CONF-8508149--4

QCD RESULTS FOR SMALL X AND FOR HEAVY FLAVOR PRODUCTION

John C. Collins*

Institute for Advanced Study.
Princeton, NJ 08540.
U.S.A.

MASTER

ABSTRACT

I review the work done at the SSC workshop on the small x problem and on heavy-quark production.

CONF-8508149--4

DE86 004963

1. INTRODUCTION

This talk covered two areas of perturbative QCD that received much attention at the Oregon workshop:

1. Hard collisions of partons whose momentum is a small fraction. x, of the total center-of-mass energy. The cross-sections are interestingly large, and nevertheless amenable to a suitably powerful perturbative treatment.
2. Production of heavy strongly-interacting particles, such as ordinary heavy quarks, or squarks or gluinos. The problem here is that the production of the first known heavy quark, viz. charm, is notoriously hard to get right using standard perturbative methods.

Progress at the workshop was made in both areas. For small x this ranged from Monte-Carlo calculations of multiple scattering, notably by Sjostrand, to theoretical calculations of parton-parton shadowing effects by Mueller. As for heavy quarks, Soper, Sterman and myself showed that ordinary perturbative methods should indeed apply. There was also some work on how to define heavy-quark distribution functions, which I will not cover in this talk. The work in these areas was part of on-going research whose final results will only emerge later.

Although many of the results have the names of particular authors attached, it should be emphasized that many other participants also contributed to the frequent discussions on these subjects — in particular S. Brodsky, K. Ellis, J. Gunion, I. Hinchliffe, etc.

2. SMALL X

The treatment of hard scattering processes at small x, i.e., when the incoming partons have a small fraction of the momenta of their parent hadrons is an important topic at SSC energies, as was seen at last summer's Snowmass workshop[1].

---

* Permanent address: Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616, U.S.A.

Talk given at Conference on Super-High-Energy Physics, Eugene, OR, Aug. 9-10, 1985.

AC02-80ER10712

Now, if one builds an accelerator like the SSC in order to go to very high energies, it might appear that to worry about small x phenomena is to go in exactly the opposite direction, back to low energies. However, there are a number of cogent reasons why one should study these phenomena in connection with the SSC. Among them are:

1. The interesting new physics at the SSC can reasonably be considered to start around the W-mass. This mass is much less than the center-of-mass energy.
2. The jet cross-section can be comparable to, or even larger than, the total cross-section, in a region that is nevertheless amenable to perturbative treatment. This not only means that the phenomena are easy to study experimentally, but also that they form significant backgrounds to other processes. At the Sp$\bar{p}$S collider the inclusive cross-section for making jets of many GeV transverse energy is at least in the millibarn region[2].
3. By going to an extremely high energy, we find a whole set of phenomena that can be treated by perturbative methods, but for which an unadorned low-order treatment is not very useful. In effect, we have to treat the sums of many graphs. We will have the opportunity to learn a lot more about QCD.
4. The total energy available is so high that even a very small value of x, e.g. 10^{-4}, can correspond to a collision with a subenergy of several GeV, where one might reasonably try to use perturbative methods.

The large cross-sections arise because of the number of gluons inside a hadron at small x.

One can see where the theoretical problems come from when one remembers that in the coefficients of the perturbation expansion there are large logarithms of the small ratio x. Thus the "convergence" and applicability of the expansion is, a priori, doubtful. The standard methods must indeed be improved to cope with the small-x region.

Perhaps the most important paper on this subject is the review by Gribov, Levin and Ryskin[3], which I will call GLR. Their work forms the basis of ours, and the first essential in working on the small-x problem is to understand their paper.

More details of the work on small-x that was associated with the present workshop can be found in papers by: Mueller and Qiu[4], Durand and Putikka[5], Ametller, Paver and Treleani[6], Humpert and Odorico[7], and Sjostrand[8]. The exposition below summarizes the understanding that we came to after many discussions.

2.1 Standard factorization

The usual factorization theorem[9] of perturbative QCD asserts that the cross-section for a process like jet production is given as a convolution of a hard scattering with parton distribution functions. Schematically, we have:

$$\sigma_{jet} = \int f(x_A;Q) f(x_B;Q) \sigma_{parton-jet}. \tag{1}$$

In this formula there are integrations over all the relevant phase-space, Q denotes a typical virtuality of the hard scattering, and the Q-dependence of the parton distributions is given by the Altarelli-Parisi equation[10]. The hard-scattering cross section, $\sigma_{parton->jet}$, can be calculated perturbatively, in an expansion in powers of $\alpha_s(Q)$.

The two basic theoretical problems that arise are:

1. Is the factorization formula (1) actually true when the x's involved are small? The standard derivation assumes that the hard collision takes a significant fraction of the total energy.

2. Even if the formula is true, the omnipresent logarithms of $1/x$ in perturbative coefficients mean that a straightforward perturbative expansion will not be useful. The largeness of $\ln(1/x)$ will overcome the smallness of α_s.

2.2 Basic structure of small-x processes

The combination of the factorization property and the Altarelli-Parisi equation implies that the most important regions of momentum-space for Feynman graphs for jet production and other similar processes have the form of Fig. 1. The subgraph for the hard scattering is joined to each incoming hadron by a ladder. Higher order corrections can all be included as corrections to the rungs of the ladder and to the hard scattering. Then the corrections have no large logarithms, and perturbation theory can be applied, provided that Q is large enough. In the usual situation, with moderate x, the ladders evolve the partons from small virtuality in the hadrons to large virtuality at the hard scattering.

