MASTER

FINAL REPORT

Magnetotelluric Interpretations in a Crustal Environment

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Abstract

Problems that arise in interpreting magnetotelluric measurements in complex geologic environments are approached by considering the low frequency approximation where the upper crust can be considered a thin layer of varying conductivity. The formulation of the thin layer problem, originally presented by Price, is modified to allow resistive coupling between the upper crust and the mantle. This is an important modification that is necessary to realistically model situations such as those that arise in geoelectric exploration for geothermal resevoirs. A new parameter arises, called the adjustment distance, which is the distance required for the current to readjust itself between the upper crust and the mantle when changes in crustal conductivity are encountered. This distance is given by the square root of the product of the integrated conductivity and the integrated resistivity across the crust (and some of the upper mantle). Layered media interpretations made on magnetotelluric data from areas that are closer to a conductivity boundary than the adjustment distance can be very misleading. Since the adjustment distance is often greater than 100 km, one has to consider more complex modelling in many situations. In order to simplify the modelling of the large region dictated by the adjustment distance we have applied the concept of a multiple scale analysis to the thin layer calculations. This phase of the study is not finished, however.

A simple interpretation of actual field data from the Snake River Plains is given which illustrates the adjustment distance effect on magnetotelluric fields.

Introduction

A technique for dealing with the electromagnetic response of a thin conducting zone was introduced by A.T. Price over thirty years ago (Price, 1949). His method needed certain modifications, however, to be applied to real geologic situations. These modifications must account for a finite resistivity of the lower crust and upper mantle. When such modifications are made one finds that low frequency magnetotelluric responses in complex geologic settings are sensitive to the resistivity of the lower crust. This comes about because the crustal current levels are modified by resistive coupling to the mantle, and the distance required for the current adjustments to take place depend in part on the crustal resistivity. Thus modelling the lower crustal resistivities becomes as important as modelling the upper crustal conductivities. We have developed such a modelling for low frequencies when both the upper and lower crust can be considered thin. This method we call a generalized thin sheet analysis.

In the first section we review the basis of the generalized thin sheet analysis and present approximate analytic solutions for simple geometries. In the second section we briefly introduce the concept of a multiple scale analysis to lessen the computations needed to solve realistic models. In the third section we illustrate the applications of these methods by considering magnetotelluric data obtained in the Snake River Plains (Stanley, Bochl, Bostick, and Smith, 1977).

I. The generalized thin sheet approximation

Starting from Maxwell's equations for low frequencies (no displacement currents) and an e^{-iwt} time dependence

$$\nabla \times \hat{E} = i\mu\omega \hat{H} \tag{1}$$

$$\nabla \times \hat{H} = \sigma \hat{E} \tag{2}$$

and assuming a transversly isotropic conductivity which is obtained with layered media

$$\sigma = \begin{bmatrix} \sigma_{tt} & 0 & 0 \\ 0 & \sigma_{tt} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \tag{3}$$

we can write from equation (1)

$$\frac{\partial \hat{E}_s}{\partial z} = -i\mu\omega \hat{i}_z \times \hat{H}_s + \nabla_s (E_z) \tag{4}$$

$$H_z = (\nabla_s \times E_s) \cdot \hat{i}_z / i\mu\omega \tag{5}$$

and from equation (2)

$$\frac{\partial \hat{H}_s}{\partial z} = -\sigma_{tt} \hat{i}_z \times \hat{E}_s + \nabla_s (H_z) \tag{6}$$

$$E_z = (\nabla_s \times \hat{H}_s) \cdot i_z / \sigma_{zz} \tag{7}$$

where

$$\hat{E}_s = \hat{i}_x E_x + \hat{i}_y E_y$$
$$\hat{H}_s = \hat{i}_x H_x + \hat{i}_y H_y$$
$$\nabla_s = \hat{i}_x \frac{\partial}{\partial x} + \hat{i}_y \frac{\partial}{\partial y}$$
$$\nabla_s \times = \hat{i}_z \left(\frac{\partial}{\partial x} \hat{i}_y - \frac{\partial}{\partial y} \hat{i}_x \right)$$

