\right)^{1/2} \frac{\hat{\epsilon} r_0}{m} \frac{\bar{\beta} \omega_c v_A}{(T/M)^{1/2} R} \sum_{\ell} a_{\ell} \\ & \cdot \int \frac{dv v^5}{(2T/M)^3} \exp\left(-\frac{mv^2}{2T}\right) \\ & \cdot \left[\frac{\frac{n\omega^*}{\omega_0} - 1 - \frac{R_0 T}{r_0 m v^2}}{\Omega_{\ell}} + \frac{(n\kappa_{\xi} \Omega_{\xi} - \ell \Omega_b \kappa_b) \frac{R}{r_0} \frac{2T}{M v^2}}{\Omega_{\ell}^2} \right] \end{aligned} \quad (7)$$

where $a_{\ell} = \frac{\ell^2}{\ell^2 - 1/4} \approx 1$ (set $a_{\ell} = 1$ henceforth),

$$\kappa_{\xi} = \frac{r_0}{R \Omega_{\xi}} \frac{\partial \Omega_{\xi}}{\partial \lambda} \Big|_{\lambda=\pi/2, s=0} = 1.32, \quad \kappa_b = \frac{r_0}{R} \frac{1}{\Omega_b} \frac{\partial \Omega_b}{\partial \lambda} \Big|_{\lambda=\pi/2, s=0} = .23$$

In the r_0 -integration, we neglect the radial dependence in Ω_{ℓ} . In the λ -integration, we take F to be a delta-function in λ and we integrate by parts in λ . Note that $\frac{\partial}{\partial \lambda} \sim \frac{R_0}{r}$, and the effects of particle anisotropy are amplified by a factor R_0/r .

If ω is not close to $\omega_{n,\ell}$, we can use Eq. (2), where $\omega = \omega_0$, and we have

$$\frac{1}{\Omega_{\ell}} = -i\pi \delta(\Omega_{\ell}), \quad \frac{1}{\Omega_{\ell}^2} = i\pi \frac{\partial}{\partial \omega_0} \delta(\Omega_{\ell}).$$

We then find for the imaginary shift of frequency,

$$\begin{aligned}
\frac{\omega - \omega_0}{\omega_0} &= i\hat{\gamma} \sum_{\ell} \left(\frac{\omega_0^2}{-\omega_0\omega_{n,\ell} + \omega_{n,\ell}^2} \right)^{1/2} \bar{v}_{n,\ell} \\
&\cdot \sum_{j=\pm} \left\{ (\bar{v}_{\ell}^{(j)})^5 \exp \left(-(\bar{v}_{\ell}^{(j)})^2 \right) \cdot \left[\frac{n\Omega_{\xi}}{\omega_0} \left(\frac{r}{R_0} \frac{\omega^*}{\Omega_{\xi}} \right) - \left(\frac{r_0}{R_0} + \frac{1}{2(\bar{v}_{\ell}^{(j)})^2} \right) \right. \right. \\
&\left. \left. - \frac{\omega_{n,\ell}}{2(\bar{v}_{\ell}^{(j)})^2} \frac{(\kappa_{\xi} n \Omega_{\xi}^{(j)} - \kappa_{\beta} \ell \Omega_{\xi}^{(j)})}{(-\omega_0\omega_{n,\ell} + \omega_{n,\ell}^2)} \right] \right. \\
&\left. - \frac{\partial}{\partial \omega_0} \left((\bar{v}_{\ell}^{(j)})^5 \exp \left(-(\bar{v}_{\ell}^{(j)})^2 \right) (\kappa_{\xi} n \Omega_{\xi}^{(j)} - \kappa_{\beta} \ell \Omega_b^{(j)}) \right) \right\} \quad (8)
\end{aligned}$$

with

$$\begin{aligned}
\hat{\gamma} &= \frac{32}{3(2\pi)^{1/2}} \left(\frac{r_0}{R} \right)^{1/2} \frac{r_0}{m} \frac{\omega_c \bar{\beta} q}{(T/M)^{1/2}} \\
\bar{v}_{\ell}^{(j)} &= \frac{v_{\ell r}^{(j)}}{(2T/M)^{1/2}} \\
\bar{v}_{n,\ell} &= \frac{v_{n,\ell}}{(2T/M)^{1/2}}
\end{aligned}$$