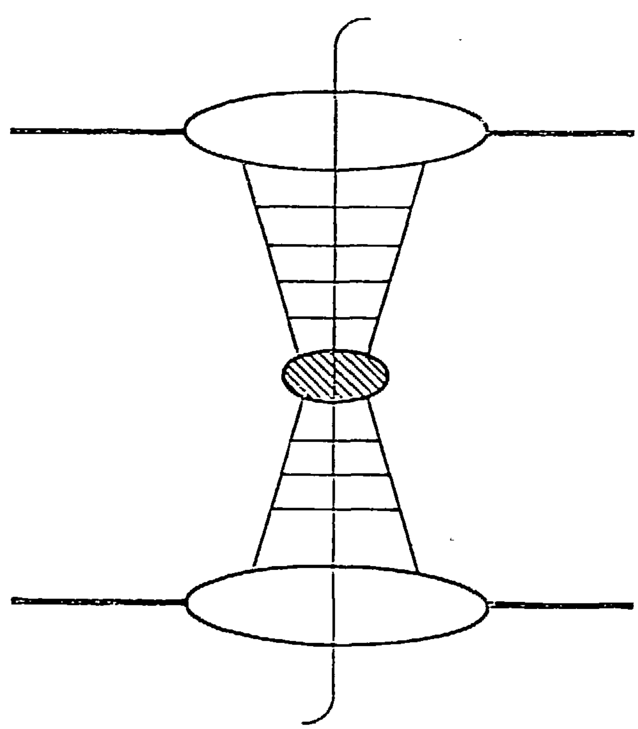

Fig. 1. Leading graphs for jet production.

GLR tell us that when one goes to small x the leading logarithmic regions do have the same form. The evolution along the ladders is from large x to small x, as well as from small virtuality to large virtuality. GLR show us exactly how one is to sum these contributions. Then, all too briefly, they sketch arguments as to the form of the corrections. The corrections are supposed to change the precise form of the rungs and sides of the ladders, but not the overall structure.

To obtain this generalized ladder structure, it is necessary to combine sets of graphs by Ward identities: the leading regions for individual graphs are quite different, in normal gauges. The necessary manipulations are not at all easy to follow in the absence of a physical picture. Luckily, GLR also provide an intuitive coordinate-space picture which at least in general terms, follows from the Feynman graphs that they study. I will summarize this in the next subsection.

There is still a lot of work that needs to be done before we we fully understand what is going on in the small-x region.

One more important topic that GLR treat is that of discovering the limits of validity of perturbative methods of the sort that we are using. Recent work has been aimed at uncovering more of the details of these limits, and I will discuss this under the heading of "parton overcrowding".

2.3 Physical picture, according to GLR

To understand what is going on at small-x, one must remember that a fast moving hadron is Lorentz contracted in the direction of motion. Thus, at least as far as the valence partons are concerned, the hadron is in the shape of a pancake. Hence the impulse approximation is appropriate for discussing hard collisions of the partons.

The partons that participate in a hard scattering of scale Q have a radius of $1/Q$, so that one has a head-on view of a hadron that is depicted in Fig. 2. It is also useful to have the side-on view of a high energy hadron collision shown in Fig. 3.

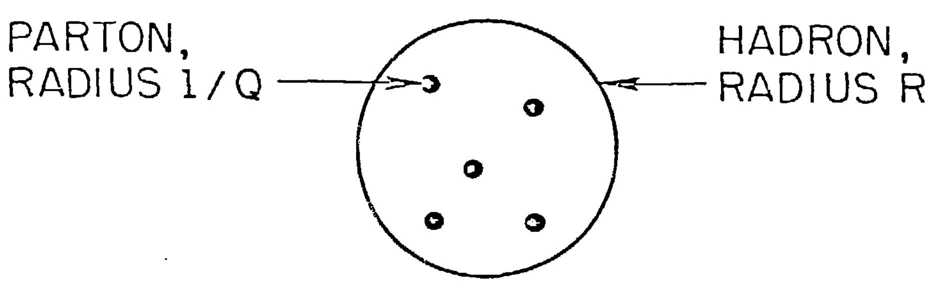

Fig. 2. Head-on view of a hadron in a high-energy collision.

If factorization for the hard collision in its normal sense is to occur, then the partons participating in the hard scattering must be independent of the other partons, <u>over the time-scale of the hard scattering</u>. Thus the partons of radius $1/Q$ must not overlap.

Let us make the approximation that the partons are uniformly spread over the face of the hadron. Then we may define a relative density of partons by

$$W(x, Q) = \alpha_s(Q) \frac{xG(x, Q)}{Q^2 R_H^2}. \tag{2}$$

Here, $G(x, Q)$ denotes the gluon distribution function, and R_H denotes the hadron radius (of the order of 1 fm). There is a factor of x multiplying G because one considers

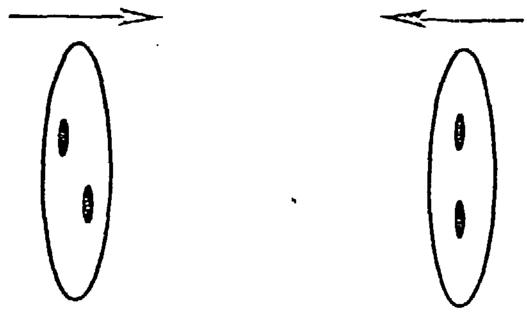

Fig. 3. Side view of high energy collision.

the number of partons in a finite rapidity interval about the particular value of interest. The criterion for the validity of normal factorization is $W \ll 1$.

