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An Optical Model Analysis of $^{16}O + ^{12}C$ Elastic Scattering at E/A = 94 MeV

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Abstract

An analysis of experimental data for $^{16}O + ^{12}C$ elastic scattering at an ^{16}O bombarding energy of 1503 MeV is presented, including the use of "model-independent" and "semi-model-independent" techniques. We conclude that the data unambiguously indicate an optical potential whose real part is approximately 80 MeV deep in the interior, but which is more attractive in the surface region than that given by a folding model with a "realistic" nucleon-nucleon interaction.

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The elastic scattering of two nuclei is frequently interpreted in terms of the scattering by a complex interaction potential, $U(r) = V(r) + i\,W(r)$, which depends only upon the distance r between the centres of mass of the two nuclei. This is called the optical model[1]. A popular choice is the Woods-Saxon (WS) form,

$$V(r) = -V_o/\left(1 + e^{x_V}\right), \qquad x = (r-R_V)/a_V, \tag{1}$$

with a similar form for W(r). There is also interest in a more microscopic approach[1] which obtains at least the real part by folding a nucleon-nucleon potential over the distributions of the nucleons in each nucleus, in analogy to the electrostatic potential between two charge distributions.

Another, "model-independent", approach[1] is valuable in order to study whether the experimental scattering data can provide information on deviations of the shape of the potential from either of the two forms just mentioned. The particular technique used here[2] takes the folded potentials as a reference potential and adds a correction term ΔV. We then treat as variables the values of ΔV at specified radii; between these "knots" a cubic spline interpolation is used. This has been used successfully, for example, to analyse the scattering over a range of energies of ^{16}O ions by ^{28}Si nuclei[3]. This procedure can result in many variable parameters which have to be adjusted to provide an optimum fit to the data, and consequently can be very demanding of computer time. A supercomputer can be very useful in these circumstances, making feasible a careful analysis and a detailed study of the multiparameter space. For example, for the case considered here a typical 'spline' fit may take 10 to 20 hours on a VAX 11/785, but only ... minutes on the CYBER/205.

An intermediate approach, also used here, takes a specified form for the correction ΔV, thus reducing considerably the number of free parameters compared to the spline method. (Typically, the form chosen for ΔV will be suggested by preliminary spline studies[3,4]). Here we use the sum of two terms, each of which is a derivative of the WS form (1),

$$\Delta V = -\sum_{i=1,2} 4a_i \, dV_i(r,R_i,a_i)/dr \tag{2}$$

thus introducing six parameters (V_i,R_i,a_i), with i=1,2.

The case we study here is the scattering of $^{16}O + {}^{12}C$ at an energy per nucleon of E/A = 94 MeV. This energy is sufficiently high, and the data[5] for this light system are sufficiently detailed and extensive (see fig. 1), that we may hope to deduce unambiguously some characteristics of the appropriate optical potential. For the (real) reference potential we took the folding model with the "realistic" DDM3Y nucleon-nucleon effective interaction[3] (see fig. 2). The imaginary potential W(r) was taken to have the Woods-Saxon form (1). This model by itself has been shown[6] to give a reasonable description of the scattering of $^{12}C + {}^{12}C$ and $^{16}O + {}^{12}C$ at a number of energies; the optimum fit to the present data, after varying the parameters of the imaginary potential, is shown in fig. 1. We used the conventional measure of the goodness of fit,

$$\chi^2 = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{\sigma_{ex}(\theta_i) - \sigma_{th}(\theta_i)}{\Delta\sigma_{ex}(\theta_i)} \right]^2 ,$$

where n = 65 is the number of experimental cross sections $\sigma_{ex}(\theta_i)$, whose uncertainties are $\Delta\sigma_{ex}(\theta_i)$, while $\sigma_{th}(\theta_i)$ are the theoretical cross sections. (Only statistical counting uncertainties are included in the $\Delta\sigma$; no attempt is made to estimate other experimental errors.) We see that,

although the qualitative agreement between theory and experiment is satisfactory, there are noticeable deviations and $\chi^2 = 8.4$ is rather large. Renormalizing the folded potential by an overall factor N led to no improvement; the optimum N = 1.0 to better than 1%. We also increased by a factor of 10, the uncertainties for the seven data in the deep minimum at 2.5°, where the finite angular aperture of the detectors (not taken into account in the theoretical fit) is likely to have most effect. This led to $\chi^2 = 4.8$, but no significant change in the potential. Using other forms for the imaginary potential also had little effect; for example, adding a term which is a derivative of the WS (1), thus adding three more parameters, only reduced χ^2 to 6.6.