Notice that the growth rate gets appreciably larger than the standard estimate in the case of counter-propagating TAE modes with negative frequencies $\omega_0 < 0$ when $|\omega_0 - \omega_{n\ell}| \ll -\omega_{n,\ell}$, as then the anisotropy contribution to the growth rate scales as $\frac{1}{(-\omega_0\omega_{n,\ell} + \omega_{n,\ell}^2)^{3/2}}$. To avoid the divergence of the growth rate, we need to keep the frequency shift of ω in $1/\Omega_{\ell}$ and $1/\Omega_{\ell}^2$ in Eq. (7), while the other terms can be evaluated at their resonance positions (however, when such a procedure is needed, all other ω -dependent terms are accurately approximated by evaluating them at $\omega = \omega_{n,\ell}$). Then, for $\omega = \omega_{n,\ell}$, the dispersion relation becomes

$$\begin{aligned}
\frac{\omega - \omega_0}{\omega_0} &= 2i\hat{\gamma} \left[\left(\frac{-\omega_{n,\ell}}{\omega - \omega_{n,\ell}} \right)^{1/2} \left\{ \frac{n\Omega_{\xi}}{\omega_0} \left(\frac{r_0}{R_0} \frac{\omega^*}{\Omega_{\xi}} \right) - \left(\frac{r_0}{R_0} + \frac{1}{2\bar{v}_{\ell}^2} \right) \right\} \right. \\
&\left. + \frac{1}{2} \left(\frac{-\omega_{n,\ell}}{\omega - \omega_{n,\ell}} \right)^{3/2} \frac{(\kappa_{\xi} n \Omega_{\xi \ell} - \kappa_{\beta} \ell \Omega_{b \ell})}{(-\omega_{n,\ell} \bar{v}_{\ell}^2)} \right] \bar{v}_{\ell}^6 \exp(-\bar{v}_{\ell}^2) \quad (9)
\end{aligned}$$

where $\Omega_{\xi\ell}$, $\Omega_{b\ell}$ and \bar{v}_ℓ are evaluated at $v = v_{n,\ell}$. Note that if there were no anisotropy, the universal drive term proportional to ω^* is stabilizing for counter-propagation since $\omega^*/\omega_0 < 0$ (however with a hollow energetic particle distribution, where ω^* changes sign, there can be an additional growth rate enhancement).

The anisotropy term is dominant at $\omega_0 = -\omega_{n\ell}$. Taking account of the finite frequency shift, we obtain the following estimate for the growth rate, γ ,

$$\gamma \approx \left[-\omega_{n\ell}^3 \tilde{\gamma}^2 (\kappa_\xi n \Omega_{\xi\ell} - \kappa_b \ell \Omega_{b\ell})^2 \bar{v}_\ell^4 \exp(-\bar{v}_\ell^2) \right]^{1/5}.$$

The linear growth rate is a function of the parameters $\bar{\beta}$, r_0/a , a/R_0 , r_0/m , $q(r_0)$, S , $\hat{\epsilon}$, $\frac{a}{\bar{\beta}} \frac{d\bar{\beta}}{dr}$, σ , $\frac{(2T/M)^{1/2}}{v_A}$, $\omega_0/\omega_{n\ell}$. We calculate the growth rate for the Tokamak Fusion Test Reactor (TFTR) experiment in which TAE modes destabilized by ICRF heating of minority hydrogen ions have been observed.⁷ Typical parameter values are: $R_0 = 262$ cm, $a = 96$ cm, magnetic field $B \sim 34$ kilogauss, safety factor $q \sim 1.25$ at $r_1/a \sim 0.31$, minority hydrogen ion temperature $T_i = mv_0^2/2 \sim 0.5$ MeV, and mode frequency $|\omega_0| \sim (1.35)10^6$ radians per sec.