If the gluon distribution were a constant times $1/x$, as a naive approach based on Regge theory used to suggest, then one would never get out of the region $W \ll 1$. However, over the range of x that is of interest, the distributions are substantially steeper than $1/x$, and they are increasing with Q.[11]

This intuitive picture summarizes the results of the analysis of Feynman graphs. When W becomes of order unity, non-ladder graphs start to be important, and it is not known how to make a perturbative analysis. One can find the limits of validity of factorization by determining when multi-ladder couplings become significant. This is the object of the studies of parton recombination summarized in Sec. 2.7.

2.4 Landmarks

As one goes from large values of x to small values, one can identify two landmarks, where the behavior of jet cross-sections reveals new features. The first is where σ_{jet} becomes of the same order as σ_{tot}, the total cross-section. Below this value of x, hard collisions become common — almost every collision of hadrons contains one. The second is where the relative density of partons, W, becomes of order unity. Below that value of x, factorization becomes inapplicable, as we have already seen, and a more appropriate way of describing the physics might, for example, be a hydrodynamic or thermodynamic model.

These limits can be reached while Q is still large enough for perturbation theory to be applicable, since the Altarelli-Parisi evolution greatly increases the number of gluons at small x as Q is increased.

An interesting region is where the jet cross-section is larger than the total cross-section. At the Sp$\bar{\text{p}}$S[2], where $\sqrt{s} = 540\text{GeV}$, this point is reached when the jet transverse energy gets down to a few GeV (Fig. 4). One might still hope to apply perturbation theory at such an energy. However, it has been suggested that the jet cross-section cannot exceed the total cross-section without violating unitarity. This is not so: the jet cross-section, calculated according to the usual factorization formula, is in fact an inclusive cross-section. Thus the jet cross-section is the total hadronic cross-section times the average number of jets per hadron-hadron collision. When the jet cross-section exceeds the total cross-section, that is merely an indication that typically there is more than one hard scattering per event or that each hard scattering produces many jets instead of two.