When the layer is thin we can change equations (4) and (6) into difference equations and using the following boundary conditions

at the base of the thin layer

$$\hat{E}_s = Z \hat{H}_s$$

at the surface

$$\hat{H}_s = Y' \hat{E}_s + \hat{H}_s^{\circ}$$

where Z is the mantle magnetotelluric impedance

Y' is the air magnetotelluric admittance for all but the source wavelength

obtain, after eliminating H_s, the following equation for the surface E field

$$\hat{E}_s - Z^* Y^* \hat{E}_s - \nabla_s \left(\rho_s \nabla_s \cdot \sigma_s \hat{E}_s \right) - Z^* \frac{\Delta z}{i\mu\omega} \nabla_s \left[(\nabla_s \times E_s) \cdot \hat{i}_z \right] = Z^* \hat{H}_s^{\circ} \qquad (8)$$

where

$$Z^* = \left(Z + i\mu\omega\Delta z \, \hat{i}_z \times \right)$$
$$Y^* = \left(Y' - \sigma_s \, \hat{i}_z \times \right)$$
$$\rho_s = \int \sigma_{zz}^{-1} \, dz$$
$$\sigma_s = \int \sigma_{\tau\tau} \, dz$$

We can illustrate the behavior of solutions to equation (8) by considering a two dimensional model with a source field polarized with E perpendicular to strike which is taken in the Y direction. If the source field is uniform in the Y direction both $Y'E_s$ and $\nabla_s \times E_s$ are identically zero and equation 8 simplifies to

$$E_x - \frac{\partial}{\partial x} \rho_s \frac{\partial}{\partial x} \sigma_s E_x = Z^*_{xy} \left(H^o_y - \sigma_s E_x \right) \tag{9}$$

The right hand side is essentially the Cagniard solution for the local conductivity, E_x°. In a uniform region where ρ_s and σ_s are constant we can write

$$E_x - \rho_s \sigma_s \frac{\partial^2}{\partial x^2} E_x \cong E^o_x \tag{10}$$

with solutions $$E_x = E^o_x + A_{\pm} \exp\left(\frac{\pm x}{\sqrt{\rho_s \sigma_s}} \right) \tag{11}$$

These solutions show the adjustment distance effect. If two such regions abut each other we must impose the boundary conditions of continuity of $\sigma_s E_x$ and continuity of $\rho_s \sigma_s \frac{\partial E_x}{\partial x}$. The former is due to the continuity of H_y across the boundary and the latter is due to the continuity of E_z. These conditions determine the magnitude of A in the adjacent regions as follows

$$A_1 = \frac{\sigma_{s2} E^o_{x2} - \sigma_{s1} E^o_{x1}}{\sqrt{\rho_{s1} \sigma_{s1}} \left(\sigma_{s1} / \sqrt{\rho_{s1} \sigma_{s1}} + \sigma_{s2} / \sqrt{\rho_{s2} \sigma_{s2}} \right)} \tag{12}$$

