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J. C. Simpson

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Pacific Northwest Laboratory
Richland, Washington 99352

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AN APPROACH FOR TESTING ATTAINMENT OF SOIL BACKGROUND STANDARDS AT SUPERFUND SITES

Richard O. Gilbert
Jeanne C. Simpson
Pacific Northwest Laboratory[a]
P. O. Box 999
Richland, WA 99352

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ABSTRACT

After the soil at a Superfund site has been remediated it is necessary to determine if the remediation effort has been successful. This determination involves comparing concentrations in soil at the remediated site with cleanup standards. The cleanup standard may be based on technological capabilities, a risk assessment, or site-specific background concentrations. In this paper we discuss an approach for using two complementary nonparametric tests, the Wilcoxon Rank Sum (WRS) test and the Quantile test, to assess attainment of site-specific background standards at remediated Superfund sites. The tests are complementary in the sense that the WRS test is more powerful than the Quantile test to detect shift alternatives, i.e., to detect when the remedial action failed more or less uniformly throughout the Superfund site, whereas the Quantile test has more power than the WRS test to detect mixture alternatives, i.e., to detect when remedial action was inadequate in only a portion of the site. The approximate minimum power of the Quantile test is obtained and used to develop sample size tables for the test. A simple "hot spot" test is also used to insure that remedial action is conducted at least locally when any measurement exceeds a specified upper-limit value H_S.

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1. INTRODUCTION

After the soil at a Superfund site has been remediated it is necessary to determine if the remediation effort has been successful by comparing concentrations in soil at the remediated site with a cleanup standard. The cleanup standard may be a single number, assumed known with no uncertainty, that was derived on the basis of a risk assessment. USEPA (1989) provides statistical tests to assess if a soils-remediation effort has succeeded at attaining this type of risk-based standard.

If the contaminant(s) at the Superfund site also occur naturally or have been deposited from regional or world-wide activities of man, then site-specific background standards may be used in addition to or instead of risk-based standards. This paper presents a strategy for using nonparametric tests to assess if a soils remediation effort has succeeded in attaining site-specific background standards. The strategy is intended to provide statistical sampling and testing procedures that are simple to use yet provide reasonable power to detect when remedial action has not fully succeeded.

The main features of the testing strategy are as follows.

1. Relatively simple sampling and testing procedures are used to promote wider understanding and use by nonstatisticians.

2. The null and alternative hypotheses that are used are

   H_o: Cleanup Standard Achieved
   H_a: Cleanup Standard Not Achieved (1)

   These hypotheses are the reverse of those that are used in USEPA (1989) for risk-based standards:

   H_o: Cleanup Standard Not Achieved (2)
   H_a: Cleanup standard Achieved

   The hypotheses in Equation 2 are not used here for background standards because they would require that the Superfund-site measurements lie demonstrably <u>below</u> the background measurements before accepting H_a that the background cleanup standard had been achieved.

3. The remediated superfund site may be divided into cleanup units for which separate decisions will be made using statistical tests.

4. Nonparametric tests are used primarily to avoid the unrealistic normal-distribution assumption and to provide increased capability to handle moderate or large numbers of less-than data (data less than the detection limit).

5. Two complementary nonparametric tests are used: the Wilcoxon Rank Sum (WRS) test (Conover 1980) and the Quantile test (Johnson et al. 1987).

This dual testing is done so that non-attainment of standards from both uniform or partial failure of the remedial action can be detected.

6. A simple "hot spot" test is used in addition to the WRS and Quantile tests. Specifically, if any measurement at the remediated site exceeds a prespecified value H_S, then some type of additional remedial action is required, at least locally, regardless of the outcome of the WRS and Quantile tests. The value chose for H_S might be determined by negotiation between the U.S. Environmental Protection Agency or a similar state agency and the Superfund site owner/operator. The value of H_S might be based on a risk assessment or on the distribution of background measurements.

7. The number of samples required to achieve specified power levels will differ for the WRS and Quantile tests. The following rules are suggested for deciding the number of samples to collect:

   i) If failure of remedial action is likely to leave concentrations uniformly too large, use the number of samples required for the WRS test.

   ii) If failure of remedial action is likely to leave concentrations too large in only some areas, use the number of samples required for the Quantile test.

   iii) If there is little difference between the number of samples determined for the two tests, or if there is little or no information about patterns of contamination that may remain, use the larger of the two sets of sample sizes determined for the two tests.

8. Random-start triangular grid sampling is used except when the remedial action method is expected or known to leave contamination in a pattern that could be missed by a triangular grid, in which case unaligned systematic grid sampling (Cochran 1977; Gilbert 1987) is recommended.

9. Resampling and retesting, rather than multiple-comparison tests (Hochberg and Tamhane 1987; Miller 1981), are used to maintain Type I error rates of individual tests to acceptable levels. Multiple-comparison tests are not used because they have reduced power to detect non-attainment of background standards. High power is needed to protect the public (see the hypotheses in Equation 1).

The above strategy can be applied in combination with more computationally intensive geostatistical methods (USEPA 1989) when the data base is sufficient to support those methods. Also, an evaluation of the number of samples required to detect hot spots of a given size (Gilbert 1987) can also provide a perspective on the probability of missing hot spots when the number of samples required for the WRS and Quantile tests are collected.

Section 2 gives definitions, assumptions, and the two contamination scenarios considered in this paper. Sections 3 and 4 discuss the WRS test and the Quantile test, respectively. A summary is given in Section 5.

2.0 DEFINITIONS, ASSUMPTIONS, AND SCENARIOS

2.1 DEFINITIONS

Cleanup Units: geographical areas of specified size and shape at the remediated Superfund site for which separate decisions will be made whether each unit attains the site-specific background standard for the designated contaminant (pollution parameter).

Background Region: the geographical region from which background areas will be selected.

Background Areas: the site-specific geographical areas from which representative background samples will be selected for comparison with samples from specific cleanup units at the remediated Superfund site.

At some Superfund sites a single background area may not be suitable for all cleanup units because the units may have different physical, chemical, or biological characteristics. The number and size of cleanup units in the remediated Superfund site will depend on many factors including the size of the site, past and future use of the site, whether the type of remedial action is likely to leave residual contamination in a known pattern, and geographical features (hill-sides, ponds, flat land, etc.).

2.2 ASSUMPTIONS

The following assumptions are used in this document (taken in part from Liggett 1984).

1. The background area contains no contamination from the Superfund site being tested.

2. The background area does not differ from the cleanup unit in physical, chemical, or biological characteristics that are extraneous to the question of excess contamination in the cleanup unit but that might cause measurements in the background area and the cleanup unit to differ.

3. There is no requirement that the site be remediated to levels less than those in the site-specific background area.

4. Contaminant concentrations in the background area and cleanup units do not cycle or change significantly during the sampling period.

5. Measurements are not spatially correlated.

2.3 CONTAMINATION SCENARIOS

Suppose that contamination above background levels is still present in the remediated cleanup unit being tested. Two types of contamination scenarios for the cleanup unit are considered in this paper:

1. Shift Alternative Scenario:

   Remedial action has failed more or less uniformly throughout the cleanup unit so that the distribution of measurements in the cleanup unit is simply shifted to the right (to higher values) of the background distribution for the pollution parameter of interest.

   The Wilcoxon Rank Sum (WRS) test is used for this contamination scenario (Section 3).

2. Mixture Alternative Scenario:

   Concentrations in only a proportion, ϵ, of the soil in the remediated cleanup unit still exceed concentrations in the applicable background area. That is, the distribution of $100\epsilon\%$ of the measurements in the remediated cleanup unit are shifted to the right of the distribution in the background area.

   The Quantile test has greater power than the WRS test for this contamination scenario (Section 4).

3.0 UNIFORM RESIDUAL CONTAMINATION AND THE WILCOXON RANK SUM TEST

If the remedial action procedure has potential for leaving concentrations in the cleanup unit that are more or less uniformly higher than background levels, then a test that is designed to detect such a "shift" is needed. The Wilcoxon test is known to have good power for this case (Conover 1980, p. 225; Lehmann 1975; Helsel and Hirsch 1987). Also, the Wilcoxon test can be used when less-than data are present by treating those data as having a single value less than the smallest measured value.

3.1 HYPOTHESES

When using the WRS test, the hypotheses in Equation 1 can be stated as

$$H_o: \; P_r = 1/2 \qquad (3)$$
$$H_a: \; P_r > 1/2$$

where

P_r = probability that a measurement of a sample collected at a random location in the cleanup unit is greater than a measurement of a sample collected at a random location in the background area. (4)

The hypotheses are stated in the form given by Equation 3 rather than in the more commonly used shift form primarily because (1) the shape or form of the distribution of measurements in the cleanup unit need not be specified, and (2) the formula used to approximate the number of samples (Equation 5 below) is expressed in terms of P_r.

The "cleanup standard" for the WRS test is the value of P_r given in the H_0, namely $P_r = 1/2$. If the distribution of measurements at the remediated cleanup unit is identical to that in the applicable background area then $P_r = 1/2$. If $P_r > 1/2$, then some or all of the distribution of measurements in the remediated cleanup-unit lies to the right of the distribution for the background area.

3.2 NUMBER OF REQUIRED SAMPLES

Let m and n denote the number of measurements in the background area and the cleanup unit being compared with the background area. Let N = m + n. Then N may be determined using the approximate formula (from Noether 1987)

$$N = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{12c(1 - c)(P_r - 0.5)^2} \qquad (5)$$

where α and β are the specified Type I and II error rates, $Z_{1-\alpha}$ and $Z_{1-\beta}$ are the standard normal deviates that cut off $(100\alpha)\%$ and $(100\beta)\%$ of the upper tail of the standard normal distribution, $c = m/N$, and P_r was defined in Equation 4. Using N from Equation 5 we can compute

$$m = cN \quad \text{and} \quad n = N - m \qquad (6)$$

3.2.1 Specifying c

The parameter c must be specified to use Equations 5 and 6. One way to determine c is to assume that the distribution of measurements in both the background and cleanup units is Gaussian and that the ratio of standard deviations, $v = \sigma_b/\sigma_c$, is known or can be estimated, where σ_b and σ_c are the standard deviations for the background area and the cleanup units, respectively. Then we may compute c as follows (Hochberg and Tamhane 1987, p. 202):

$$c = v^2h^{1/2}/(v^2h^{1/2} + 1) \qquad (7)$$

where h is the number of cleanup units that will be compared with the background area.

Equation 7 minimizes the sum (or the maximum) of the variances of the differences in background and cleanup-unit means. If only one cleanup unit is being compared with background, then Equation 7 gives $c = 0.5$ if $v = 1$.

In practice, the distributions are not likely to be Gaussian and v will be unknown. Hence, Equation 7 is a rough approximation at best, but the data so obtained can be used to estimate v and evaluate the form of the distributions that are actually present.

3.2.2 Alternative Expressions of P_r

A value of P_r must also be specified to use Equation 5. As discussed by Noether (1987) it may be easier to fix in ones mind a value of the odds ratio, d, rather than a value of P_r, where

$$d = \frac{\text{probability a cleanup-unit measurement is larger than a background measurement}}{\text{probability a cleanup-unit measurement is smaller than a background measurement}} \tag{8}$$

$$= P_r/(1 - P_r).$$

Hence, if d is specified, we can compute

$$P_r = d/(1 - d)$$

for use in Equation 5.