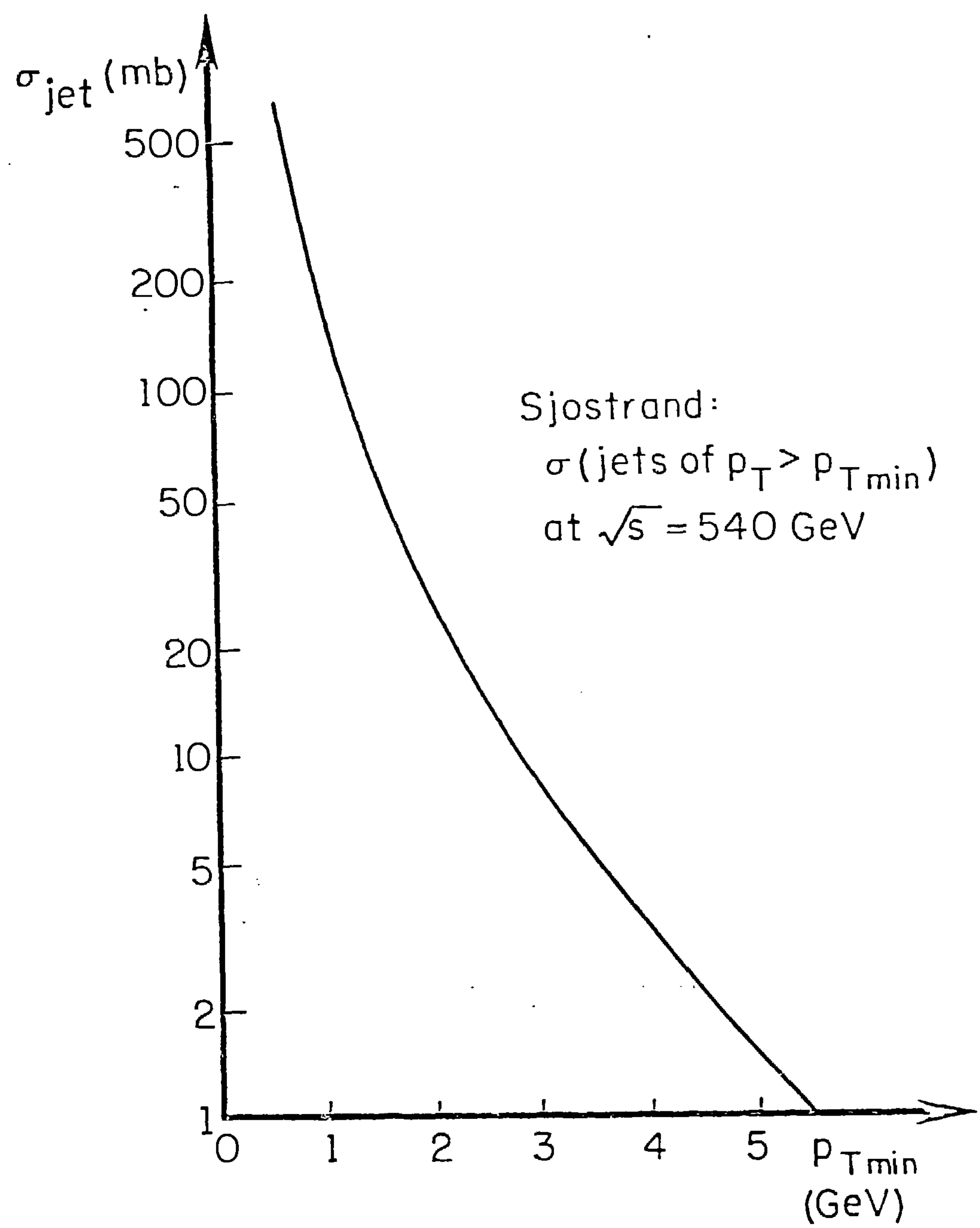

Fig. 4. Cross-section for jet production at the $Sp\bar{p}S$ as a function of transverse momentum p_T. The calculation is from Sjostrand's Monte-Carlo.

Even when more than one hard scattering occurs per event. the calculation of the inclusive cross-section is still correct. The point is that to prove factorization[12]. one explicitly considers the cross-section to make the observed jets. summed over everything else that happens in the collision[13]. This is accord with the physical picture of hadrons in a hard collision as given in Figs. 2 and 3. A similar phenomenon can be seen in the the AGK cutting rules for multiple Pomeron exchange. where an inclusive cross-section for some particle can be calculated while ignoring rescattering effects. If one examines Fig. 3. it is evident that the total cross-section can be driven by the geometrical size of hadrons, and the fact that the jet cross-section exceeds the total cross-section should not even be thought of as necessarily driving the increase in the total cross-section.

2.5 Multiple (parallel) hard scattering

It has been realized for some time[14] that σ_{jet} can be larger than σ_{tot} in a region of practical importance. and that therefore there are many events in which several hard scattering occur in parallel.

In connection with the workshop. Sjostrand modified his Monte-Carlo for hadron-hadron scattering to include the effects of these parallel hard scatterings. His result for the distribution of the number of hard scatterings per event is shown in Fig. 5. He then investigated the KNO and the E_T distributions. Without the multiple hard-scattering effects incorporated. he was unable to reproduce the distributions that are seen in experiment. With the multiple hard-scattering in. he had little trouble.

These results are rather nice. Unfortunately, the model is rather easy to criticize.

First. and most seriously. the results are strongly dependent on the lower cut-off, p_{Tmin}, on the produced jets. This is an indication that phenomena below the cut-off have not been taken into account correctly. In a proper treatment, which has not yet been worked out, but should be. there would be an upper cut-off of the same value on the low-p_T physics, and the two cut-off dependences would cancel. The cut-off would then be an arbitrary parameter chosen by the user of the program. It would merely indicate the scale below which one chooses not to use perturbative calculations. An appropriate model for the low-p_T physics would be some kind of multi-Pomeron exchange. such as already used for the "minimum-bias" physics in Paige's Monte-Carlo ISAJET. This would nicely fit in with the diagrammatic structure of the multiple hard scatterings.

The other serious problem is that the value of the hadron radius is inconsistent between different calculations.

There are also assumptions about the two-parton distribution within a hadron, but these can be tested and corrected by comparison with experiment. Indeed. the two-parton distribution is a quantity just like the single-parton distribution. It is cannot be calculated perturbatively, but must be measured.

Despite these obvious criticisms, one must not loose sight of the importance of such results for the qualitative features of hadron-hadron collisions at very high energies, whether at the Sp$\bar{p}$S, the Tevatron or the SSC. Hard scattering will be common. so that a minimum bias event will no longer be purely an effect of old-fashioned low-p_T physics. The tails of the KNO distribution and the E_T distributions may well be driven by fluctuations due to multiple hard scattering.

One can see similar effects in a slightly different way in a plot, shown in Fig. 6, that is due to Ametller. Paver, and Treleani[6]. This compares the contribution of $2 \to 4$ scattering with that from two separate $2 \to 2$ scatterings. now at $\sqrt{s} = 40\text{TeV}$, as is appropriate for the SSC.

2.6 Corrections to basic picture

The physical picture presented above is rather appealing. The various calculations that