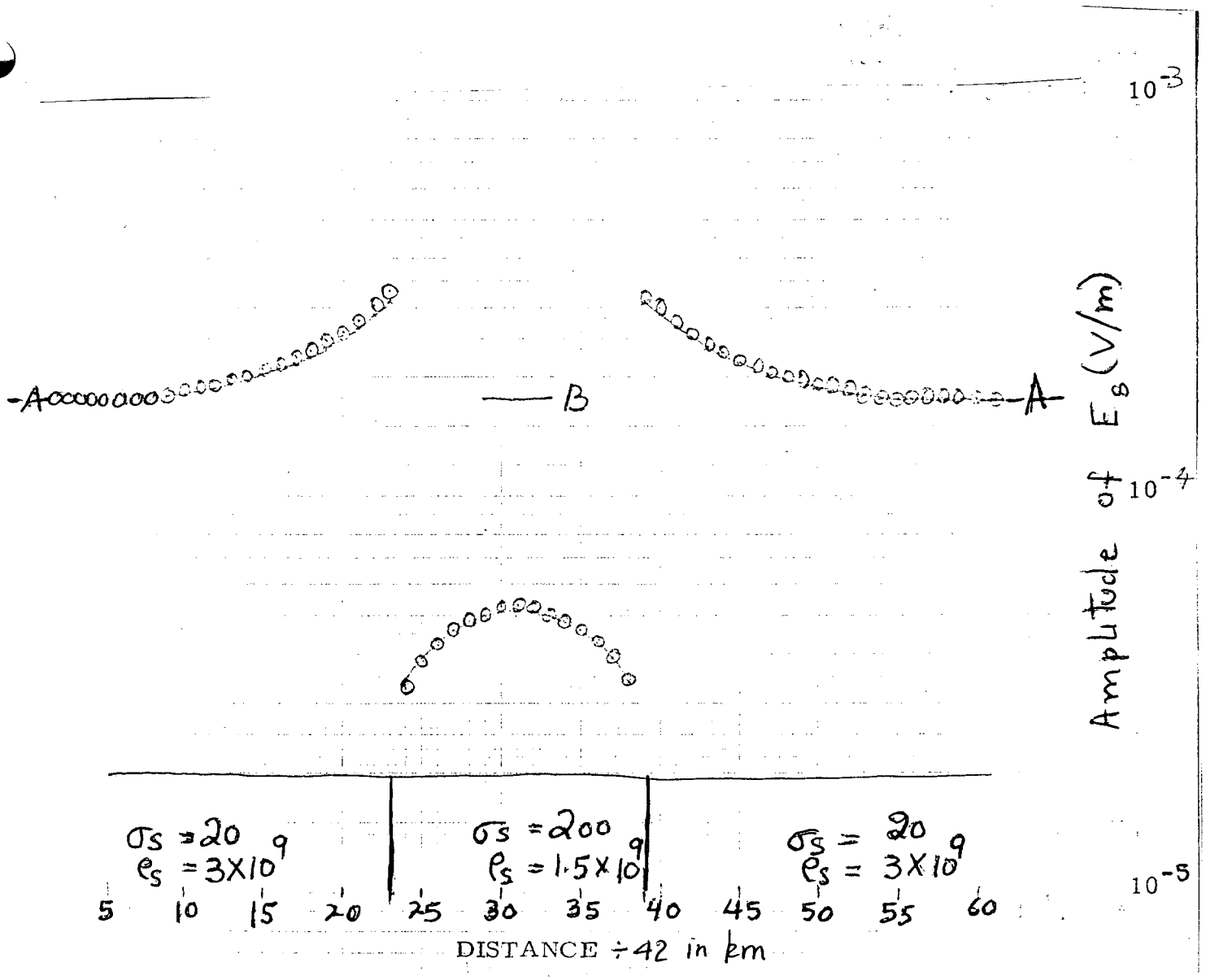


Fig 1 Adjustment distance effects in two dimensional model for $E_\perp$. Frequency 10^{-4} Hertz. $H_s^o = 796\gamma$

A and B are Cagniard solution for the respective zones

$$A_2 = \frac{-\left(\sigma_{s2} E^{o}_{x2} - \sigma_{s1} E^{o}_{x1}\right)}{\sqrt{\rho_{s2}\sigma_{s2}}\left(\sigma_{s1}/\sqrt{\rho_{s1}\sigma_{s1}} + \sigma_{s2}/\sqrt{\rho_{s2}\sigma_{s2}}\right)} \tag{13}$$

Thus the boundary anomaly is due to variations in σ_s, but the partitioning of the anomaly between the regions and the geometry of the anomaly depends on ρ_s as well as σ_s.