Alternatively, for the case of Gaussian distributions when the cleanup-unit distribution is simply shifted to the right of the background distribution by the amount Δ (in units of standard deviation), then

$$P_r = \phi(0.707\Delta)$$

where $\phi(0.707\Delta)$ is the probability that a standard normal deviate is less than 0.707Δ. The parameter Δ is discussed further in Section 4.1.

3.2.3 Computing the WRS Test

The WRS test is well known and described in many statistics books, e.g., Conover (1980). The WRS test statistic when m and n are large is

$$Z_{rs} = \frac{W_{rs} - n(N + 1)/2}{\left\{(nm/12)\left[N + 1 - \sum_{j=1}^{g} t_j(t_j^2 - 1)/N(N+1)\right]\right\}^{1/2}} \tag{9}$$

where W_{rs} is the sum of the ranks of the n samples from the cleanup unit, g is the number of tied groups, t_j is the number of tied data in the jth group, and n, m, and N have been previously defined. When $m > 10$ and $n > 10$, Z_{rs} is referred to the standard normal distribution to decide whether to reject H_0. When $m < 10$ and $n < 10$, the exact distribution of W_{rs} is used to test H_0 using tables in, e.g., Hollander and Wolfe (1973).

Example 1. Measurements of 1,2,3,4-Tetrachlorobenzene (TcCB)

This example is based on measurements of 1,2,3,4-Tetrachlorobenzene (TcCB) (ppb) taken at a contaminated site and a site-specific background area. m = 47 and n = 77 for a total of N = 124.

1. The ordered background-area and cleanup-unit measurements are as follows:

Background		Cleanup Unit		tj
Data	Rank	Data	Rank	
		ND	1	
		0.09	2.5	2
		0.09	2.5	
		0.12	4.5	2
		0.12	4.5	
		0.14	6	
		0.16	7	
		0.17	9	3
		0.17	9	
		0.17	9	
		0.18	11	
		0.19	12	
		0.20	13.5	2
		0.20	13.5	
		0.21	15.5	2
		0.21	15.5	
0.22	18.5	0.22	18.5	4
		0.22	18.5	
		0.22	18.5	
0.23	21.5	0.23	21.5	2
		0.24	23	
		0.25	25.5	4
		0.25	25.5	
		0.25	25.5	
		0.25	25.5	
0.26	28.5	0.26	28.5	2
0.27	30			
0.28	32.5	0.28	32.5	4
0.28	32.5	0.28	32.5	
0.29	35.5	0.29	35.5	2
		0.31	37	
0.33	39.5	0.33	39.5	4
		0.33	39.5	
		0.33	39.5	
0.34	42.5	0.34	42.5	2
0.35	44			
		0.37	45	
0.38	46.5	0.38	46.5	2
0.39	49	0.39	49	3
0.39	49			

		0.40	51	
0.42	52.5			2
0.42	52.5			
0.43	55	0.43	55	3
		0.43	55	
0.45	57			
0.46	58			
		0.47	59	
0.48	61	0.48	61	3
		0.48	61	
		0.49	63	
0.50	64.5			2
0.50	64.5			
0.51	67	0.51	67	3
		0.51	67	
0.52	69			
0.54	70.5	0.54	70.5	2
0.56	72.5			2
0.56	72.5			
0.57	74.5			2
0.57	74.5			
0.60	76.5	0.60	76.5	2
		0.61	78	
0.62	79.5	0.62	79.5	2
0.63	81			
0.67	82			
0.69	83			
0.72	84			
0.74	85			
		0.75	86	
0.76	87			
0.79	88			
0.81	89			
0.82	90.5	0.82	90.5	2
0.84	92			
		0.85	93	
0.89	94			
		0.92	95	
		0.94	96	
		1.05	97	
		1.10	98.5	2
		1.10	98.5	
1.11	100			
1.13	101			
1.14	102.5			2
1.14	102.5			
		1.19	104	
1.20	105			
		1.22	106	
1.33	107.5	1.33	107.5	2
		1.39	109.5	2
		1.39	109.5	

1.52	111
1.53	112
1.73	113
2.35	114
2.46	115
2.59	116
2.61	117
3.06	118
3.29	119
5.56	120
6.61	121
18.40	122
51.97	123
168.64	124

W_{rs} = Sum of Ranks of Cleanup Unit = 4585

2. As ties are present we compute $\sum_{j=1}^{g} t_j(t_j^2 - 1)/2$ for use in the denominator of the WRS test (Equation 9):

   There are g = 30 groups of ties: 21 groups with $t_j = 2$; 5 groups with $t_j = 3$; and 4 groups with $t_j = 4$. Therefore,

t_j	Number of Groups	$t_j(t_j^2 - 1)$	Product of Column 2 and Column 3
2	21	6	126
3	5	24	120
4	4	60	240
		Sum =	486

   Hence, $\sum_{j=1}^{g} t_j(t_j^2 - 1)/2 = 486.$

3. Using Equation 9 we obtain

$$Z_{rs} = \frac{4585 - 77(124 + 1)/2}{\left\{(77*47/12)\left[124 + 1 - 486/(124(124+1)\right]\right\}^{1/2}}$$

$$= -1.17 \quad ,$$

   which is non-significant at the $\alpha = 0.05$ level since $-1.17 < 1.645$.

4. Conclusion: The WRS test provides no evidence that remedial action has failed to attain the background cleanup standard of $P_r = 1/2$.

5. In Section 4. we will see that the Quantile test <u>is</u> statistically significant for this data set.

4.0 "SPOTTY" RESIDUAL CONTAMINATION AND THE QUANTILE TEST

The nonparametric "Quantile test" was developed by Johnson et al. (1987) to detect changes in a small proportion of a treated population. However, this test can also be used to test for attainment of background standards.

Briefly, the Quantile test is performed by first ordering the combined background and cleanup-unit data. Then a count is made of the number of measurements from the cleanup unit that are in the largest 100(1-q)% of the combined set of N = m + n measurements (defined above) and q is a specified proportion of the distribution of combined measurements. Before proceeding we need to define:

r = number of largest measurements among the N combined background-area and cleanup-unit measurements that must be examined,

k = number of measurements from the cleanup unit that are among the r largest measurements

= test statistic for the Quantile test.

To illustrate the Quantile test, if X_q is the 90th quantile of the combined data set, i.e., if q = 0.90, and if m = n = 50, then the test statistic for the Quantile test is the number, k, of measurements from the cleanup unit that are among the r = (1 - q)(m + n) = 0.10(50 + 50) = 10 largest measurements of the combined data set. If k is sufficiently large, as determined using the hypergeometric distribution or (for large r) the standard normal distribution, then we conclude that the cleanup unit has not attained the cleanup standard.

4.1 HYPOTHESES

The null and alternative hypotheses used for the Quantile test are

$$H_0: \epsilon = 0, \Delta = 0 \qquad (10)$$
$$H_a: \epsilon > 0, \Delta > 0$$

where

ϵ = proportion of the soil in the cleanup unit that has not been remediated to the background standard, and

Δ = amount (in units of standard deviation) that the distribution of 100ϵ% of the measurements in the remediated cleanup unit is shifted to the right (to higher measurements) of the distribution in the background area.

The "cleanup standard" for the Quantile test is $\epsilon = 0$ and $\Delta = 0$.

Figures 1 and 2 illustrate the distribution of measurements for a hypothetical pollution parameter in the remediated cleanup unit and the background area to which it is being compared. In Figure 1, $\epsilon = 0.10$ and $\Delta = 4$, i.e., the measurements of the pollution parameter in $100(\epsilon)\% = 10\%$ of the cleanup unit have a distribution that is shifted to the right of the distribution of that pollution parameter in the background area by $\Delta = 4$ standard-deviation units. As seen in Figure 1, when Δ is this large, the distribution of measurements for the entire cleanup unit has a distinct bimodal appearance. The Quantile test has more power than the WRS test for this situation.

In Figure 2, $\epsilon = 0.25$ and $\Delta = 1$. Figure 2 illustrates that when Δ is small, the distribution of measurements for the entire cleanup unit does not have a bimodal appearance. The WRS test has more power than the Quantile test for this situation.

When $\epsilon = 1$, then the distribution of measurements in the cleanup unit is the same as the distribution in the background area except that it is shifted to the right by the amount $\Delta > 0$. In that case, and more generally whenever ϵ is close to 1, the WRS test will have more power than the Quantile test.

4.2 CONDUCTING THE QUANTILE TEST WHEN q IS PRESPECIFIED

When q is prespecified the quantile test is conducted as follows (from Johnson et al. 1987):

1. Select a value of q greater than 0.5 and less than 1.0 (q = 0.90 is recommended; see Example 2 below).

2. Determine $R_1, R_2, \ldots, R_n$, which are the ranks of the n cleanup-unit measurements in the combined set of $N = m + n$ background plus cleanup-site measurements. (Johnson et al. 1987, used average ranks when ties were present.)

3. Compute the quantile test statistics k, where

$$k = \sum_{i=1}^{n} J[R_i/(m+n+1)] \tag{11}$$

where

$$\begin{aligned} J[R_i/(m+n+1)] &= 1 \quad \text{if } q < R_i/(m+n+1) < 1 \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{12}$$

From Equations 11 and 12 we see that k is the number of cleanup-unit measurements for which $R_i/(m+n+1)$ exceeds q.