We evaluate the growth rate by numerically solving Eq. (5) for $\gamma = \text{Im}(\omega)$. We take a distribution function sharply peaked in λ ($\sigma = 0.01r_0/R_0$), and we consider $n = 2$, $m = 2$, $\hat{\epsilon} = 0.11$, $(\frac{a}{\bar{\beta}} \frac{d\bar{\beta}}{dr}) = 2$, and $\bar{\beta} = 0.004$. We calculate $\gamma/|\omega_0|$ for both co-propagation and counter-propagation, and we compare the relative growth rate magnitudes with and without particle anisotropy.

In Fig. 1, we plot the growth rate $\gamma/|\omega_0|$ versus $(2T/M)^{1/2}/v_A$ for co-propagating TAE modes with positive frequency $\omega_0 = \frac{v_A}{2qR_0} = 1.35 \times 10^6$ radians per second. The growth rate $\gamma/|\omega_0| \sim 0.03$ is a maximum at $(2T/M)^{1/2}/v_A \sim 1.4$ corresponding to a mean particle energy of $T = 0.5$ MeV. The magnitude of the growth rate is determined by the sum of several bounce harmonic resonances, although the main contribution at the growth rate maximum is due to the $\ell = 0$ bounce harmonic resonance.

For comparison, we also plot the growth rate $\gamma/|\omega_0|$ for an isotropic distribution function ($\sigma \rightarrow \infty$). The threshold for instability occurs at $(2T/M)^{1/2}/v_A = 0.9$ corresponding to $n\omega^*/\omega_0 = 1$. We note that the growth rate for the anisotropic particle distribution is considerably larger, and it is positive over a somewhat wider frequency band.

For our parameters, the critical frequency for counter-propagating TAE modes is $\omega_{n=2,\ell=2} = -(1.35)10^6$ radians per second at the $\ell = 2$ bounce harmonic resonance. In Fig. 2, we plot the growth rate $\gamma/|\omega_0|$ versus $(2T/M)^{1/2}/v_A$ for four values of the frequency ratio $\omega_0/\omega_{n=2,\ell=2} = 1.003, 0.999, 0.993, 0.979$. When $\omega_0/\omega_{n=2,\ell=2} = 0.999$, the growth rate $\gamma/|\omega_0| \sim 0.1$ is a maximum at $(2T/M)^{1/2}/v_A \sim 1.55$ corresponding to a mean particle energy of $T = 0.65$ MeV. The growth rate maximum is a very sensitive function of $\omega_0/\omega_{n=2,\ell=2}$, and it is significant only when the mode frequency ω_0 is close to the critical frequency $\omega_{n,\ell}$. These curves imply the following for our numerical example: (1) the growth rate maximum for counter-propagating TAE modes is more than three times larger than the growth rate maximum for co-propagating TAE modes; (2) the parameter limits within which counter-propagating TAE modes are preferentially destabilized are rather narrow, and this can result in sporadic experimental observation of counter-propagating modes destabilized by particle anisotropy.

These results predict finite TAE growth rates for modest values of $\bar{\beta} \sim 0.004$, consistent with the observation in TFTR (and other Tokamak experiments) of unstable TAE modes with low toroidal mode numbers destabilized by ICRF heating.⁷ In the case of co-propagation, particle anisotropy enhances the universal instability drive and reduces the threshold for the onset of instability.

In the case of counter-propagation, the effects of particle anisotropy is considerably enhanced when the wave frequency is close to certain critical frequencies, and this enhancement can overcome the now stabilizing role of spatial gradients (non-hollow profiles) to preferentially destabilize counter-propagating TAE modes.