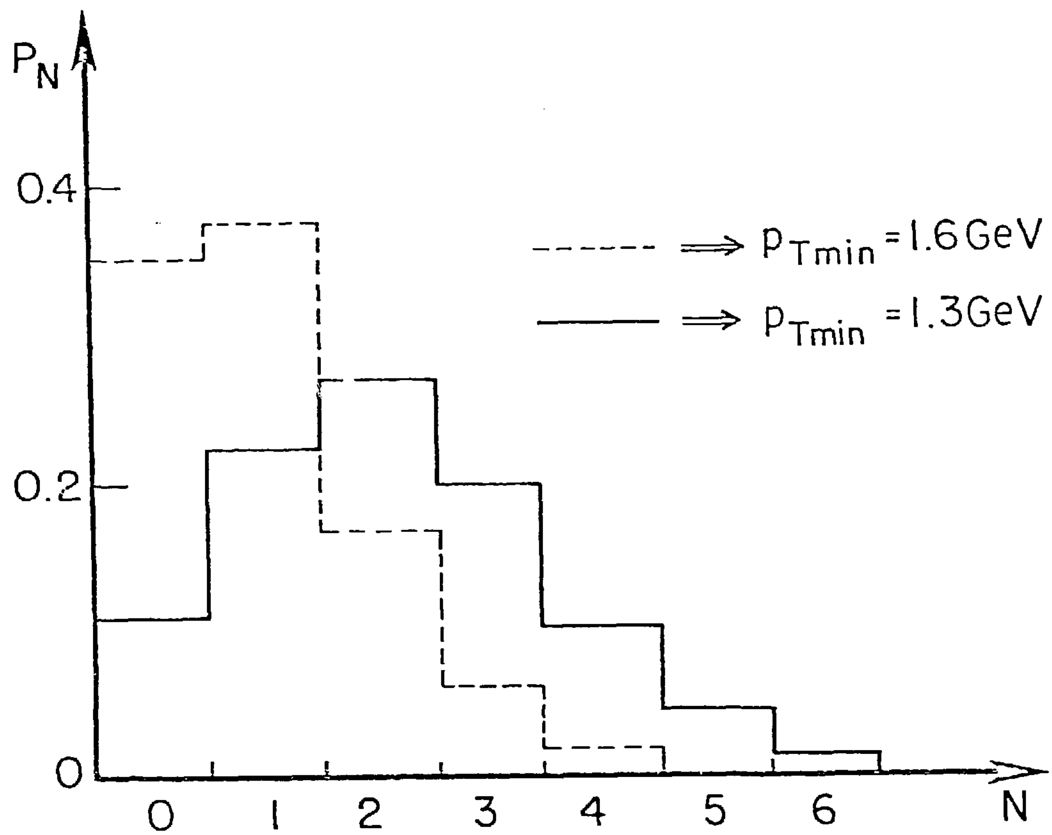

Fig. 5 Distribution of number of hard scattering per event at $\sqrt{s} = 540\text{GeV}$, according to Sjostrand. The curves are given for two values of the minimum p_T for an observed jet.

have been done are at least correct in the rough size of the predicted effects. However, to make the theoretical basis really reliable, we must understand the corrections to the leading logarithm approximation. The most intriguing are those that are due to parton overcrowding: these we will discuss in the next section. The other kind are those that correct the ladder graphs: they are probably the more important corrections, and I will discuss them first, following GLR's treatment.

The leading-logarithmic contributions come from ladders in which the momenta in the rungs are strongly ordered in both Q and x. There are some corrections that we are used to. For example, if we consider a rung without the strong ordering in x, then we simply have a piece of the same Altarelli-Parisi evolution that we have at larger x. If we lose the strong orderings in both x and Q, then we are in the middle of an ordinary higher order correction to the kernel, with no large logarithms to bother us.

If we have a rung with only the strong ordering in x, then we are building up a $\ln x$ in a higher-order graph for the Altarelli-Parisi kernel. One must perform a resummation of these logarithmic corrections, for otherwise the perturbation expansion makes no sense. According to GLR, these corrections have the effect of a Reggeization of the gluon, and their effect is small. The Reggeization amounts to a change of order α_s in the power of x, at small x.

Kwiecinski[15] has numerically calculated the effect of these corrections in the range of x

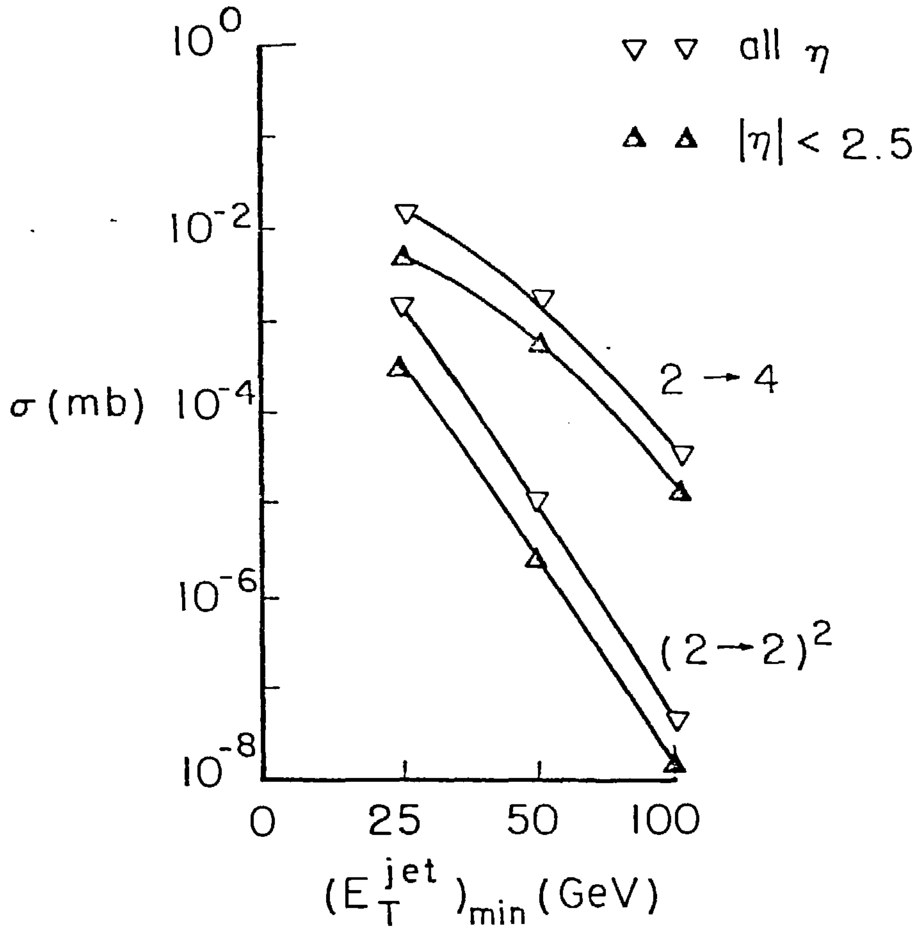

Fig. 6. Multiple jet production at the SSC according to Ref. 6].