4. Compute the probability, P, of obtaining a value of k as large or larger than the observed k if, in fact, the H_0 is true:

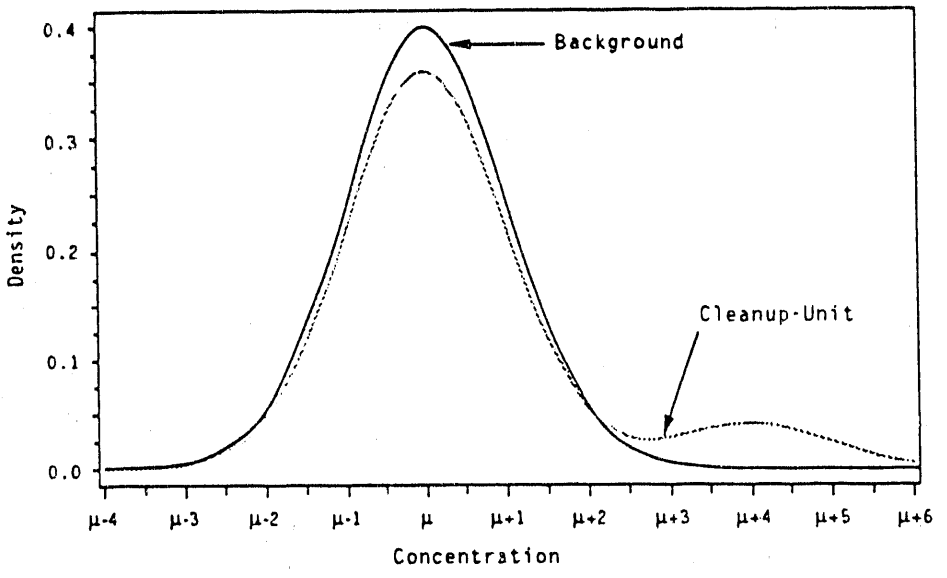

FIGURE 1. Hypothetical Distribution of Measurements for a Pollution Parameter in the Background Area and in a Remediated Cleanup Unit

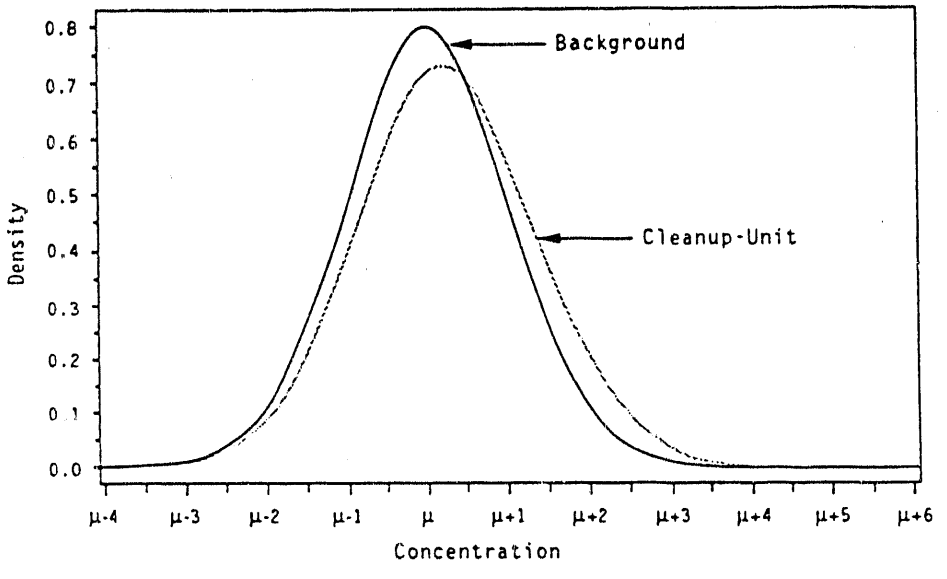

FIGURE 2. Hypothetical Distribution of Measurements for A Pollutant in the Background Area and for a Remediated Cleanup Unit When 25% (ϵ = 0.25) of the Unit is Still Contaminated by the Amount Δ/σ = 1 (in Units of Standard Deviation) Greater than the Average Background Level (μ)

$$P = \sum_{i=k}^{r} \frac{\binom{N-r}{n-i}\binom{r}{i}}{\binom{N}{n}} \tag{13}$$

where $r = (1-q)N$.

5. If r is large, say $r > 20$, then the normal distribution may be used to conduct the test. Compute

$$Z = \frac{k - 0.5 - XBAR}{SD}$$

where

$$XBAR = \frac{nr}{m + n}$$

= mean of the hypergeometric distribution

and

$$SD = \left[\frac{mnr(m + n - r)}{(m + n)^2(m + n - 1)} \right]^{1/2}$$

= standard deviation of the hypergeometric distribution.

Refer Z to the standard normal tables to determine the significance of the test result.

Example 2. The Quantile Test

This example (from Johnson et al. 1987) illustrates the computation of k for a data set for which m = n = 10. Johnson et al. (1987) used q = 0.8.

i	R_i	$R_i/(m+n+1)$	$J[R_i/(m+n+1)$
1	1	0.0476	0
2	5	0.238	0
3	7	0.333	0
4	11.5	0.548	0
5	14	0.666	0
6	16	0.762	0
7	17	0.810	1
8	18	0.857	1
9	19	0.905	1
10	20	0.952	1

Quantile Statistic: k = 4

Using Equation 13 , Johnson et al. (1987) computed P = 0.042, which implies that $H_0: \epsilon = 0$, $\Delta = 0$ is rejected at the 0.05 significance level.

4.3 SPECIFYING THE VALUE OF q

In Example 2, note that if q = 0.9 had been used, then r = 2, k = 2, and Equation 13 would have given P = 0.237. Hence, a test at the 0.05 significant level could not have been conducted. To assure that 0.05 or 0.01 tests can be conducted when m and n are small we suggest the following rules.

1. If m < 20 and n < 20, then a Quantile test with $\alpha = 0.05$ cannot be achieved with a q as large as 0.90. To achieve an approximate $\alpha = 0.05$ test when m < 20 and n < 20 set r = 4.

2. For similar reasons, to achieve an approximate $\alpha = 0.01$ test when m < 20 and n < 20, set r = 6.

If m > 20 and n > 20, we recommend using q = 0.90 to ensure that only the upper tail of the combined data distribution is "examined" by the Quantile test. That is, when q = 0.90, the Quantile test will be "looking at" the data in a different way than the WRS test, which is desirable. Also, as noted by Johnson et al. (1987), when q = 0.50, the Quantile test is equivalent to the median test.

Example 3. Quantile Test Applied to the TcCB Data

In this example we illustrate the Quantile test using the TcCB (ppb) measurements in Example 1 (Section 3.2.3). We have that m = 47, n = 77, so N = 124. Suppose we specify $\alpha = 0.01$.

1. Since m > 20 and n > 20 we set q = 0.90. Hence, r = (1 - q)N = 13.

2. Of the r = 13 largest measurements among the N = 124 total measurements, all 13 are from the cleanup unit. That is, k = 13.

3. Since r < 20 we compute P using Equation 13. Since k = r we have

$$P = \frac{\binom{124-13}{77-13}\binom{13}{13}}{\binom{124}{77}} = 0.00134$$

4. Since 0.00134 < 0.01, we reject H_0 and conclude that the cleanup unit has not attained the cleanup standard of $\epsilon = 0$ and $\Delta = 0$.

   Note that for these same data, the WRS test did <u>not</u> reject H_0 (see Example 1), because only a small proportion of the cleanup-unit measurements were greater than the largest background measurements, a situation where the WRS test has less power than the Quantile test. This example illustrates why we recommend that both the WRS and Quantile tests should be conducted.

4.4 CONDUCTING THE QUANTILE TEST USING TABLES OF THE APPROXIMATE POWER

As discussed in Section 4.2, the Quantile test is easily conducted when a specified value of q is used. In this section a table look-up procedure is given for determining the number of samples and conducting the Quantile test that does not depend on prespecifying a value of q. This procedure uses Tables 1 and 2 that give the approximate minimum power of the Quantile test for different combinations of ϵ, Δ, m and n for approximate significance levels of α = 0.01 and 0.05, respectively, when m = n. The approximate minimum power was obtained by computing the power for a wide variety of distribution shapes for the cleanup unit. The following distributions were used: standard normal, exponential, gamma with parameters (β, α) = (1, 2), (1, 3) and (1, 4) (using notation in Rothschild and Logothetis 1986), Student's t with 3 degrees of freedom (df), Student's t with 7 df, and the F distribution with 1 and 5 degrees of freedom.

The procedure for conducting the Quantile test making use of Tables 1 and 2 is:

1. Specify α, ϵ, Δ, and the required minimum power, 1 - β.

2. Use Table 1 or 2, depending of the chosen value of α, to determine m, r, and k, where r and k have been defined above, and

   m_{bc} = number of measurements needed from both the background area and the cleanup unit.

3. Order from smallest to largest the $N = 2m_{bc}$ measurements.

4. If the rth largest measurement (counting down from the maximum measurement) is among a group of tied measurements, then increase r to include the entire set of tied measurements. Also increase k by the same amount. For example, suppose from Step 2 that r = 10, k = 7 and that the 7th through 12th largest measurements (counting from the maximum measurement) have the same value. Then increase r from 10 to 12 and k from 7 to 9.

5. Reject H_0 and accept H_a (Equation 10) if k or more of the largest r measurements in the combined data set of N_f measurements are from the cleanup unit. Note that the Quantile test can be conducted when there are as many as N_f - r less-than data.

In Step 3, if measurements less than the detection limit are present in either the background-area or cleanup-unit data sets, consider them to have a value less than the smallest measured value from the respective area or unit.

In Step 4, increasing k by the <u>same</u> amount as r insures that the true α remains less than the specified alpha. However, a smaller increase in k may still give an α less than the specified alpha. The optimum value of k can be determined by computing P (Equation 13) for different values of k. The optimum k (that gives the highest power) is the largest k that still gives P $\leq \alpha$.

5.0 CONCLUSIONS

The use of the Wilcoxon Rank Sum (WRS) Test in combination with the Quantile test is proposed as a method of increasing the power of detecting when remedial action at Superfund sites have not attained site-specific background standards. The WRS test has more power than the Quantile test to detect when the remedial action failed more or less uniformly throughout the Superfund site, whereas the Quantile test has more power than the WRS test to detect when remedial action was inadequate in only a portion of the site. Tables 1 and 2 allow one, for the special case of m = n, to (1) easily determine the number of samples required for the Quantile test for significance levels of 0.01 and 0.05, respectively, and (2) to conduct the test. At the present time no tables equivalent to Tables 1 and 2 exist for conducting the Quantile test when the number of samples in the background and cleanup units are unequal. However, once m and n are determined, the Quantile test is easily performed.

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Table 1. Power and Number of Measurements for the Quantile Test for Type I Error Rate α = 0.01 for when m = n. m and n are the Number of Required Measurements From the Background Area and the Cleanup Unit, respectively.

					delta							
alpha	m=n	r	k	e	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.005	10	6	6	0.10	0.007	0.009	0.011	0.013	0.014	0.015	0.015	0.015
	10	6	6	0.20	0.009	0.014	0.022	0.030	0.037	0.042	0.045	0.048
	10	6	6	0.30	0.011	0.021	0.040	0.063	0.086	0.105	0.115	0.120
	10	6	6	0.40	0.013	0.031	0.065	0.115	0.169	0.214	0.240	0.255
	10	6	6	0.50	0.016	0.042	0.097	0.181	0.277	0.360	0.408	0.440
	10	6	6	0.60	0.019	0.056	0.135	0.257	0.397	0.511	0.584	0.633
	10	6	6	0.70	0.022	0.072	0.177	0.334	0.502	0.630	0.721	0.783
	10	6	6	0.80	0.026	0.089	0.219	0.398	0.568	0.703	0.800	0.865
	10	6	6	0.90	0.030	0.107	0.258	0.441	0.608	0.739	0.832	0.894
	10	6	6	1.00	0.034	0.126	0.290	0.474	0.637	0.761	0.848	0.905
0.008	15	6	6	0.10	0.011	0.015	0.020	0.027	0.033	0.037	0.039	0.039
	15	6	6	0.20	0.013	0.023	0.041	0.067	0.098	0.125	0.141	0.150
	15	6	6	0.30	0.016	0.034	0.069	0.129	0.205	0.277	0.325	0.358
	15	6	6	0.40	0.019	0.046	0.104	0.200	0.325	0.437	0.525	0.587
	15	6	6	0.50	0.022	0.060	0.139	0.267	0.417	0.558	0.669	0.749
	15	6	6	0.60	0.026	0.074	0.172	0.319	0.483	0.632	0.748	0.830
	15	6	6	0.70	0.030	0.089	0.204	0.364	0.532	0.677	0.786	0.063
	15	6	6	0.80	0.034	0.105	0.235	0.404	0.571	0.709	0.810	0.880
	15	6	6	0.90	0.039	0.121	0.265	0.439	0.604	0.735	0.829	0.892
	15	6	6	1.00	0.043	0.137	0.293	0.471	0.632	0.757	0.844	0.902