Next we added the correction term ΔV to the real potential, using the spline technique. (For these calculations we used the modified uncertainties.) The knots were taken to be at r = 0(0.5)8.0 fm. Preliminary studies suggested that a fit could be obtained by constraining ΔV to be +50 MeV (i.e., repulsive) at the first three knots. Varying the remaining values, together with the imaginary parameters (a total of 12), resulted in $\chi^2 = 1.06$ (with the modified uncertainties), an improvement by a factor of about 4 over what was obtained without ΔV $(\chi^2 = 4.8)$. The corresponding real potential is shown in fig. 3 ("constrained"), and the fit to the measured cross sections is similar to the dashed curve shown in fig. 1.

These results indicate an added attraction in the surface (r ~ 3 to 6 fm), but a considerable weakening of the real potential (from -130 MeV to about -80 MeV) at small r, compared to the folding model. To study the latter point, we looked for other solutions in which the spline potential was allowed to vary freely at small r (thus a total of 19 parameters). The

only one obtained (with $\chi^2 = 0.51$) is included in fig. 3 ("free"); it oscillates about the constrained solution but maintains the same overall characteristics. The improvement in fit to the data, over that obtained with the constrained potential, is perceptible but it is not perfect and may not be significant in view of our lack of knowledge of other possible experimental errors. Consequently, although such behaviour might be the consequence of representing a non-local and angular momentum-dependent potential by a local, ℓ-independent one, it would be unwise to conclude that the oscillations in the potential at large r have any physical significance. We seem to be at the limit of what can be learned from the data.

Since these studies indicated the need for repulsion in the interior and additional attraction in the surface, compared to the folding model, next we considered the model (2) for ΔV, giving a total of nine variables. One result is shown in fig. 2; the corresponding fit to the data is included in fig. 1 ($\chi^2 = 1.20$ with the original uncertainties; a similar fit with the modified errors yielded $\chi^2 = 0.81$) and is comparable to those obtained with the spline technique. Other calculations with just one derivative term for ΔV resulted in some improvements over the folded potential alone, either with a repulsive term at small r or an attractive one in the surface ($r \approx 4$ fm), but clearly both features are needed to give the best fit to the data.

We conclude from these studies that, while the folded potential gives qualitative agreement with these data, closer examination reveals that large corrections are needed, namely a substantial repulsive component in the interior and some added attraction in the surface region. The folding model gives the correct potential for distances beyond about 6 fm.

As an ironic postscript, we note that previous analyses[5)] of these data employed the six-parameter WS form (1) and found a preference for a potential with a depth $V_o \approx 80$ MeV. We confirmed this preference, obtaining the potential WS shown as dotted curves in figs. 2 and 3. This yields $\chi^2 =$ 1.95 with the original uncertainties and 1.07 with the modified ones; the fit to the data is similar to that shown for "FOLD + ΔV" in fig. 1. Consequently, the additional six parameters of the constrained spline potential have produced no improvement in χ^2, while the unconstrained one reduces χ^2 by a factor of 2 but may have introduced insignificant and/or unphysical features. This serves to emphasise that such powerful techniques must be applied with caution and their use guided by physical intuition and a sense of what is reasonable. Nonetheless, these studies have served to establish that a potential whose main features are embodied in the WS one is, within the uncertainties, uniquely determined by the data.

Acknowledgments

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Table 1. Optical Potential Parameters

Potential type	W_o (MeV)	R_W (fm)	a_W (fm)	χ^2	σ_A [a] (mb)
WS [b]	23.9	5.070	0.813	1.95	1290
WS [c]	29.7	4.843	0.769	1.07 [d]	1237
FOLD [e]	15.8	5.664	0.785	8.4	1321
FOLD + WSD [f]	19.0	5.443	0.793	1.20	1323
FOLD + SPLINE [g] (CONSTRAINED)	20.0	5.235	0.882	1.06 [d]	1345
FOLD + SPLINE [g] (FREE)	21.5	5.285	0.725	0.51 [d]	1246

[a] Total absorption, or reaction, cross section.

[b] Real part as eq. (1) with $V_o = 80$, $R_V = 4.259$, $a_V = 0.773$.

[c] Real part as eq. (1) with $V_o = 80$, $R_V = 4.259$, $a_V = 0.776$.

[d] Obtained using the modified uncertainties; see text.