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FIGURE CAPTIONS

FIG. 1. growth rate of co-propagating TAE modes for anisotropic and isotropic particle distribution functions: $\gamma/|\omega_0|$ vs. $v_0/v_A \equiv (\frac{2T}{M})^{1/2}/v_A$ for $n = 2$, $|\omega_0/\omega_{2,2}| = 1.0$

FIG. 2. growth rate of counter-propagating TAE modes destabilized by particle anisotropy: $\gamma/|\omega_0|$ vs. $v_0/v_A \equiv (\frac{2T}{M})^{1/2}/v_A$ for $n = 2$, $\omega_0/\omega_{2,2} = 1.003, 0.999, 0.993, 0.979$

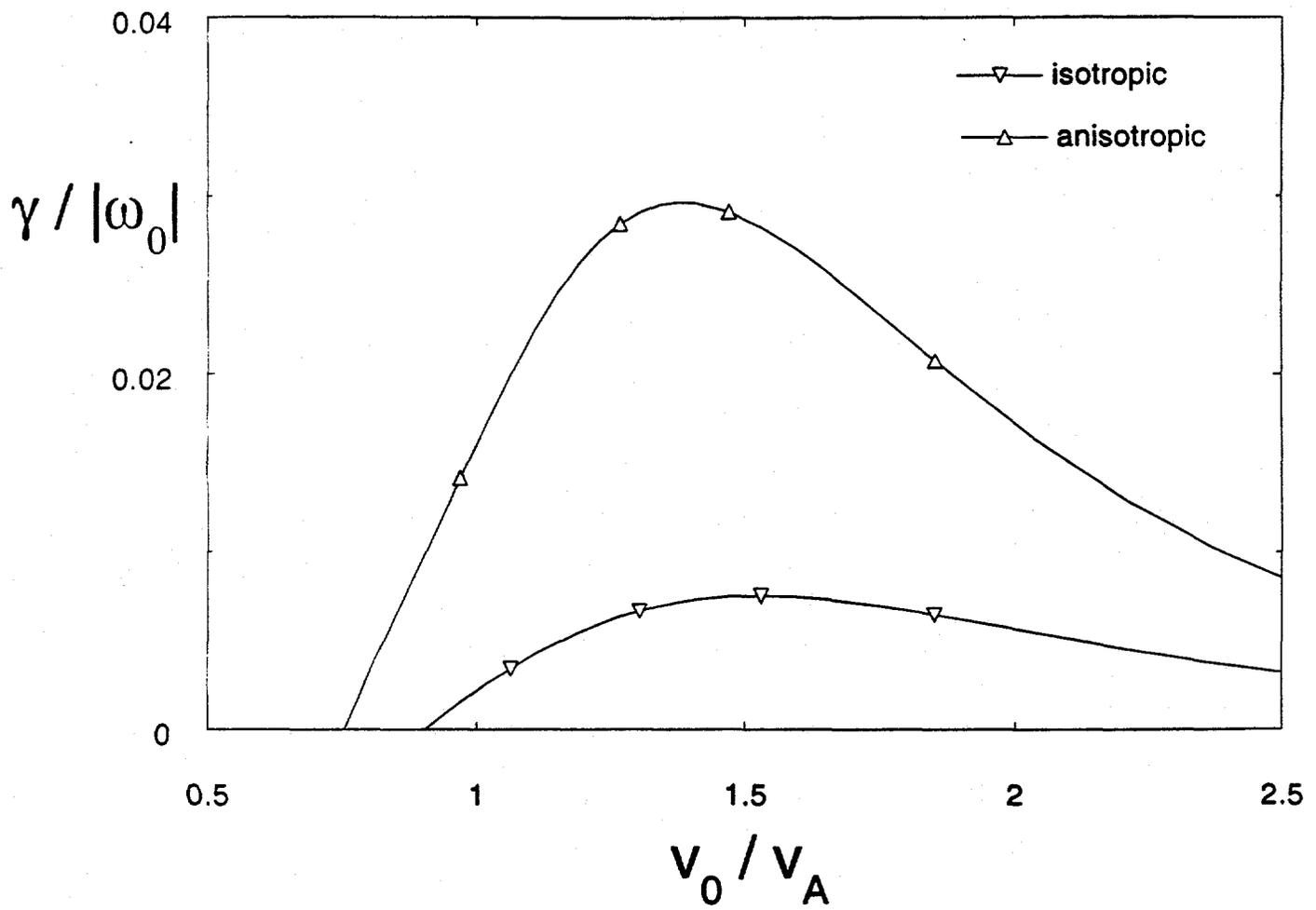


Fig. 1

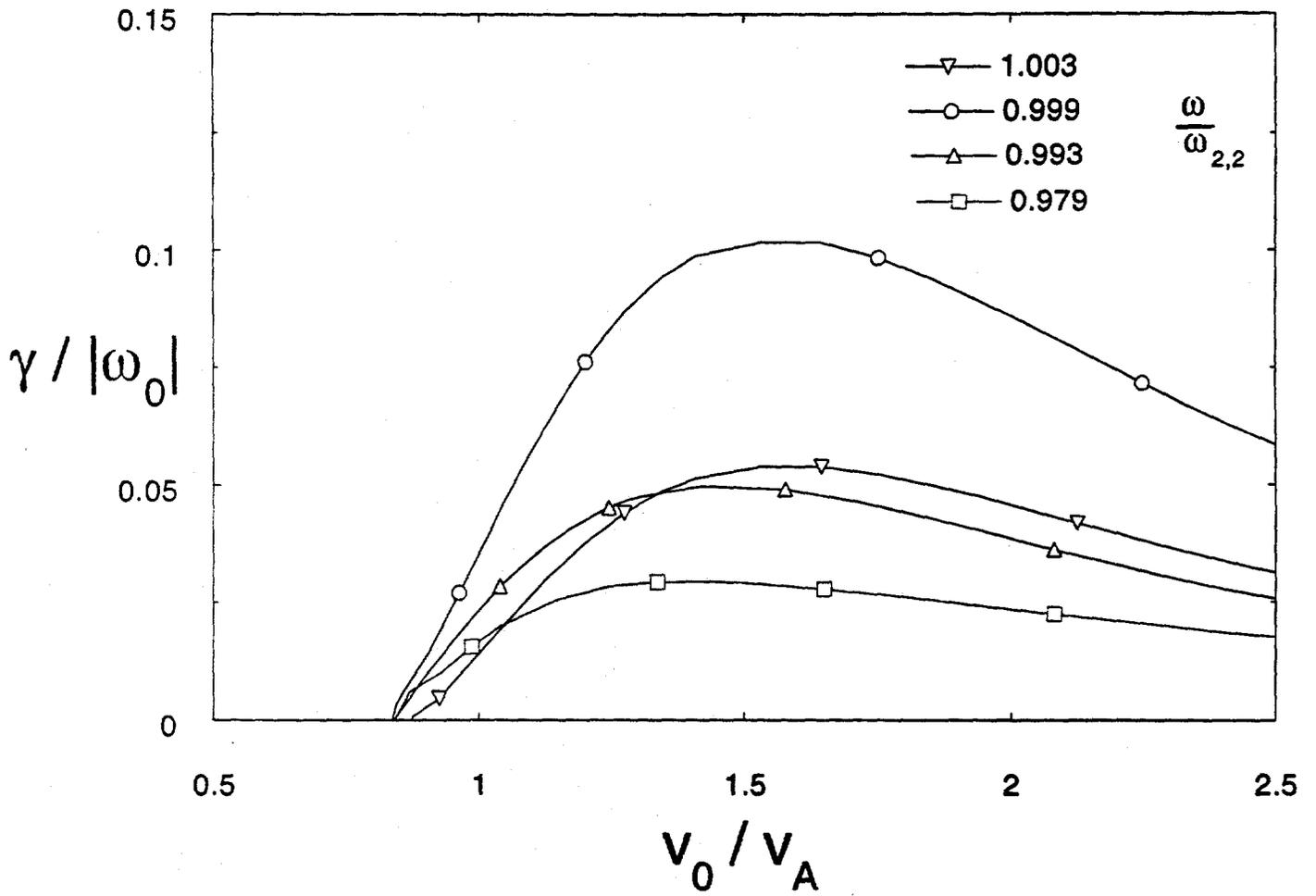


Fig. 2