TABLE 1. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.010	20	6	6	0.10	0.013	0.017	0.026	0.038	0.052	0.064	0.070	0.073
0.010	20	9	8	0.10	0.013	0.019	0.027	0.035	0.041	0.045	0.046	0.047
	20	6	6	0.20	0.015	0.026	0.050	0.092	0.150	0.209	0.253	0.284
	20	9	8	0.20	0.017	0.032	0.060	0.100	0.135	0.161	0.174	0.179
	20	6	6	0.30	0.018	0.037	0.079	0.157	0.266	0.381	0.480	0.553
	20	9	8	0.30	0.021	0.050	0.110	0.205	0.313	0.391	0.430	0.446
	20	6	6	0.40	0.022	0.050	0.111	0.217	0.359	0.507	0.634	0.730
	20	9	8	0.40	0.026	0.072	0.173	0.334	0.511	0.644	0.707	0.735
	20	6	6	0.50	0.025	0.064	0.143	0.272	0.429	0.584	0.713	0.808
	20	9	8	0.50	0.032	0.098	0.241	0.452	0.660	0.799	0.873	0.908
	20	6	6	0.60	0.029	0.078	0.176	0.320	0.485	0.636	0.756	0.842
	20	9	8	0.60	0.038	0.125	0.303	0.534	0.737	0.868	0.937	0.968
	20	6	6	0.70	0.033	0.094	0.207	0.364	0.530	0.675	0.785	0.863
	20	9	8	0.70	0.045	0.153	0.357	0.595	0.785	0.899	0.956	0.982
	20	6	6	0.80	0.038	0.109	0.238	0.403	0.569	0.706	0.808	0.877
	20	9	8	0.80	0.052	0.183	0.408	0.646	0.820	0.918	0.966	0.986
	20	6	6	0.90	0.042	0.125	0.267	0.438	0.601	0.732	0.827	0.880
	20	9	8	0.90	0.060	0.212	0.454	0.688	0.847	0.932	0.972	0.989
	20	6	6	1.00	0.047	0.141	0.294	0.470	0.630	0.754	0.842	0.882
	20	9	8	1.00	0.069	0.241	0.496	0.724	0.868	0.943	0.977	0.991
0.012	25	9	8	0.10	0.015	0.022	0.034	0.050	0.063	0.072	0.076	0.077
0.009	25	12	10	0.10	0.013	0.019	0.028	0.036	0.042	0.046	0.047	0.047
	25	9	8	0.20	0.019	0.037	0.075	0.138	0.213	0.275	0.305	0.317
	25	12	10	0.20	0.017	0.035	0.069	0.113	0.150	0.174	0.186	0.189
	25	9	8	0.30	0.024	0.056	0.131	0.259	0.421	0.558	0.632	0.667
	25	12	10	0.30	0.022	0.058	0.136	0.255	0.366	0.436	0.470	0.482
	25	9	8	0.40	0.030	0.079	0.190	0.374	0.581	0.743	0.840	0.887
	25	12	10	0.40	0.028	0.088	0.223	0.425	0.620	0.731	0.774	0.788
	25	9	8	0.50	0.035	0.103	0.248	0.461	0.674	0.828	0.916	0.959
	25	12	10	0.50	0.036	0.124	0.316	0.577	0.793	0.903	0.941	0.952
	25	9	8	0.60	0.042	0.130	0.305	0.533	0.736	0.870	0.942	0.976
	25	12	10	0.60	0.044	0.163	0.404	0.678	0.865	0.952	0.983	0.993
	25	9	8	0.70	0.049	0.158	0.359	0.593	0.782	0.897	0.956	0.982
	25	12	10	0.70	0.053	0.202	0.473	0.740	0.901	0.968	0.991	0.998
	25	9	8	0.80	0.057	0.187	0.408	0.643	0.817	0.916	0.965	0.986
	25	12	10	0.80	0.062	0.242	0.535	0.788	0.924	0.977	0.994	0.998
	25	9	8	0.90	0.065	0.216	0.454	0.685	0.844	0.931	0.971	0.988
	25	12	10	0.90	0.073	0.283	0.589	0.825	0.941	0.983	0.995	0.999
	25	9	8	1.00	0.073	0.245	0.495	0.720	0.866	0.942	0.976	0.989
	25	12	10	1.00	0.084	0.322	0.636	0.854	0.953	0.987	0.997	0.999

TABLE 1. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.011	30	12	10	0.10	0.015	0.022	0.036	0.051	0.062	0.069	0.072	0.073
0.008	30	15	12	0.10	0.011	0.018	0.026	0.035	0.041	0.044	0.045	0.045
	30	12	10	0.20	0.019	0.041	0.087	0.159	0.233	0.280	0.304	0.312
	30	15	12	0.20	0.016	0.035	0.073	0.118	0.157	0.181	0.191	0.195
	30	12	10	0.30	0.025	0.066	0.162	0.322	0.499	0.620	0.669	0.685
	30	15	12	0.30	0.021	0.062	0.152	0.289	0.399	0.469	0.500	0.510
	30	12	10	0.40	0.032	0.096	0.250	0.484	0.714	0.852	0.907	0.925
	30	15	12	0.40	0.028	0.097	0.260	0.493	0.688	0.777	0.813	0.824
	30	12	10	0.50	0.039	0.130	0.329	0.595	0.812	0.928	0.973	0.988
	30	15	12	0.50	0.036	0.141	0.375	0.668	0.868	0.946	0.964	0.969
	30	12	10	0.60	0.047	0.167	0.404	0.675	0.861	0.954	0.986	0.996
	30	15	12	0.60	0.046	0.192	0.483	0.775	0.930	0.983	0.995	0.998
	30	12	10	0.70	0.057	0.206	0.472	0.736	0.898	0.967	0.991	0.998
	30	15	12	0.70	0.056	0.241	0.583	0.831	0.954	0.990	0.998	0.999+
	30	12	10	0.80	0.067	0.246	0.533	0.784	0.922	0.976	0.994	0.998
	30	15	12	0.80	0.068	0.290	0.630	0.872	0.968	0.994	0.999	0.999+
	30	12	10	0.90	0.077	0.285	0.587	0.822	0.939	0.982	0.995	0.999
	30	15	12	0.90	0.081	0.340	0.687	0.901	0.977	0.996	0.999	0.999+
	30	12	10	1.00	0.089	0.324	0.634	0.851	0.951	0.986	0.996	0.999
	30	15	12	1.00	0.094	0.388	0.735	0.923	0.983	0.997	0.999	0.999+
0.010	40	15	12	0.10	0.014	0.023	0.040	0.062	0.078	0.089	0.093	0.095
0.009	40	20	15	0.10	0.014	0.024	0.037	0.050	0.059	0.063	0.065	0.065
	40	15	12	0.20	0.019	0.045	0.105	0.207	0.318	0.384	0.416	0.427
	40	20	15	0.20	0.021	0.051	0.111	0.178	0.234	0.267	0.280	0.283
	40	15	12	0.30	0.026	0.074	0.200	0.413	0.636	0.769	0.814	0.827
	40	20	15	0.30	0.029	0.093	0.234	0.422	0.554	0.627	0.655	0.664
	40	15	12	0.40	0.033	0.111	0.300	0.583	0.821	0.936	0.972	0.980
	40	20	15	0.40	0.040	0.149	0.390	0.674	0.840	0.901	0.921	0.926
	40	15	12	0.50	0.042	0.153	0.396	0.691	0.889	0.971	0.993	0.998
	40	20	15	0.50	0.052	0.216	0.538	0.836	0.963	0.988	0.993	0.994
	40	15	12	0.60	0.052	0.199	0.483	0.769	0.928	0.983	0.997	0.999
	40	20	15	0.60	0.067	0.289	0.658	0.908	0.986	0.998	0.999+	0.999+
	40	15	12	0.70	0.063	0.247	0.580	0.825	0.951	0.989	0.998	0.999+
	40	20	15	0.70	0.083	0.358	0.738	0.942	0.992	0.999	0.999+	0.999+
	40	15	12	0.80	0.075	0.295	0.626	0.866	0.965	0.993	0.999	0.999+
	40	20	15	0.80	0.101	0.426	0.800	0.962	0.995	0.999+	0.999+	0.999+
	40	15	12	0.90	0.088	0.343	0.682	0.896	0.975	0.995	0.999	0.999+
	40	20	15	0.90	0.121	0.490	0.846	0.975	0.997	0.999+	0.999+	0.999+
	40	15	12	1.00	0.102	0.390	0.729	0.919	0.982	0.997	0.999	0.999+
	40	20	15	1.00	0.143	0.549	0.882	0.983	0.998	0.999+	0.999+	0.999+

TABLE 1. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.011	50	15	12	0.10	0.016	0.026	0.048	0.085	0.125	0.149	0.161	0.165
0.011	50	20	15	0.10	0.017	0.030	0.052	0.079	0.099	0.110	0.115	0.116
0.008	50	23	17	0.10	0.013	0.023	0.040	0.056	0.069	0.075	0.078	0.078
	50	15	12	0.20	0.021	0.048	0.117	0.259	0.445	0.588	0.641	0.658
	50	20	15	0.20	0.024	0.062	0.148	0.284	0.396	0.462	0.490	0.499
	50	23	17	0.20	0.019	0.052	0.125	0.225	0.304	0.351	0.370	0.376
	50	15	12	0.30	0.028	0.078	0.206	0.441	0.705	0.875	0.941	0.958
	50	20	15	0.30	0.034	0.107	0.288	0.555	0.769	0.849	0.878	0.886
	50	23	17	0.30	0.028	0.098	0.271	0.517	0.680	0.759	0.788	0.797
	50	15	12	0.40	0.036	0.115	0.303	0.582	0.822	0.945	0.986	0.997
	50	20	15	0.40	0.045	0.163	0.434	0.753	0.933	0.984	0.991	0.992
	50	23	17	0.40	0.038	0.160	0.440	0.758	0.924	0.963	0.974	0.977
	50	15	12	0.50	0.045	0.157	0.397	0.688	0.887	0.970	0.994	0.999
	50	20	15	0.50	0.058	0.226	0.554	0.848	0.970	0.996	0.999+	0.999+
	50	23	17	0.50	0.051	0.231	0.592	0.885	0.983	0.998	0.999	0.999
	50	15	12	0.60	0.055	0.203	0.483	0.765	0.925	0.982	0.996	0.999
	50	20	15	0.60	0.073	0.293	0.654	0.904	0.984	0.998	0.999+	0.999+
	50	23	17	0.60	0.066	0.305	0.696	0.933	0.992	0.999	0.999+	0.999+
	50	15	12	0.70	0.067	0.250	0.558	0.822	0.949	0.989	0.998	0.999+
	50	20	15	0.70	0.089	0.361	0.733	0.938	0.991	0.999	0.999+	0.999+
	50	23	17	0.70	0.083	0.381	0.776	0.960	0.996	0.999+	0.999+	0.999+
	50	15	12	0.80	0.079	0.298	0.623	0.863	0.964	0.993	0.999	0.999+
	50	20	15	0.80	0.108	0.427	0.794	0.959	0.995	0.999+	0.999+	0.999+
	50	23	17	0.80	0.103	0.454	0.835	0.976	0.998	0.999+	0.999+	0.999+
	50	15	12	0.90	0.092	0.345	0.679	0.893	0.974	0.995	0.999	0.999+
	50	20	15	0.90	0.128	0.489	0.841	0.973	0.997	0.999+	0.999+	0.999+
	50	23	17	0.90	0.124	0.522	0.878	0.985	0.999	0.999+	0.999+	0.999+
	50	15	12	1.00	0.106	0.391	0.726	0.916	0.981	0.996	0.999	0.999+
	50	20	15	1.00	0.149	0.547	0.877	0.981	0.998	0.999+	0.999+	0.999+
	50	23	17	1.00	0.147	0.584	0.910	0.990	0.999	0.999+	0.999+	0.999+