[e] Real part is folded potential shown in fig. 2.

[f] ΔV as eq. (2) with $V_1 = -53.4$, $R_1 = 0.549$, $a_1 = 0.493$, $V_2 = 9.5$, $R_2 = 3.866$, $a_2 = 0.451$.

[g] ΔV as in fig. 3.

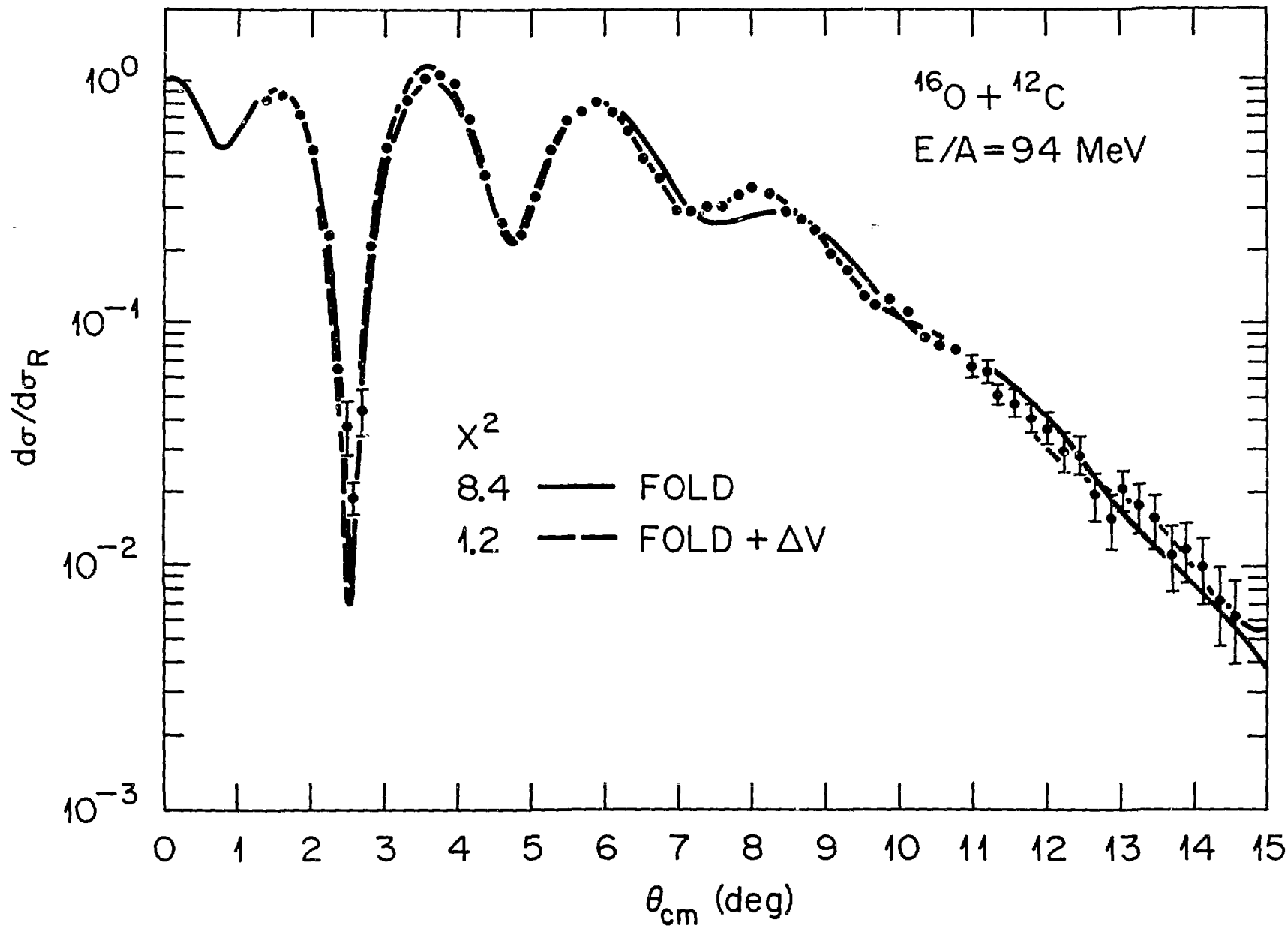
Figure Captions

Fig. 1. Comparison of the measured differential[5] cross sections for $^{16}O + ^{12}C$ at E/A = 94 MeV with the fits obtained with the folded potential alone (FOLD) and with the addition of ΔV with the form (2) (FOLD + ΔV).