Figure 1 shows such solutions for a conductive zone sandwiched between less conductive zones assuming an excitation field of 1 amp m^{-1} (795.82) at 10^{-4} Hertz.

These solutions are quite frequency independent so that magnetotelluric profiles within the adjustment zone would all be similar except for the changes in level. One dimensional interpretations of these data would be completely misleading however.

II. Multiple scale calculation methods

The fact that the adjustment distance can often be hundreds of kilometers means that realistic modelling of magnetotelluric references must often include a very large region around the measurement sites. This would seem to make the modelling problem a huge undertaking. On the other hand one does not need to know the solution in the outer regions in great detail, only their average effects are important. One approach then to such modelling is a multiple scale method. The region in question can be subdivided into large blocks and the average property of each block determined by a small scale calculation. A large scale calculation is then made on the model using the average properties for each block. A small scale calculation is then made for the block of specific interest, using the large scale results for all the other blocks as knowns. Figures

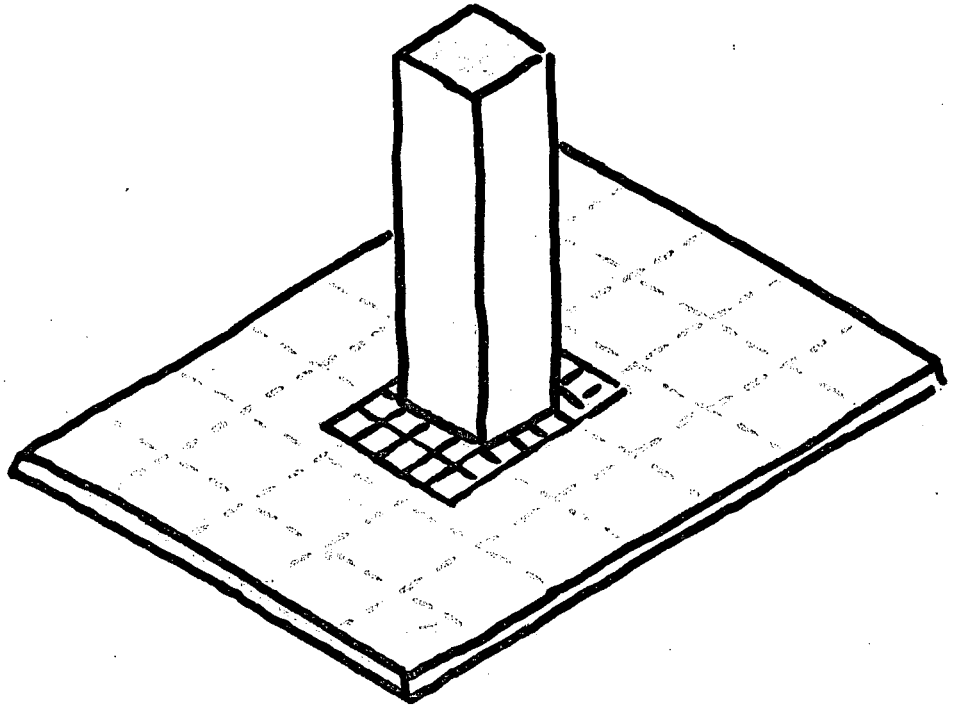


Fig 2

2 Scale Resistivity Model

Histogram of conductivity structure given as $1/\sigma_s$

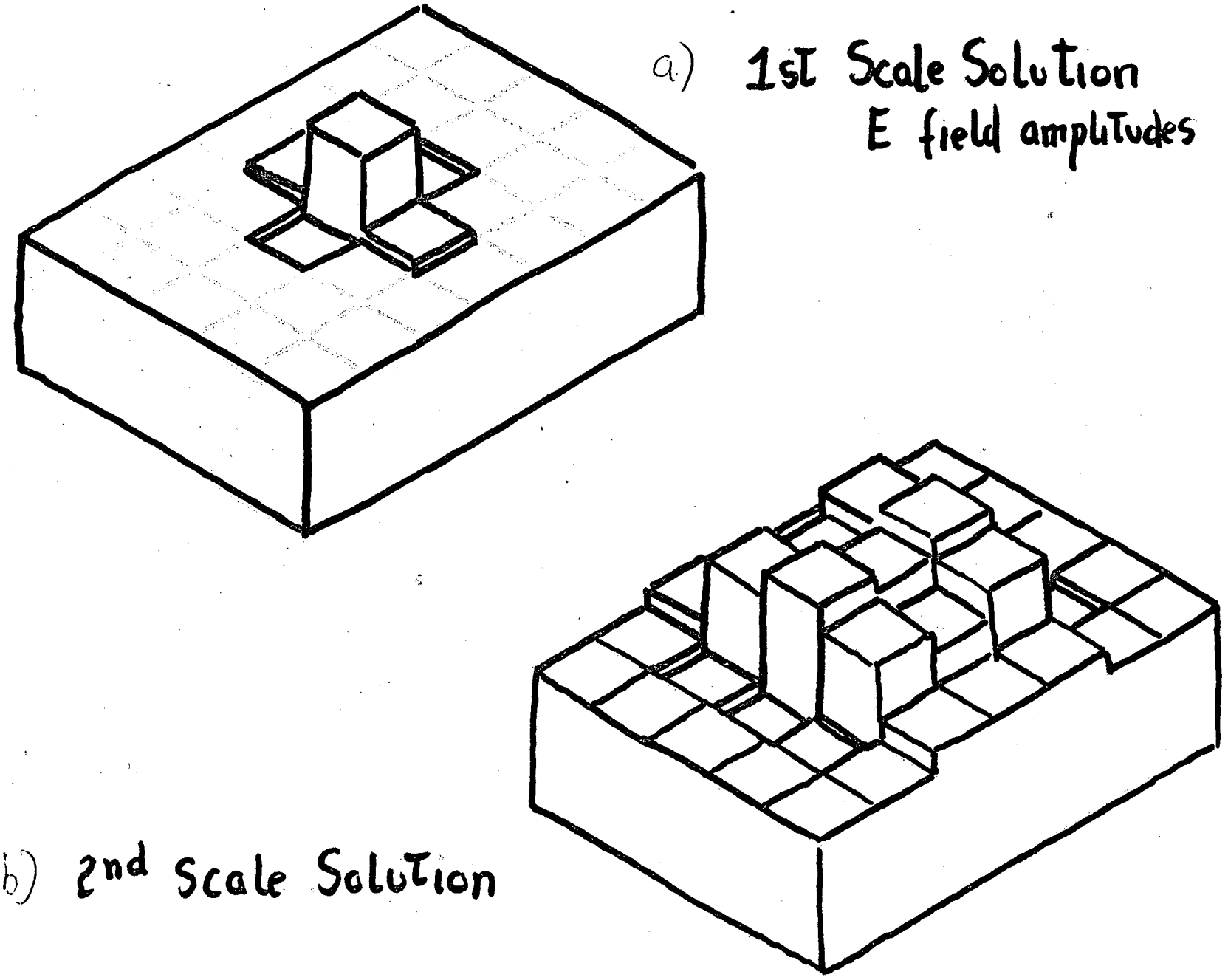


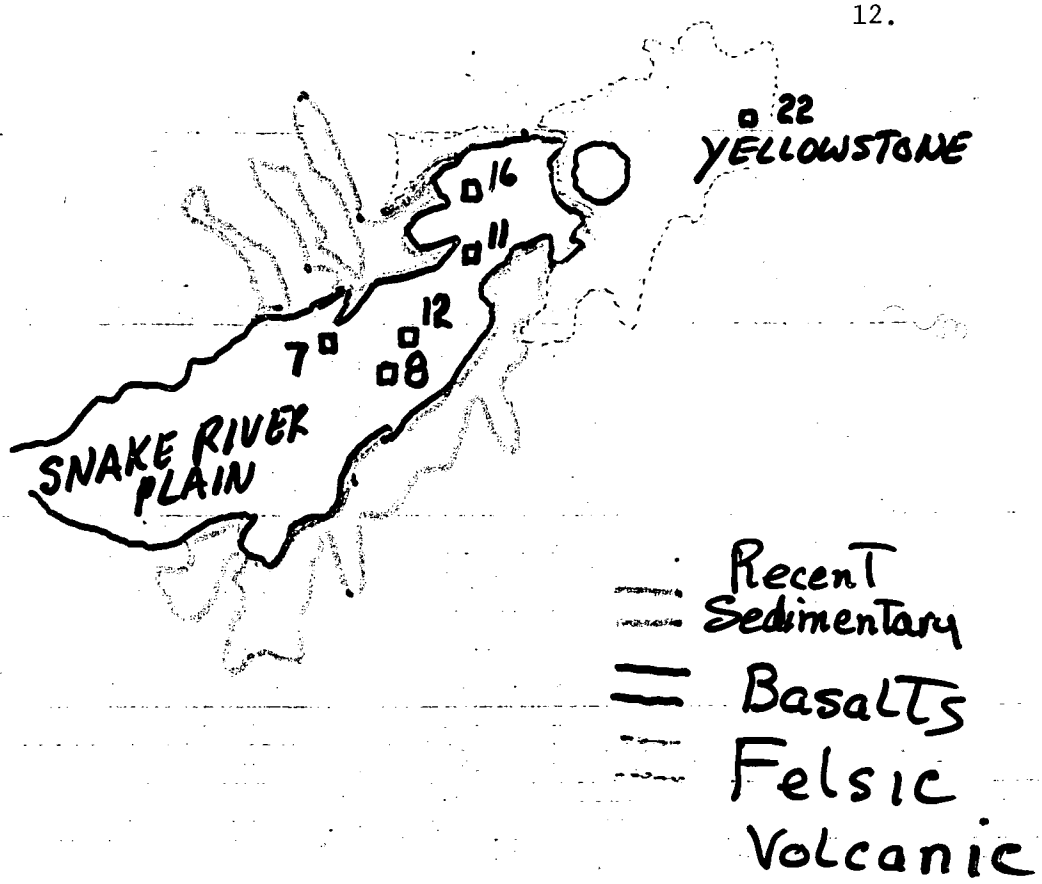
Fig 3 Multiple scale backwards solutions, E field amplitudes plotted as histogram

2 and 3 illustrate this concept. Figure 2 is a histogram of the resistivity model which represents a resistive 'island' surrounded by a conductive 'ocean'. The whole region is represented by 35 large blocks. The inner region of interest is represented by 35 small blocks. In this case the average property of each large block is known since they were homogeneous, and Figure 3a shows the large scale solution. The inner group is then solved again fixing the electric field values outside this region at their values as obtained in the large scale solution. The solution for the inner region is shown in Figure 3b.

These methods work very well for two dimensional problems or for problems where the region of interest was more resistive than the outer regions, but severe problems arose in other situations. The difficulty mainly stems from the Y' operator which is sensitive to the details of the outer region solutions. The problem is akin to the Gibbs phenomena due to the sharp cutoff in wavenumber that occurs when a jump in scale is made. We have not solved this problem and it may be necessary to reformulate the problem so as to avoid impedance conditions at the surface.