TABLE 1. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.008	60	10	9	0.10	0.011	0.016	0.028	0.054	0.110	0.196	0.277	0.329
0.008	60	13	11	0.10	0.011	0.017	0.032	0.068	0.130	0.197	0.237	0.249
0.010	60	18	14	0.10	0.014	0.024	0.048	0.090	0.135	0.162	0.175	0.180
0.010	60	23	17	0.10	0.015	0.027	0.051	0.083	0.107	0.121	0.127	0.129
	60	10	9	0.20	0.014	0.027	0.060	0.134	0.272	0.465	0.656	0.794
	60	13	11	0.20	0.015	0.032	0.076	0.182	0.371	0.593	0.751	0.827
	60	18	14	0.20	0.020	0.048	0.127	0.294	0.507	0.653	0.704	0.720
	60	23	17	0.20	0.022	0.059	0.156	0.318	0.458	0.534	0.568	0.578
	60	10	9	0.30	0.018	0.042	0.102	0.227	0.417	0.626	0.795	0.901
	60	13	11	0.30	0.019	0.051	0.136	0.311	0.555	0.776	0.910	0.968
	60	18	14	0.30	0.027	0.082	0.231	0.500	0.776	0.926	0.971	0.978
	60	23	17	0.30	0.031	0.106	0.310	0.611	0.833	0.908	0.931	0.937
	60	10	9	0.40	0.022	0.059	0.150	0.317	0.530	0.727	0.862	0.938
	60	13	11	0.40	0.024	0.075	0.205	0.431	0.682	0.862	0.952	0.985
	60	18	14	0.40	0.035	0.124	0.343	0.651	0.880	0.973	0.995	0.999
	60	23	17	0.40	0.042	0.166	0.461	0.794	0.957	0.994	0.997	0.998
	60	10	9	0.50	0.026	0.079	0.201	0.398	0.617	0.793	0.901	0.944
	60	13	11	0.50	0.030	0.103	0.276	0.533	0.768	0.910	0.971	0.992
	60	18	14	0.50	0.045	0.173	0.450	0.758	0.932	0.987	0.998	0.999+
	60	23	17	0.50	0.055	0.235	0.589	0.881	0.982	0.998	0.999+	0.999+
	60	10	9	0.60	0.031	0.101	0.251	0.469	0.683	0.838	0.920	0.948
	60	13	11	0.60	0.036	0.134	0.345	0.616	0.827	0.938	0.981	0.993
	60	18	14	0.60	0.056	0.226	0.545	0.831	0.960	0.993	0.999	0.999+
	60	23	17	0.60	0.070	0.309	0.693	0.930	0.991	0.999	0.999+	0.999+
	60	10	9	0.70	0.036	0.124	0.300	0.530	0.735	0.870	0.925	0.951
	60	13	11	0.70	0.044	0.167	0.410	0.682	0.868	0.956	0.987	0.994
	60	18	14	0.70	0.069	0.282	0.626	0.880	0.975	0.996	0.999+	0.999+
	60	23	17	0.70	0.088	0.382	0.772	0.958	0.996	0.999+	0.999+	0.999+
	60	10	9	0.80	0.042	0.148	0.347	0.583	0.775	0.888	0.929	0.953
	60	13	11	0.80	0.052	0.201	0.470	0.735	0.897	0.967	0.989	0.994
	60	18	14	0.80	0.083	0.337	0.694	0.914	0.984	0.998	0.999+	0.999+
	60	23	17	0.80	0.108	0.454	0.831	0.974	0.998	0.999+	0.999+	0.999+
	60	10	9	0.90	0.049	0.173	0.391	0.628	0.807	0.894	0.932	0.956
	60	13	11	0.90	0.060	0.236	0.524	0.778	0.919	0.975	0.989	0.995
	60	18	14	0.90	0.098	0.391	0.749	0.937	0.989	0.999	0.999+	0.999+
	60	23	17	0.90	0.129	0.521	0.874	0.984	0.999	0.999+	0.999+	0.999+
	60	10	9	1.00	0.055	0.198	0.431	0.666	0.832	0.899	0.935	0.958
	60	13	11	1.00	0.069	0.271	0.572	0.812	0.935	0.980	0.990	0.995
	60	18	14	1.00	0.114	0.443	0.794	0.953	0.993	0.999	0.999+	0.999+
	60	23	17	1.00	0.152	0.582	0.906	0.989	0.999	0.999+	0.999+	0.999+

TABLE 1. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.009	75	10	9	0.10	0.011	0.017	0.029	0.055	0.114	0.219	0.350	0.457
0.009	75	13	11	0.10	0.012	0.018	0.034	0.070	0.150	0.271	0.367	0.412
0.011	75	18	14	0.10	0.016	0.026	0.052	0.112	0.199	0.269	0.299	0.309
0.011	75	23	17	0.10	0.017	0.030	0.063	0.120	0.177	0.210	0.225	0.230
	75	10	9	0.20	0.015	0.028	0.062	0.136	0.273	0.467	0.666	0.818
	75	13	11	0.20	0.016	0.033	0.079	0.184	0.374	0.611	0.803	0.908
	75	18	14	0.20	0.022	0.051	0.130	0.308	0.576	0.797	0.888	0.903
	75	23	17	0.20	0.024	0.063	0.175	0.397	0.638	0.760	0.802	0.814
	75	10	9	0.30	0.018	0.043	0.104	0.228	0.417	0.625	0.794	0.901
	75	13	11	0.30	0.020	0.053	0.139	0.312	0.554	0.775	0.910	0.970
	75	18	14	0.30	0.029	0.085	0.234	0.499	0.776	0.934	0.986	0.997
	75	23	17	0.30	0.034	0.111	0.317	0.642	0.891	0.978	0.991	0.993
	75	10	9	0.40	0.022	0.061	0.152	0.317	0.530	0.726	0.861	0.918
	75	13	11	0.40	0.025	0.077	0.207	0.431	0.681	0.861	0.951	0.985
	75	18	14	0.40	0.038	0.126	0.344	0.649	0.877	0.972	0.995	0.999
	75	23	17	0.40	0.045	0.171	0.461	0.790	0.956	0.995	0.999+	0.999+
	75	10	9	0.50	0.027	0.080	0.202	0.398	0.616	0.792	0.886	0.924
	75	13	11	0.50	0.031	0.105	0.277	0.532	0.767	0.908	0.970	0.988
	75	18	14	0.50	0.048	0.177	0.450	0.755	0.930	0.986	0.998	0.999+
	75	23	17	0.50	0.059	0.239	0.587	0.877	0.981	0.998	0.999+	0.999+
	75	10	9	0.60	0.032	0.102	0.252	0.469	0.682	0.836	0.894	0.929
	75	13	11	0.60	0.038	0.136	0.346	0.614	0.825	0.937	0.979	0.989
	75	18	14	0.60	0.059	0.229	0.544	0.828	0.958	0.993	0.999	0.999+
	75	23	17	0.60	0.075	0.312	0.689	0.927	0.991	0.999	0.999+	0.999+
	75	10	9	0.70	0.038	0.125	0.301	0.530	0.734	0.847	0.900	0.934
	75	13	11	0.70	0.045	0.169	0.410	0.681	0.867	0.955	0.981	0.990
	75	18	14	0.70	0.072	0.284	0.625	0.877	0.974	0.996	0.999+	0.999+
	75	23	17	0.70	0.093	0.384	0.768	0.956	0.995	0.999+	0.999+	0.999+
	75	10	9	0.80	0.043	0.149	0.347	0.582	0.774	0.856	0.906	0.937
	75	13	11	0.80	0.053	0.203	0.470	0.734	0.896	0.965	0.982	0.991
	75	18	14	0.80	0.086	0.339	0.691	0.912	0.983	0.998	0.999+	0.999+
	75	23	17	0.80	0.112	0.454	0.827	0.973	0.998	0.999+	0.999+	0.999+
	75	10	9	0.90	0.050	0.174	0.391	0.627	0.788	0.864	0.911	0.940
	75	13	11	0.90	0.062	0.238	0.523	0.776	0.918	0.968	0.984	0.991
	75	18	14	0.90	0.101	0.392	0.746	0.935	0.989	0.999	0.999+	0.999+
	75	23	17	0.90	0.134	0.520	0.871	0.983	0.999	0.999+	0.999+	0.999+
	75	10	9	1.00	0.056	0.199	0.431	0.665	0.798	0.870	0.915	0.943
	75	13	11	1.00	0.071	0.273	0.571	0.811	0.934	0.970	0.985	0.992
	75	18	14	1.00	0.118	0.444	0.791	0.952	0.992	0.999	0.999+	0.999+
	75	23	17	1.00	0.157	0.580	0.903	0.989	0.999	0.999+	0.999+	0.999+