Fig. 2. The real parts of the potentials corresponding to the fits shown in fig. 1, together with the optimum WS potential with the form (1).

Fig. 3. Comparison of the real potentials consisting of the folded one of fig. 2 plus the spline ΔV terms, with the WS one.

ORNL-DWG 87-11748



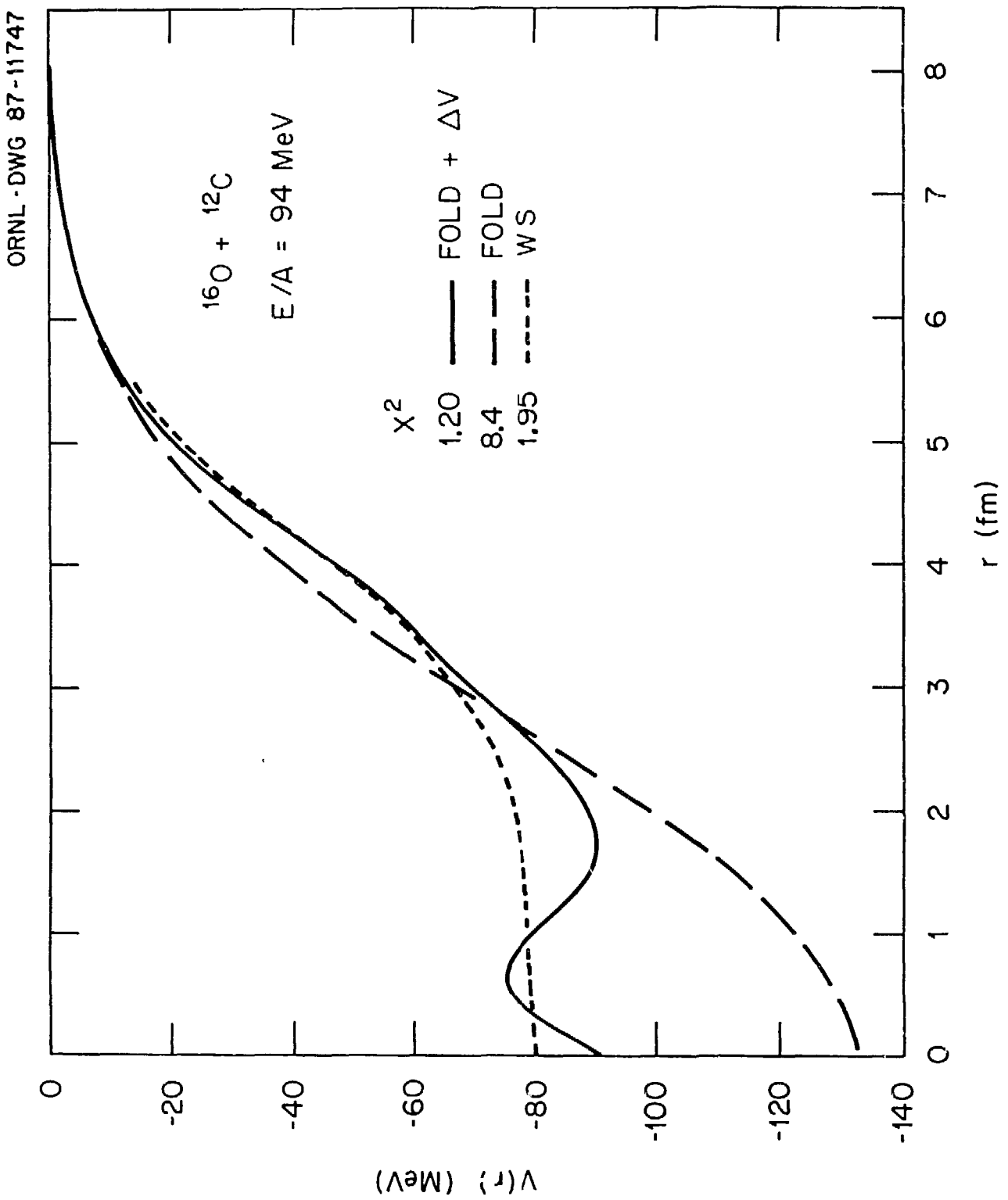

ORNL-DWG 87-11747
$^{16}O + ^{12}C$
E/A = 94 MeV
χ^2
1.20 FOLD + ΔV
8.4 FOLD
1.95 WS
V(r) (MeV)
r (fm)
0
-20
-40
-60
-80
-100
-120
-140
0
1
2
3
4
5
6
7
8