down to 10^{-5} and of Q up to 100 GeV. He compared the case of the $O(\alpha_s)$ kernel with the case in which the extra Reggeization terms from higher orders were also included. Only below $x = 10^{-4}$ was there a significant effect, of more than 10%. It would nevertheless be useful to incorporate this Reggeization into other calculations of the evolution of the parton distributions. (It should be noted that Kwiecinski also studied the effects of combining the Reggeization corrections with the logarithmic part of the kernel instead of with the full $O(\alpha_s)$ kernel. In that case the effect was bigger — up to a factor of 2 in the range he considered. So it is not at all obvious that the effects are necessarily small.)

Although the form of the result for the Reggeization looks rather simple, it is far from trivial to derive it, and it is crucial to improve our understanding. The difficulty is that the regions of momentum-space for individual Feynman graphs do not manifestly have the form of ladders, even when the sum does. Moreover, the precise form of these regions is strongly gauge-dependent. To get the final result, one must sum whole classes of graphs using Ward identities similar to those used in the recent proof of Drell-Yan factorization[12]. GLR's discussion is quite terse. More work is needed here.

2.7 Parton overcrowding

Mueller and Qiu [4] have presented a calculation of the correction to the evolution of the parton distributions that is caused by the parton overcrowding. They consider the coupling of three ladders, as shown in Fig. 7. There the lower ladder represents the partons that go into the hard scattering, while the upper ladders represent the partons that are "recombining". This three-ladder coupling is related to the triple-Pomeron coupling, that was so well-known in the heyday of Regge theory. Mueller and Qiu present their result as a $1/Q^2$ correction to the Altarelli-Parisi equation. The correction is of order α_s^2 times $1/Q^2 R_H^2$ compared with the leading term. Here, R_H is a hadron radius: it is a scale that appears in the joint distribution of two gluons in a hadron.

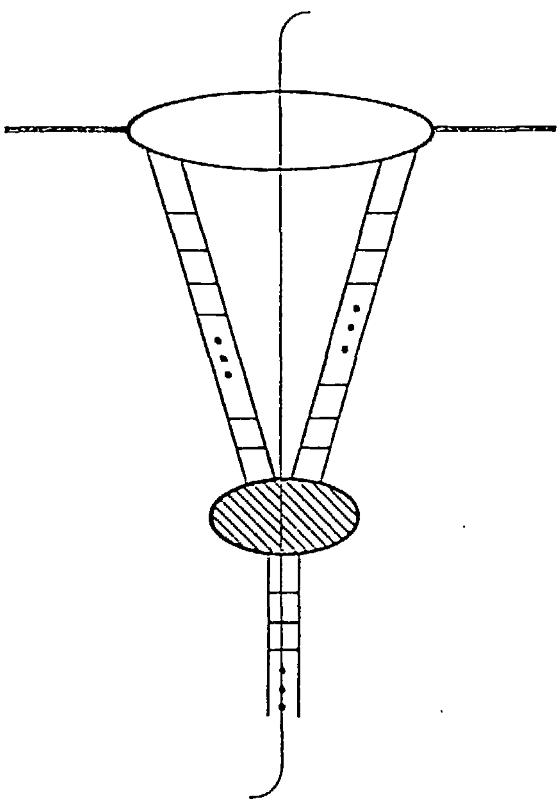

Fig. 7. Typical graphs calculated by Mueller and Qiu.

The result is:

$$\frac{d}{d\ln Q^2}G(x,Q) = \alpha_s \int \frac{dx'}{x'}P(x/x')G(x') - \frac{4\pi\alpha_s{}^2 C_A{}^2}{(N^2-1)}\frac{1}{Q^2}\int \frac{dx'}{x'}x'^2 G^{(2)}(x',x',Q^2). \quad (3)$$

Here, $P(x/x')$ is the ordinary Altarelli-Parisi kernel. N is the number of colors, and $G^{(2)}(x_1,x_2)$ is the two gluon distribution function. Mueller and Qiu estimate the numerical size of the correction and find it to be small under all practical circumstances where one would wish to trust perturbation theory at all. GLR[21] discuss a corresponding result, but they work with an equation for the x-dependence of the distribution function, rather than for the Q-dependence.

Durand and Putikka[5] have computed what ought to be the same quantity, but by a different method that they suggest is the correct generalization of the ordinary parton model. Their correction is of order α_s, so that the numerical effect is bigger than for Mueller and Qiu's correction, which is of order α_s^2.

It is important to resolve the discrepancy between the two calculations of the parton overcrowding effect.

Another problem that needs work is that if one defines the single-gluon distribution as a matrix element of an appropriate renormalized operator, as one often does, then the Altarelli-Parisi equation is exactly linear: it is the renormalization group equation. So to get the non-linear effects that are so obviously present on physical grounds, one must be careful to choose a definition of the parton distributions that is more suited to the small-x region.