TABLE 1. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.009	100	10	9	0.10	0.012	0.018	0.030	0.057	0.115	0.227	0.393	0.566
0.009	100	13	11	0.10	0.012	0.019	0.035	0.072	0.155	0.307	0.492	0.623
0.012	100	18	14	0.10	0.017	0.028	0.054	0.118	0.257	0.432	0.533	0.557
0.012	100	23	17	0.10	0.019	0.033	0.068	0.158	0.291	0.391	0.431	0.444
0.010	100	26	19	0.10	0.015	0.028	0.063	0.144	0.252	0.314	0.344	0.354
	100	10	9	0.20	0.015	0.029	0.063	0.137	0.274	0.467	0.666	0.820
	100	13	11	0.20	0.016	0.035	0.081	0.186	0.374	0.611	0.811	0.926
	100	18	14	0.20	0.023	0.054	0.134	0.310	0.578	0.823	0.947	0.984
	100	23	17	0.20	0.027	0.067	0.179	0.416	0.720	0.914	0.967	0.973
	100	26	19	0.20	0.022	0.062	0.179	0.436	0.747	0.906	0.939	0.947
	100	10	9	0.30	0.019	0.044	0.105	0.229	0.417	0.624	0.793	0.869
	100	13	11	0.30	0.021	0.055	0.141	0.313	0.554	0.773	0.909	0.970
	100	18	14	0.30	0.031	0.088	0.236	0.499	0.773	0.933	0.986	0.998
	100	23	17	0.30	0.037	0.116	0.320	0.639	0.889	0.981	0.998	0.999+
	100	26	19	0.30	0.032	0.112	0.332	0.672	0.915	0.989	0.999	0.999+
	100	10	9	0.40	0.023	0.062	0.153	0.318	0.529	0.724	0.825	0.881
	100	13	11	0.40	0.027	0.079	0.209	0.431	0.679	0.859	0.950	0.975
	100	18	14	0.40	0.040	0.131	0.346	0.647	0.875	0.971	0.995	0.999
	100	23	17	0.40	0.049	0.176	0.461	0.786	0.954	0.994	0.999+	0.999+
	100	26	19	0.40	0.043	0.175	0.485	0.819	0.969	0.997	0.999+	0.999+
	100	10	9	0.50	0.028	0.082	0.203	0.398	0.615	0.762	0.839	0.890
	100	13	11	0.50	0.033	0.107	0.279	0.531	0.765	0.907	0.959	0.978
	100	18	14	0.50	0.050	0.180	0.450	0.752	0.928	0.986	0.998	0.999+
	100	23	17	0.50	0.063	0.243	0.585	0.873	0.979	0.998	0.999+	0.999+
	100	26	19	0.50	0.057	0.248	0.616	0.901	0.988	0.999	0.999+	0.999+
	100	10	9	0.60	0.033	0.104	0.253	0.469	0.672	0.779	0.851	0.898
	100	13	11	0.60	0.039	0.138	0.347	0.613	0.824	0.930	0.963	0.980
	100	18	14	0.60	0.062	0.232	0.543	0.825	0.957	0.993	0.999	0.999+
	100	23	17	0.60	0.079	0.314	0.686	0.924	0.990	0.999	0.999+	0.999+
	100	26	19	0.60	0.074	0.325	0.720	0.945	0.995	0.999+	0.999+	0.999+
	100	10	9	0.70	0.039	0.127	0.302	0.529	0.693	0.794	0.860	0.904
	100	13	11	0.70	0.047	0.171	0.411	0.679	0.865	0.937	0.967	0.982
	100	18	14	0.70	0.075	0.286	0.623	0.875	0.973	0.996	0.999	0.999+
	100	23	17	0.70	0.097	0.386	0.764	0.954	0.995	0.999+	0.999+	0.999+
	100	26	19	0.70	0.092	0.403	0.798	0.969	0.998	0.999+	0.999+	0.999+
	100	10	9	0.80	0.045	0.151	0.348	0.572	0.711	0.806	0.869	0.910
	100	13	11	0.80	0.055	0.205	0.469	0.732	0.889	0.942	0.970	0.983
	100	18	14	0.80	0.089	0.340	0.689	0.909	0.982	0.997	0.999	0.999+
	100	23	17	0.80	0.117	0.454	0.823	0.971	0.997	0.999+	0.999+	0.999+
	100	26	19	0.80	0.113	0.477	0.854	0.982	0.999	0.999+	0.999+	0.999+
	100	10	9	0.90	0.051	0.175	0.391	0.594	0.727	0.817	0.876	0.915
	100	13	11	0.90	0.064	0.240	0.523	0.775	0.808	0.947	0.972	0.985
	100	18	14	0.90	0.105	0.393	0.744	0.933	0.988	0.998	0.999	0.999+
	100	23	17	0.90	0.139	0.519	0.867	0.981	0.998	0.999+	0.999+	0.999+
	100	26	19	0.90	0.136	0.546	0.895	0.989	0.999	0.999+	0.999+	0.999+
	100	10	9	1.00	0.058	0.200	0.431	0.613	0.740	0.826	0.882	0.919
	100	13	11	1.00	0.073	0.274	0.570	0.809	0.906	0.951	0.974	0.986
	100	18	14	1.00	0.121	0.444	0.788	0.950	0.992	0.998	0.999	0.999+
	100	23	17	1.00	0.162	0.578	0.900	0.988	0.999	0.999+	0.999+	0.999+
	100	26	19	1.00	0.160	0.608	0.924	0.993	0.999+	0.999+	0.999+	0.999+

Table 2. Power and Number of Measurements for the Quantile Test for Type I Error Rate α = 0.05 for when m = n. m and n are the Number of Required Measurements From the Background Area and the Cleanup Unit, respectively.

					delta							
alpha	m=n	r	k	e	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.043	10	4	4	0.10	0.051	0.062	0.077	0.093	0.105	0.113	0.117	0.119
	10	4	4	0.20	0.058	0.084	0.121	0.166	0.210	0.242	0.259	0.270
	10	4	4	0.30	0.066	0.108	0.172	0.255	0.338	0.401	0.443	0.470
	10	4	4	0.40	0.075	0.133	0.225	0.342	0.460	0.551	0.615	0.659
	10	4	4	0.50	0.083	0.159	0.276	0.418	0.553	0.662	0.741	0.794
	10	4	4	0.60	0.092	0.183	0.318	0.474	0.618	0.733	0.815	0.871
	10	4	4	0.70	0.101	0.207	0.356	0.519	0.683	0.774	0.853	0.905
	10	4	4	0.80	0.110	0.230	0.391	0.555	0.695	0.800	0.873	0.920
	10	4	4	0.90	0.119	0.253	0.422	0.587	0.721	0.819	0.886	0.929
	10	4	4	1.00	0.128	0.275	0.451	0.614	0.743	0.835	0.896	0.936
0.050	15	4	4	0.10	0.057	0.071	0.093	0.122	0.153	0.177	0.191	0.200
	15	4	4	0.20	0.065	0.093	0.142	0.214	0.296	0.368	0.423	0.461
	15	4	4	0.30	0.074	0.117	0.191	0.298	0.[illegible]20	0.533	0.623	0.686
	15	4	4	0.40	0.082	0.141	0.238	0.368	0.511	0.640	0.741	0.813
	15	4	4	0.50	0.091	0.166	0.281	0.426	0.575	0.702	0.800	0.868
	15	4	4	0.60	0.100	0.190	0.321	0.475	0.622	0.743	0.832	0.893
	15	4	4	0.70	0.109	0.213	0.358	0.516	0.660	0.773	0.853	0.907
	15	4	4	0.80	0.118	0.236	0.391	0.552	0.691	0.796	0.870	0.909
	15	4	4	0.90	0.127	0.257	0.421	0.583	0.717	0.815	0.883	0.911
	15	4	4	1.00	0.136	0.278	0.449	0.610	0.739	0.831	0.890	0.913

TABLE 2. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.053	20	4	4	0.10	0.061	0.074	0.098	0.136	0.183	0.227	0.260	0.282
0.046	20	7	6	0.10	0.055	0.073	0.100	0.131	0.157	0.171	0.178	0.181
	20	4	4	0.20	0.069	0.097	0.146	0.224	0.326	0.433	0.523	0.591
	20	7	6	0.20	0.066	0.106	0.174	0.267	0.361	0.428	0.459	0.470
	20	4	4	0.30	0.077	0.121	0.194	0.303	0.436	0.570	0.683	0.769
	20	7	6	0.30	0.077	0.142	0.258	0.412	0.571	0.681	0.737	0.763
	20	4	4	0.40	0.086	0.145	0.240	0.370	0.515	0.650	0.760	0.841
	20	7	6	0.40	0.089	0.179	0.332	0.528	0.706	0.826	0.891	0.922
	20	4	4	0.50	0.095	0.169	0.283	0.426	0.574	0.703	0.802	0.873
	20	7	6	0.50	0.102	0.217	0.400	0.608	0.781	0.890	0.947	0.973
	20	4	4	0.60	0.104	0.193	0.322	0.474	0.620	0.741	0.831	0.881
	20	7	6	0.60	0.115	0.255	0.460	0.670	0.827	0.919	0.965	0.986
	20	4	4	0.70	0.112	0.216	0.358	0.515	0.658	0.771	0.852	0.805
	20	7	6	0.70	0.129	0.293	0.514	0.719	0.860	0.937	0.974	0.990
	20	4	4	0.80	0.122	0.238	0.391	0.550	0.689	0.794	0.858	0.888
	20	7	6	0.80	0.142	0.329	0.561	0.758	0.884	0.949	0.979	0.992
	20	4	4	0.90	0.131	0.260	0.421	0.581	0.714	0.813	0.862	0.890
	20	7	6	0.90	0.156	0.363	0.602	0.790	0.902	0.958	0.983	0.993
	20	4	4	1.00	0.140	0.280	0.448	0.608	0.736	0.827	0.865	0.893
	20	7	6	1.00	0.171	0.396	0.638	0.816	0.917	0.965	0.986	0.993
0.049	25	7	6	0.10	0.059	0.077	0.110	0.155	0.201	0.233	0.247	0.253
0.048	25	12	9	0.10	0.062	0.084	0.113	0.136	0.152	0.160	0.163	0.164
	25	7	6	0.20	0.070	0.110	0.185	0.303	0.435	0.543	0.601	0.629
	25	12	9	0.20	0.078	0.134	0.220	0.304	0.364	0.398	0.411	0.415
	25	7	6	0.30	0.082	0.146	0.261	0.430	0.611	0.753	0.837	0.879
	25	12	9	0.30	0.096	0.194	0.351	0.525	0.632	0.690	0.713	0.720
	25	7	6	0.40	0.094	0.183	0.334	0.529	0.714	0.848	0.924	0.961
	25	12	9	0.40	0.115	0.261	0.485	0.708	0.845	0.895	0.913	0.919
	25	7	6	0.50	0.106	0.221	0.400	0.606	0.779	0.892	0.952	0.980
	25	12	9	0.50	0.136	0.330	0.600	0.828	0.944	0.979	0.985	0.987
	25	7	6	0.60	0.119	0.258	0.460	0.667	0.824	0.910	0.965	0.986
	25	12	9	0.60	0.158	0.395	0.687	0.887	0.970	0.993	0.998	0.999
	25	7	6	0.70	0.133	0.295	0.512	0.716	0.857	0.935	0.973	0.989
	25	12	9	0.70	0.180	0.454	0.748	0.919	0.981	0.996	0.999	0.999+
	25	7	6	0.80	0.147	0.330	0.559	0.755	0.881	0.948	0.979	0.989
	25	12	9	0.80	0.204	0.509	0.796	0.941	0.987	0.998	0.999+	0.999+
	25	7	6	0.90	0.161	0.364	0.599	0.787	0.900	0.957	0.983	0.990
	25	12	9	0.90	0.228	0.559	0.834	0.956	0.991	0.998	0.999+	0.999+
	25	7	6	1.00	0.175	0.396	0.635	0.813	0.915	0.964	0.984	0.990
	25	12	9	1.00	0.253	0.605	0.864	0.966	0.993	0.999	0.999+	0.999+