III. Snake River Plains: an example of adjustment distance effects

Figure 4 shows magnetotelluric data and the station locations from the Yellowstone-Snake River Plain area. This is a subset of the data shown by Stanley et al (1977) chosen to illustrate the self similarity that is typical of adjustment distance effects, i.e., the shape of the apparent resistivity levels are similar, but the levels are displaced. Because of this one suspects that the apparent resistivity levels are not representative of a layered media model. If the Sanke River Plain region was more conductive than the surrounding regions or at least the regions to the NW, N, and NE, one would expect the apparent resistivities at the low frequencies to be



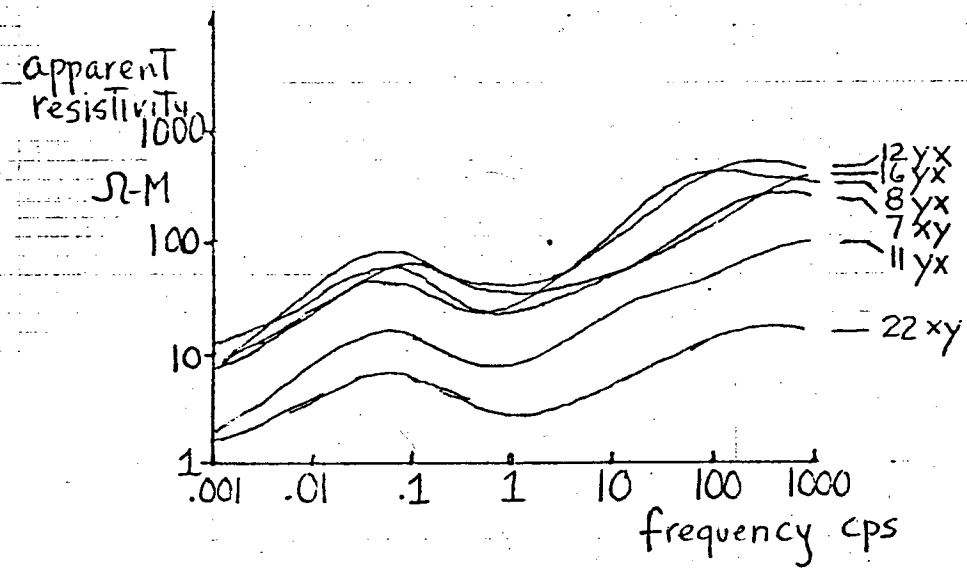
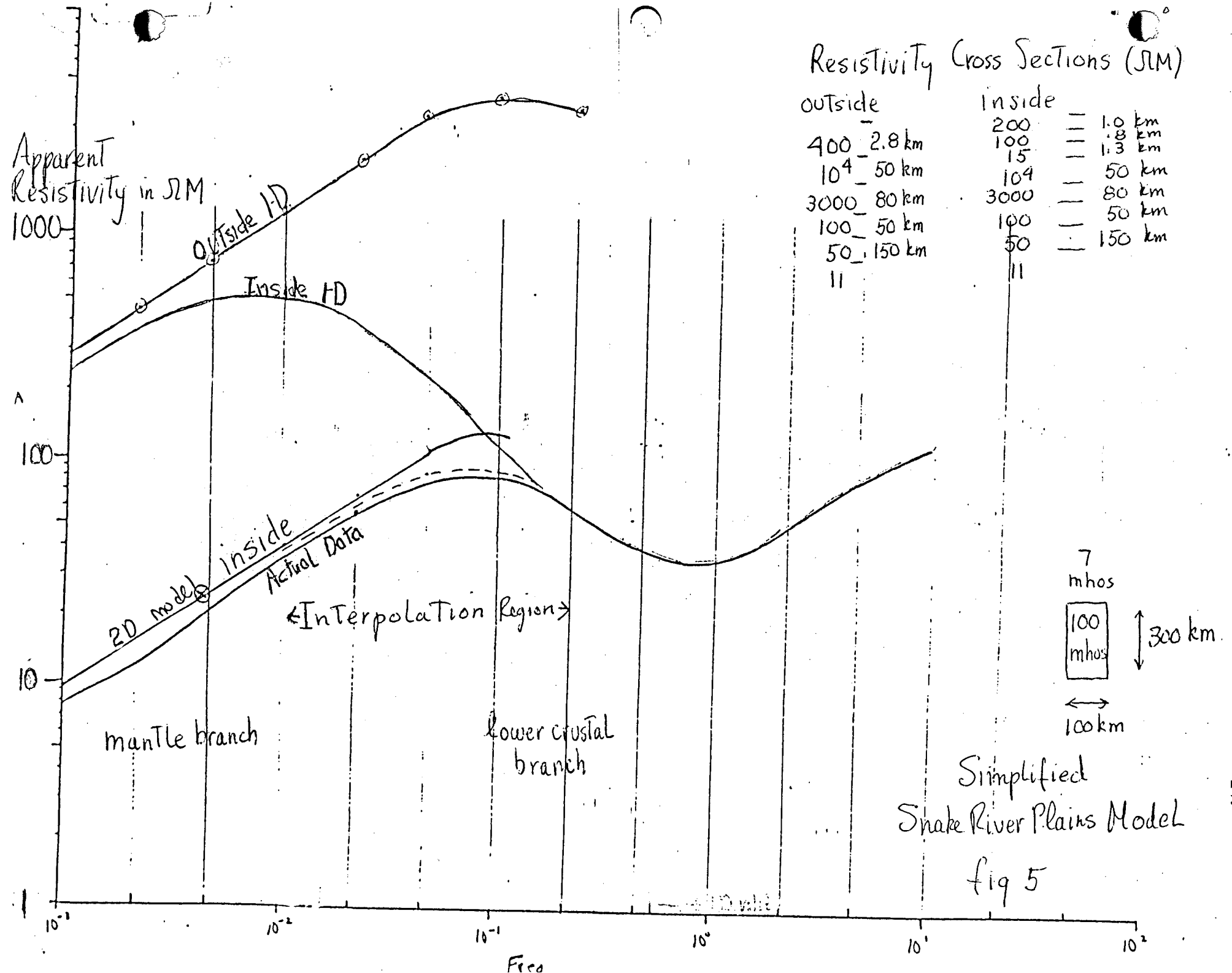