2.8 Conclusions on small x

The recombination effects seem not to be drastically important. But it is important to ensure that this statement is reliable. One way of checking it is to study the situation with heavy nuclei, where the effects are bigger.[4]

Multiple scattering effects are going to be important at collider energies, and they must be included in the Monte-Carlos. But a considerable amount of work is needed to understand the correct formulation.

For both these last problems, it is necessary to know the precise form of the two-gluon distribution in a hadron. Experimental data is needed. The two-gluon correlation function is a fundamental quantity in QCD.

Further work is still needed to understand the theory of the small-x region. In particular, we must understand the non-leading logarithms.

3 HEAVY QUARKS

Another QCD topic that received much attention at the workshop was that of how to calculate the cross-section to make heavy flavors. Here, heavy flavors may be ordinary heavy quarks, or they may be, for example, gluinos, or other supersymmetric particles. To design an experiment to see some such flavor, one must have an estimate of the cross-section. Naturally one would expect to use QCD perturbation theory, with the obvious factorized formula. Unfortunately, there is, as yet, no complete proof of factorization for this case. In general terms, we know that having heavy quarks in Feynman graphs has a similar effect to giving light quarks large transverse momentum, as far as determining when we may use perturbation theory.

One must also know whether one should look in the central region as indicated by the gluon-fusion calculations, or in the forward direction, as suggested by some of the alternatives to the standard factorization result.

Unfortunately, the predictions for the case of charm are notoriously far below the data. Typical calculations for charm cross-sections at the ISR yield tens of μb, while the quoted experimental cross-sections range up to several mb[16]. This situation has led to a mistrust of perturbative calculations for heavy flavor production. Indeed, there have been a whole plethora of alternative mechanisms proposed that produce much higher cross-sections, at the expense of the failure of the factorization theorem for heavy flavor production.

The situation has now changed.

3.1 Standard QCD applies

On the theoretical side, Soper. Sterman and myself[17] showed that ordinary perturbative methods do in fact apply to the production of heavy enough quarks. We did not give a full proof. but merely looked at low-order graphs. using our work on the Drell-Yan case[12] to show what the danger points are. Moreover. we considered the alternative mechanisms that have been proposed for heavy flavor production. Our examination of low order graphs showed that these mechanisms either fail to be the leading effect or double-count contributions that are already in the standard factorization. We concluded from this that at least one of the following must be true:

1. QCD is wrong. Quite unlikely.
2. Higher twist terms in the production of charm are abnormally large. But we know that low order perturbation theory is fairly accurate for total cross-sections at energies above a couple of GeV.
3. Charmed hadrons do not follow the distribution of charmed quarks very well. This is likely, since 1.5 GeV jets do not look especially jet-like.
4. The experiments at the ISR are wrong. or, at least, misinterpreted in their extraction of a total charm cross-section is concerned. This is likely — see below.
5. Higher order corrections are abnormally gigantic. Possible.

3.2 Data on charm production; diffractive production

At the DPF conference immediately after the workshop summary conference, Goshaw presented some data from the LEBC collaboration[18]. These were bubble chamber measurements with very good acceptance over the whole of the forward hemisphere. The charm cross-section agreed with the standard perturbative QCD picture, at least if one allows oneself a K-factor of a common size. In the forward direction there was a significant excess of D-mesons correlated in valence quark content with that of the pion in the beam. A theoretical calculation using the Lund Monte-Carlo, which models the final-state interactions by strings. reproduced this effect.

Now the largest charm cross-sections quoted[16] by experiments involve big extrapolations from measurements restricted to the extreme forward direction. The extrapolations often assume the associated charmed particles are uncorrelated and that the cross-section in the forward direction is given by the unadorned perturbative prediction. These assumptions all appear to be in error, on a combination of theoretical and experimental reasoning.

When one mentions forward production of charm. the subject of diffractive production rears its head. The ISR data indicate a significant fraction of diffractive charm production. The diffractive events are those in which one of the beam particles emerges unscathed. scattered through a small angle. There is a large rapidity gap between it and the other outgoing hadrons in the event. Now. there has been discussion of diffractive excitation of charm. According to our recent work. any such production. at the leading twist level. is actually included in the regular gluon-fusion term (and its higher order corrections). The point is that in the production process for the heavy quarks. the internal lines must be far off-shell.

However, this also implies that the diffractive production is the same for all hard processes. For example, if 20% of heavy quark production is diffractive, then the same fraction of jet production must be diffractive, modulo presumably minor differences because of the different gluon fraction of the partons initiating the hard scatterings. Just after this conference, new data[19] on photoproduction of charm was published, indicating that 20% of that cross-section is diffractive. A naive view of this suggests that around 40% of production of top quarks of mass 40 GeV at the Sp$\bar{p}$S or the Tevatron is diffractive.

Now, there is a possibility that requiring a diffractive trigger reduces backgrounds to such processes much more than it reduces the signal. Therefore it appears sensible to continue to explore the use of diffractive triggers, and to understand what QCD has to say about diffractive hard scattering. There is some recent work on this subject[20].

3.3 Higher-order corrections

We have concluded that the production of a new very heavy flavor, be it a top or a squark, for example, will be correctly given by the standard QCD calculation. The forward production that has so confused the issue in charm production is a higher twist effect, as we showed in Ref. 17]. That is, the forward production, as a fraction of the total production of the heavy flavor, goes away like an inverse power of the heavy mass. A key component in the reasoning is that, on purely kinematic grounds, there is a large rapidity difference between the heavy quarks and the incoming hadrons. It is precisely such a rapidity difference that allows perturbative methods to work. The lower bound on the rapidity difference is only one unit for charm in a baryon collision.