TABLE 2. (CONTINUED)

alpha	m=n	r	k	delta: e	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.051	30	7	6	0.10	0.062	0.080	0.114	0.170	0.236	0.290	0.319	0.330
0.052	30	12	9	0.10	0.067	0.093	0.132	0.170	0.196	0.211	0.217	0.218
	30	7	6	0.20	0.073	0.113	0.188	0.311	0.467	0.607	0.697	0.744
	30	12	9	0.20	0.083	0.147	0.252	0.381	0.476	0.528	0.550	0.558
	30	7	6	0.30	0.084	0.149	0.263	0.431	0.619	0.775	0.875	0.927
	30	12	9	0.30	0.101	0.208	0.388	0.600	0.759	0.826	0.851	0.858
	30	7	6	0.40	0.096	0.186	0.335	0.528	0.714	0.849	0.929	0.968
	30	12	9	0.40	0.121	0.271	0.513	0.755	0.905	0.961	0.974	0.977
	30	7	6	0.50	0.109	0.223	0.401	0.605	0.777	0.890	0.951	0.980
	30	12	9	0.50	0.141	0.335	0.607	0.835	0.950	0.987	0.996	0.998
	30	7	6	0.60	0.122	0.260	0.460	0.665	0.822	0.916	0.964	0.984
	30	12	9	0.60	0.163	0.396	0.683	0.884	0.969	0.993	0.999	0.999+
	30	7	6	0.70	0.136	0.297	0.512	0.714	0.855	0.934	0.972	0.985
	30	12	9	0.70	0.186	0.454	0.744	0.916	0.980	0.996	0.999	0.999+
	30	7	6	0.80	0.149	0.332	0.558	0.753	0.880	0.947	0.977	0.986
	30	12	9	0.80	0.209	0.508	0.791	0.938	0.986	0.997	0.999+	0.999+
	30	7	6	0.90	0.163	0.365	0.598	0.785	0.899	0.956	0.978	0.986
	30	12	9	0.90	0.233	0.557	0.829	0.954	0.990	0.998	0.999+	0.999+
	30	7	6	1.00	0.177	0.397	0.634	0.811	0.913	0.963	0.979	0.987
	30	12	9	1.00	0.257	0.602	0.859	0.964	0.993	0.999	0.999+	0.999+
0.054	40	7	6	0.10	0.065	0.083	0.117	0.178	0.272	0.371	0.440	0.478
0.050	40	17	12	0.10	0.067	0.098	0.145	0.186	0.214	0.230	0.235	0.236
	40	7	6	0.20	0.076	0.116	0.191	0.314	0.480	0.652	0.783	0.861
	40	17	12	0.20	0.087	0.166	0.298	0.447	0.540	0.591	0.610	0.616
	40	7	6	0.30	0.088	0.152	0.266	0.431	0.619	0.780	0.889	0.948
	40	17	12	0.30	0.109	0.248	0.474	0.706	0.838	0.886	0.903	0.908
	40	7	6	0.40	0.100	0.189	0.337	0.527	0.711	0.848	0.929	0.969
	40	17	12	0.40	0.134	0.332	0.627	0.862	0.964	0.985	0.990	0.991
	40	7	6	0.50	0.113	0.226	0.401	0.603	0.775	0.888	0.950	0.974
	40	17	12	0.50	0.161	0.414	0.732	0.928	0.988	0.998	0.999+	0.999+
	40	7	6	0.60	0.126	0.263	0.459	0.663	0.820	0.915	0.961	0.975
	40	17	12	0.60	0.190	0.492	0.806	0.957	0.994	0.999	0.999+	0.999+
	40	7	6	0.70	0.139	0.299	0.511	0.711	0.853	0.933	0.964	0.977
	40	17	12	0.70	0.220	0.562	0.859	0.974	0.997	0.999+	0.999+	0.999+
	40	7	6	0.80	0.153	0.333	0.556	0.750	0.878	0.945	0.966	0.978
	40	17	12	0.80	0.250	0.624	0.897	0.984	0.998	0.999+	0.999+	0.999+
	40	7	6	0.90	0.167	0.366	0.596	0.782	0.897	0.948	0.968	0.979
	40	17	12	0.90	0.282	0.679	0.924	0.989	0.999	0.999+	0.999+	0.999+
	40	7	6	1.00	0.181	0.397	0.631	0.808	0.912	0.951	0.969	0.980
	40	17	12	1.00	0.313	0.726	0.943	0.993	0.999	0.999+	0.999+	0.999+

TABLE 2. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.046	50	10	8	0.10	0.057	0.078	0.118	0.193	0.304	0.406	0.463	0.477
0.045	50	15	11	0.10	0.059	0.087	0.142	0.219	0.286	0.322	0.338	0.344
0.050	50	24	16	0.10	0.071	0.109	0.158	0.199	0.226	0.240	0.244	0.245
	50	10	8	0.20	0.069	0.116	0.209	0.365	0.571	0.757	0.868	0.915
	50	15	11	0.20	0.076	0.142	0.278	0.486	0.680	0.781	0.814	0.824
	50	24	16	0.20	0.097	0.197	0.354	0.489	0.576	0.619	0.633	0.637
	50	10	8	0.30	0.083	0.160	0.304	0.512	0.726	0.878	0.955	0.985
	50	15	11	0.30	0.094	0.206	0.416	0.681	0.880	0.963	0.984	0.986
	50	24	16	0.30	0.127	0.306	0.571	0.784	0.875	0.911	0.922	0.925
	50	10	8	0.40	0.097	0.206	0.394	0.625	0.818	0.929	0.977	0.993
	50	15	11	0.40	0.114	0.275	0.538	0.798	0.942	0.988	0.998	0.999+
	50	24	16	0.40	0.161	0.422	0.749	0.935	0.982	0.992	0.994	0.995
	50	10	8	0.50	0.112	0.253	0.475	0.709	0.874	0.955	0.987	0.996
	50	15	11	0.50	0.136	0.344	0.639	0.870	0.969	0.995	0.999	0.999+
	50	24	16	0.50	0.198	0.533	0.859	0.981	0.999	0.999+	0.999+	0.999+
	50	10	8	0.60	0.128	0.300	0.546	0.771	0.909	0.970	0.991	0.996
	50	15	11	0.60	0.159	0.411	0.719	0.914	0.982	0.997	0.999+	0.999+
	50	24	16	0.60	0.237	0.625	0.916	0.992	0.999+	0.999+	0.999+	0.999+
	50	10	8	0.70	0.144	0.346	0.606	0.818	0.933	0.979	0.993	0.997
	50	15	11	0.70	0.183	0.474	0.780	0.942	0.989	0.999	0.999+	0.999+
	50	24	16	0.70	0.278	0.702	0.949	0.996	0.999+	0.999+	0.999+	0.999+
	50	10	8	0.80	0.161	0.390	0.658	0.853	0.949	0.985	0.994	0.997
	50	15	11	0.80	0.208	0.532	0.828	0.960	0.993	0.999	0.999+	0.999+
	50	24	16	0.80	0.320	0.766	0.969	0.998	0.999+	0.999+	0.999+	0.999+
	50	10	8	0.90	0.178	0.431	0.702	0.880	0.960	0.988	0.994	0.997
	50	15	11	0.90	0.233	0.585	0.864	0.971	0.996	0.999	0.999+	0.999+
	50	24	16	0.90	0.362	0.817	0.980	0.999	0.999+	0.999+	0.999+	0.999+
	50	10	8	1.00	0.196	0.470	0.739	0.900	0.969	0.989	0.995	0.997
	50	15	11	1.00	0.259	0.633	0.892	0.979	0.997	0.999+	0.999+	0.999+
	50	24	16	1.00	0.404	0.857	0.988	0.999	0.999+	0.999+	0.999+	0.999+

TABLE 2. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.047	60	10	8	0.10	0.059	0.080	0.120	0.195	0.320	0.465	0.562	0.604
0.048	60	15	11	0.10	0.062	0.090	0.147	0.247	0.356	0.421	0.449	0.457
0.049	60	22	15	0.10	0.068	0.107	0.173	0.248	0.296	0.324	0.334	0.338
0.047	60	31	20	0.10	0.071	0.112	0.162	0.204	0.230	0.243	0.246	0.247
	60	10	8	0.20	0.071	0.118	0.210	0.366	0.572	0.766	0.893	0.951
	60	15	11	0.20	0.079	0.146	0.281	0.500	0.737	0.877	0.917	0.925
	60	22	15	0.20	0.090	0.187	0.370	0.588	0.728	0.787	0.809	0.816
	60	31	20	0.20	0.100	0.215	0.383	0.518	0.601	0.641	0.653	0.656
	60	10	8	0.30	0.084	0.162	0.305	0.512	0.725	0.877	0.956	0.986
	60	15	11	0.30	0.097	0.209	0.417	0.679	0.880	0.969	0.993	0.998
	60	22	15	0.30	0.116	0.281	0.567	0.832	0.953	0.979	0.985	0.986
	60	31	20	0.30	0.136	0.347	0.636	0.822	0.900	0.929	0.937	0.939
	60	10	8	0.40	0.099	0.208	0.395	0.624	0.816	0.928	0.977	0.993
	60	15	11	0.40	0.117	0.277	0.537	0.795	0.940	0.988	0.998	0.999+
	60	22	15	0.40	0.145	0.379	0.709	0.928	0.991	0.999	0.999+	0.999+
	60	31	20	0.40	0.178	0.486	0.820	0.963	0.990	0.995	0.997	0.997
	60	10	8	0.50	0.114	0.255	0.475	0.707	0.872	0.955	0.986	0.994
	60	15	11	0.50	0.139	0.345	0.637	0.867	0.968	0.995	0.999	0.999+
	60	22	15	0.50	0.177	0.474	0.808	0.965	0.997	0.999+	0.999+	0.999+
	60	31	20	0.50	0.223	0.614	0.920	0.994	0.999+	0.999+	0.999+	0.999+
	60	10	8	0.60	0.130	0.301	0.545	0.770	0.908	0.970	0.989	0.994
	60	15	11	0.60	0.162	0.412	0.716	0.912	0.982	0.997	0.999+	0.999+
	60	22	15	0.60	0.211	0.561	0.874	0.983	0.999	0.999+	0.999+	0.999+
	60	31	20	0.60	0.272	0.719	0.964	0.999	0.999+	0.999+	0.999+	0.999+
	60	10	8	0.70	0.146	0.347	0.605	0.816	0.932	0.979	0.990	0.995
	60	15	11	0.70	0.186	0.474	0.777	0.940	0.989	0.998	0.999+	0.999+
	60	22	15	0.70	0.246	0.637	0.917	0.991	0.999	0.999+	0.999+	0.999+
	60	31	20	0.70	0.322	0.795	0.982	0.999	0.999+	0.999+	0.999+	0.999+
	60	10	8	0.80	0.163	0.390	0.657	0.851	0.948	0.982	0.991	0.995
	60	15	11	0.80	0.211	0.532	0.825	0.958	0.993	0.999	0.999+	0.999+
	60	22	15	0.80	0.283	0.702	0.945	0.995	0.999+	0.999+	0.999+	0.999+
	60	31	20	0.80	0.374	0.852	0.991	0.999+	0.999+	0.999+	0.999+	0.999+
	60	10	8	0.90	0.180	0.431	0.700	0.878	0.960	0.983	0.992	0.996
	60	15	11	0.90	0.236	0.584	0.861	0.970	0.995	0.999	0.999+	0.999+
	60	22	15	0.90	0.320	0.757	0.963	0.997	0.999+	0.999+	0.999+	0.999+
	60	31	20	0.90	0.425	0.894	0.995	0.999+	0.999+	0.999+	0.999+	0.999+
	60	10	8	1.00	0.197	0.470	0.738	0.899	0.967	0.985	0.992	0.996
	60	15	11	1.00	0.262	0.631	0.890	0.979	0.997	0.999+	0.999+	0.999+
	60	22	15	1.00	0.357	0.802	0.975	0.998	0.999+	0.999+	0.999+	0.999+
	60	31	20	1.00	0.475	0.925	0.997	0.999+	0.999+	0.999+	0.999+	0.999+