Fig 4 Self-similar MagneToTelluric DaTa in Snake River Plain
(from Stanley, Boehl, Bostick, and Smith; 1977)

depressed. The similarity of the curves at the higher frequencies must indicate that local inhomogeneities also exist. Cagniard layered media interpretations of this data led to anomalously low resistivity values for the lower crust, but it is very likely that these interpretations are quite false. To see if such data could result from near surface conductivity variations we attempted to model the data with a normal lower crust and upper mantle conductivity structure. We assumed that station 12 gave the regional result and that the depression of the other curves was due to more local features. Under this assumption we can expect the high frequency behavior of station 12 to faithfully represent the layered media solution, but the low frequency response would reflect the influence of the outer regions which prevent the current levels from reaching those expected if the station was far from the boundaries. A quasi three dimensional interpretation can then be made by a) finding a conductivity structure for the Plain that satisfies that high frequency data, and b) finding a conductivity structure for the outside region that causes the appropriate decrease of apparent resistivity at the low frequencies. This is illustrated in Figure 5. The curves marked 1-D are layered media solutions and the curves marked 2-D are thin layer calculations. These curves merge well into the actual data with very reasonable conductivity models.

To interpret real data one really needs three dimensional modelling, but this example illustrates the pitfalls of ignoring the regional conductivity structure and attempting to use one dimensional interpretations of magnetotelluric data.


Apparent Resistivity in ΩM
1000
100
10
1
Outside 1-D
Inside 1-D
2D model inside
Actual Data
<Interpolation Region>
mantle branch
lower crustal branch
Resistivity Cross Sections (ΩM)
outside
400 _ 2.8 km
10^4 _ 50 km
3000 _ 80 km
100 _ 50 km
50 _ 150 km
11
inside
200 — 1.0 km
100 — .8 km
15 — 1.3 km
10^4 — 50 km
3000 — 80 km
100 — 50 km
50 — 150 km
11
7 mhos
100 mhos
300 km
100 km
Simplified Snake River Plains Model
fig 5

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