Fig. 2

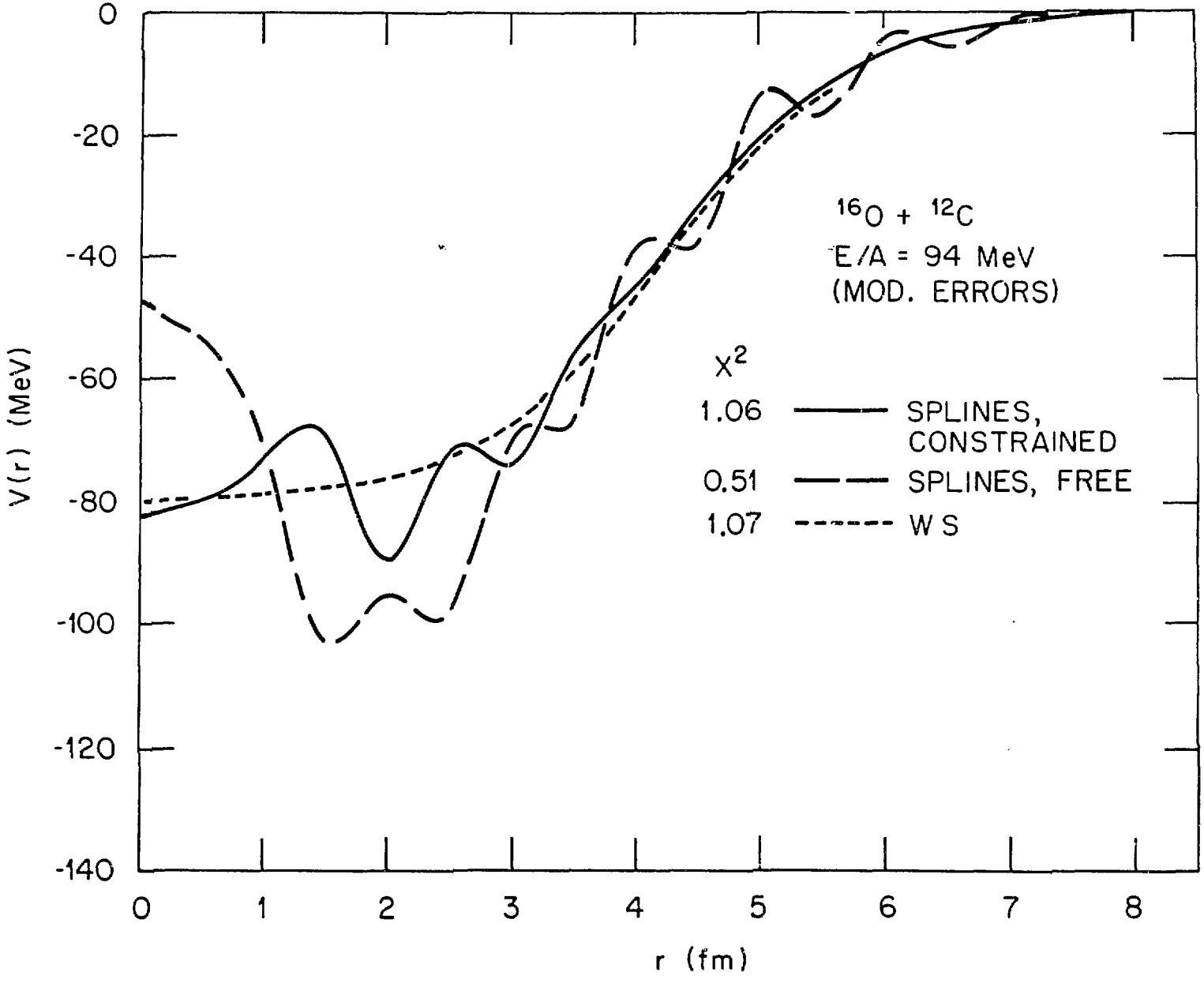

0
-20
-40
-60
-80
-100
-120
-140
V(r) (MeV)
0
1
2
3
4
5
6
7
8
r (fm)
$^{16}O + {}^{12}C$
E/A = 94 MeV
(MOD. ERRORS)
χ^2
1.06 SPLINES, CONSTRAINED
0.51 SPLINES, FREE
1.07 W S

Fig. 3