TABLE 2. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.049	75	10	8	0.10	0.060	0.081	0.122	0.197	0.325	0.499	0.652	0.737
0.050	75	15	11	0.10	0.065	0.093	0.150	0.260	0.424	0.556	0.607	0.621
0.052	75	22	15	0.10	0.072	0.112	0.193	0.310	0.407	0.454	0.475	0.482
0.049	75	29	19	0.10	0.071	0.118	0.196	0.277	0.330	0.359	0.369	0.372
	75	10	8	0.20	0.073	0.120	0.212	0.367	0.571	0.766	0.898	0.963
	75	15	11	0.20	0.082	0.149	0.283	0.499	0.741	0.905	0.968	0.983
	75	22	15	0.20	0.095	0.192	0.386	0.657	0.851	0.918	0.936	0.941
	75	29	19	0.20	0.098	0.220	0.441	0.673	0.791	0.841	0.858	0.863
	75	10	8	0.30	0.086	0.164	0.306	0.511	0.723	0.876	0.955	0.986
	75	15	11	0.30	0.100	0.212	0.417	0.677	0.878	0.969	0.994	0.999
	75	22	15	0.30	0.121	0.285	0.566	0.840	0.969	0.996	0.999	0.999
	75	29	19	0.30	0.131	0.339	0.670	0.906	0.979	0.991	0.994	0.994
	75	10	8	0.40	0.101	0.210	0.395	0.623	0.815	0.927	0.976	0.989
	75	15	11	0.40	0.120	0.279	0.537	0.792	0.938	0.987	0.998	0.999+
	75	22	15	0.40	0.150	0.381	0.705	0.924	0.990	0.999	0.999+	0.999+
	75	29	19	0.40	0.167	0.460	0.813	0.974	0.999	0.999+	0.999+	0.999+
	75	10	8	0.50	0.116	0.256	0.475	0.706	0.871	0.954	0.981	0.990
	75	15	11	0.50	0.142	0.347	0.635	0.864	0.967	0.994	0.999	0.999+
	75	22	15	0.50	0.182	0.474	0.803	0.963	0.996	0.999+	0.999+	0.999+
	75	29	19	0.50	0.207	0.571	0.896	0.991	0.999+	0.999+	0.999+	0.999+
	75	10	8	0.60	0.132	0.303	0.544	0.768	0.907	0.966	0.983	0.991
	75	15	11	0.60	0.165	0.412	0.714	0.910	0.981	0.997	0.999+	0.999+
	75	22	15	0.60	0.216	0.559	0.869	0.981	0.999	0.999+	0.999+	0.999+
	75	29	19	0.60	0.251	0.667	0.942	0.996	0.999+	0.999+	0.999+	0.999+
	75	10	8	0.70	0.148	0.348	0.604	0.814	0.931	0.970	0.985	0.992
	75	15	11	0.70	0.189	0.474	0.775	0.938	0.988	0.993	0.999+	0.999+
	75	22	15	0.70	0.251	0.634	0.913	0.990	0.999	0.999+	0.999+	0.999+
	75	29	19	0.70	0.295	0.745	0.968	0.999	0.999+	0.999+	0.999+	0.999+
	75	10	8	0.80	0.165	0.391	0.655	0.850	0.945	0.973	0.986	0.993
	75	15	11	0.80	0.214	0.531	0.822	0.957	0.993	0.999	0.999+	0.999+
	75	22	15	0.80	0.287	0.699	0.942	0.995	0.999+	0.999+	0.999+	0.999+
	75	29	19	0.80	0.341	0.807	0.982	0.999	0.999+	0.999+	0.999+	0.999+
	75	10	8	0.90	0.182	0.432	0.699	0.877	0.950	0.975	0.987	0.993
	75	15	11	0.90	0.239	0.583	0.859	0.969	0.995	0.999	0.999+	0.999+
	75	22	15	0.90	0.323	0.753	0.960	0.997	0.999+	0.999+	0.999+	0.999+
	75	29	19	0.90	0.387	0.855	0.990	0.999+	0.999+	0.999+	0.999+	0.999+
	75	10	8	1.00	0.199	0.470	0.736	0.898	0.954	0.977	0.988	0.994
	75	15	11	1.00	0.264	0.629	0.887	0.978	0.997	0.999	0.999+	0.999+
	75	22	15	1.00	0.359	0.798	0.973	0.998	0.999+	0.999+	0.999+	0.999+
	75	29	19	1.00	0.433	0.892	0.994	0.999+	0.999+	0.999+	0.999+	0.999+

TABLE 2. (CONTINUED)

alpha	m=n	r	k	e	delta 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.050	100	10	8	0.10	0.062	0.083	0.123	0.198	0.327	0.509	0.699	0.834
0.052	100	15	11	0.10	0.067	0.096	0.154	0.263	0.444	0.657	0.790	0.827
0.049	100	20	14	0.10	0.065	0.099	0.170	0.307	0.510	0.658	0.703	0.717
0.048	100	27	18	0.10	0.069	0.112	0.204	0.357	0.496	0.557	0.585	0.593
0.048	100	36	23	0.10	0.073	0.128	0.230	0.346	0.418	0.457	0.472	0.476
	100	10	8	0.20	0.074	0.122	0.214	0.367	0.570	0.765	0.898	0.964
	100	15	11	0.20	0.084	0.152	0.285	0.499	0.739	0.906	0.977	0.996
	100	20	14	0.20	0.085	0.168	0.336	0.594	0.838	0.962	0.992	0.996
	100	27	18	0.20	0.093	0.201	0.422	0.721	0.919	0.972	0.981	0.984
	100	36	23	0.20	0.105	0.248	0.518	0.781	0.896	0.933	0.945	0.948
	100	10	8	0.30	0.088	0.166	0.307	0.511	0.722	0.875	0.954	0.976
	100	15	11	0.30	0.103	0.215	0.418	0.675	0.875	0.968	0.994	0.999
	100	20	14	0.30	0.108	0.247	0.499	0.779	0.944	0.992	0.999	0.999+
	100	27	18	0.30	0.122	0.307	0.619	0.888	0.985	0.999	0.999+	0.999+
	100	36	23	0.30	0.142	0.387	0.748	0.957	0.996	0.999	0.999	0.999+
	100	10	8	0.40	0.103	0.211	0.396	0.622	0.813	0.926	0.962	0.979
	100	15	11	0.40	0.124	0.282	0.536	0.790	0.937	0.987	0.998	0.999+
	100	20	14	0.40	0.132	0.331	0.635	0.881	0.978	0.998	0.999+	0.999+
	100	27	18	0.40	0.155	0.416	0.762	0.956	0.996	0.999+	0.999+	0.999+
	100	36	23	0.40	0.185	0.523	0.876	0.990	0.999+	0.999+	0.999+	0.999+
	100	10	8	0.50	0.118	0.258	0.475	0.704	0.869	0.938	0.967	0.982
	100	15	11	0.50	0.145	0.348	0.633	0.862	0.966	0.994	0.999	0.999+
	100	20	14	0.50	0.159	0.414	0.738	0.935	0.991	0.999	0.999+	0.999+
	100	27	18	0.50	0.190	0.520	0.855	0.982	0.999	0.999+	0.999+	0.999+
	100	36	23	0.50	0.233	0.644	0.941	0.997	0.999+	0.999+	0.999+	0.999+
	100	10	8	0.60	0.134	0.304	0.544	0.766	0.895	0.945	0.971	0.984
	100	15	11	0.60	0.168	0.413	0.711	0.907	0.980	0.997	0.999	0.999+
	100	20	14	0.60	0.188	0.493	0.813	0.963	0.996	0.999+	0.999+	0.999+
	100	27	18	0.60	0.228	0.612	0.913	0.992	0.999+	0.999+	0.999+	0.999+
	100	36	23	0.60	0.284	0.741	0.972	0.999	0.999+	0.999+	0.999+	0.999+
	100	10	8	0.70	0.150	0.348	0.603	0.813	0.907	0.951	0.974	0.986
	100	15	11	0.70	0.192	0.474	0.772	0.937	0.988	0.998	0.999	0.999+
	100	20	14	0.70	0.218	0.565	0.867	0.978	0.998	0.999+	0.999+	0.999+
	100	27	18	0.70	0.268	0.691	0.947	0.996	0.999+	0.999+	0.999+	0.999+
	100	36	23	0.70	0.336	0.816	0.987	0.999+	0.999+	0.999+	0.999+	0.999+
	100	10	8	0.80	0.167	0.391	0.654	0.836	0.916	0.956	0.976	0.987
	100	15	11	0.80	0.216	0.530	0.820	0.956	0.992	0.998	0.999	0.999+
	100	20	14	0.80	0.249	0.629	0.905	0.987	0.999	0.999+	0.999+	0.999+
	100	27	18	0.80	0.309	0.756	0.968	0.998	0.999+	0.999+	0.999+	0.999+
	100	36	23	0.80	0.389	0.871	0.994	0.999+	0.999+	0.999+	0.999+	0.999+
	100	10	8	0.90	0.184	0.432	0.698	0.851	0.923	0.960	0.978	0.988
	100	15	11	0.90	0.241	0.581	0.856	0.968	0.994	0.998	0.999	0.999+
	100	20	14	0.90	0.280	0.684	0.931	0.992	0.999	0.999+	0.999+	0.999+
	100	27	18	0.90	0.350	0.809	0.980	0.999	0.999+	0.999+	0.999+	0.999+
	100	36	23	0.90	0.442	0.910	0.997	0.999+	0.999+	0.999+	0.999+	0.999+
	100	10	8	1.00	0.201	0.470	0.733	0.863	0.930	0.963	0.980	0.989
	100	15	11	1.00	0.267	0.628	0.885	0.977	0.995	0.999	0.999+	0.999+
	100	20	14	1.00	0.312	0.733	0.950	0.995	0.999+	0.999+	0.999+	0.999+
	100	27	18	1.00	0.392	0.851	0.988	0.999+	0.999+	0.999+	0.999+	0.999+
	100	36	23	1.00	0.494	0.938	0.998	0.999+	0.999+	0.999+	0.999+	0.999+

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12 / 31 / 90