Some work has also been done on the higher-order corrections to the standard gluon-fusion graphs for heavy quark production. A long while ago, Kunszt and Pietarinen[21] looked at all the subprocesses for production of jets. They found that the subprocess for which the α_s^3 term is largest relative to the α_s^2 term is precisely the gluon fusion process. They were working, however, with massless quarks; but the result suggests that the gluon fusion process is likely to have large higher order corrections. Halzen and Hoyer[22] have looked at the α_s^3 contribution to charm production and found, in a certain kinematic region, that it is a factor of ten higher than the lowest order term. However their region appears to be one that is particularly favorable to the higher order graphs and that is not the most important region for charm production. (They require that there be a jet of transverse momentum greater than 7 GeV, which may, but need not be, generated by one of the charmed quarks. The reason that they looked at such a restricted region is that a complete calculation of the order α_s^3 graphs has not yet been done.)

Clearly, we must have a full calculation of heavy quark production at order α_s^3, including the virtual graphs. The $2 \to 3$ tree graphs are already known.

Further measurements of heavy flavor production over the full kinematic range are needed. We need this not only for charm, but also for bottom- and top-production. If the measurements for the heavier known flavors agree with the scenario explained above, then this will be added confirmation that the calculations for the new flavors that we might see at the SSC are in good shape. In particular, it does not appear to be especially useful to design a special detector for looking for forward production of very heavy flavors.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under contracts DE-AC02-80ER-10712 and DE-AC02-76ER-02220. I would like to thank the organizers of the workshop for doing such a superb job, to make it both enjoyable and productive.

REFERENCES

1] Donaldson, R. and Morfin, J. (eds.), "Proceedings of 1984 Summer Study on the Design and Utilization of the Superconducting Super Collider" (Fermilab, 1984).

2] UA1 and UA2 results presented in "Proceedings of DPF Conference, Aug. 12-15, 1985" (R.C. Hwa ed.) (World Scientific, Singapore, to appear).

3] Gribov, L.V., Levin, E.M., and Ryskin, M.G., Phys. Rep. 100, 1 (1983).

4] Mueller, A.H. and Qiu, J., Columbia preprint CU-TP-322; Mueller, A.H., in Ref. 2.

5] Durand, L. and Putikka, W., Madison preprint MAD/TH/85-3.

6] Ametller, L., Paver, N; and Treleani, D; Trieste preprint IC/85/118; Paver, N. and Treleani, D; Zeit. Phys. C 28, 187 (1985).

7] Humpert, B. and Odorico, R., Phys. Lett. 154B, 211 (1985).

8] Sjostrand, T., Fermilab preprint FERMILAB-PUB-85/119-T.

9] Amati, D., Petronzio, R., and Veneziano, G., Nucl. Phys. B146, 29 (1978); Libby, S.B., and Sterman, G., Phys. Rev. D18, 3252 (1978); Mueller, A.H.. Phys. Rev. D18, 3705 (1978); Gupta, S. and Mueller, A.H., Phys. Rev. D20, 118 (1979); Ellis, R.K., Georgi, H., Machacek, M., Politzer, H.D., and Ross, G.G., Nucl. Phys. B152, 285 (1979); Collins, J.C., and Sterman, G., Nucl. Phys. B185, 172 (1981).

10] Gribov, V.N. and Lipatov, L.N., Sov. J. Nucl. Phys. 46, 438, 675 (1972), Dokshitzer, Yu.L., Sov. Phys. JETP 46, 641 (1977); Altarelli, G., and Parisi, G., Nucl. Phys. B126, 298 (1977); Johnson, P.W., and Tung, W.-K.. Phys. Rev. D16, 2769 (1977).

11] Eichten, E., Hinchliffe, I., Lane, K.. and Quigg, C., Rev. Mod. Phys. 56, 579 (1984).

12] Collins, J.C., Soper, D.E., and Sterman, G., Nucl. Phys. B261, 104 (1985); G. Bodwin, Phys. Rev. D31, 2616 (1985).

13] DeTar, C., Ellis, S.D., and Landshoff, P.V., Nucl. Phys. B87, 176 (1975), and Cardy, J.L., and Winbow, G., Phys. Lett. 52B, 95 (1974).

14] Paver, N. and Treleani, D., Nuovo Cim. 70A, 215 (1982) and 73A, 392 (1983); Humpert, B., Phys. Lett. 131B, 461 (1983).

15] Kwiecinski, J., Cambridge preprint HEP85/5.

16] Kernan, A. and VanDalen, G., Phys. Rep. 106, 297 (1985).

17] Collins, J.C., Soper, D.E., and Sterman, G., Oregon preprint OITS 292, Nucl. Phys. B (to be published).

18] LEBC Collaboration, Aguilar-Benitez, M. et al., CERN preprints CERN/EP/85-103 and CERN/EP/85-118.

19] Sliwa, K. et al. Phys. Rev. D32, 1053 (1985).

20] Ingelman, G. and Schlein, P.E., Phys. Lett. 152B, 256 (1985).

21] Kunszt, Z. and Pietarinen, E., Zeit. Phys. C2, 355 (1979).

22] Halzen, F. and Hoyer, P., Phys. Lett. 154B, 324 (1985).

DISCLAIMER