CONF 871101 22

CONF-871101--22

DE87 011646

OPTIMAL CONTROL OF NONLINEAR NUCLEAR REACTORS

by

Carlos March-Leuba

Rafael B. Perez

Center for Study of Nonlinear Phenomena
The University of Tennessee
and
Oak Ridge National Laboratory

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Paper to be submitted to the
Winter Meeting of the American Nuclear Society
Los Angeles, California
November 15-19, 1987

Research sponsored by Office of Basic Energy Sciences, U.S. Department of Energy under contract number DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

"The submitted manuscript has been authored by a contractor of the U.S. Government under contract No. DE-AC05-84OR21400. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes."

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Optimal Control of Nonlinear Nuclear Reactors.

Carlos March-Leuba and Rafael B. Perez (The University of Tennessee).

This paper presents a new formulation of a class of nonlinear optimal control problems as a solution of a system of differential equations with initial conditions. An application to the control of a nonlinear nuclear reactor is shown.

Pontryagin's theorem[1] has been successfully applied to the problem of designing control systems that are optimum in the sense that they minimize a given cost function.[2] Rosztoczy et al.[3] showed how the "maximum principle" can be used to optimize control-rod movement during transients in a nonlinear nuclear reactor by using a point kinetics reactor model with temperature feedback proportional to the flux. The problem was formulated in a way that demanded the solution of four coupled first-order nonlinear differential equations with initial conditions for the flux and final conditions for the adjoint. This particular set of boundary conditions required an iterative numerical solution of the equations that did not apply easily to real-time applications.

This paper presents a new formulation of the problem that allows on-line solution for the control by transforming it to an initial value problem.

The equations describing a nuclear reactor with temperature feedback can be expressed as

$$dn/dt = (\rho n - \alpha n_0 n^2 - \beta n)/\Lambda + \lambda c \qquad (1)$$

$$dc/dt = \beta n/\Lambda - \lambda c \qquad (2)$$

where n is the relative neutron density, n_0 the initial density, c is the normalized precursors concentration, ρ is the control (rod-reactivity), α is the temperature feedback, Λ is the neutron generation time, and β and λ are the delayed-neutron fraction and decay constant.

Given a demand, d(t), we want to find the control, $\rho(t)$, such that n(t) follows that demand and, at the same time, the amount of control used is minimized in some sense. The formulation of the problem is not unique. The approach taken in this work has been to minimize a cost function, J, defined as

$$J = 1/2 \int_{to}^{T} [\ (d(t)-n(t))^2\ Q + (\rho-\alpha n_0 d(t))^2\ R\ ]\ dt \qquad (3)$$

where Q and R are positive weighting parameters which can be interpreted as time constants of the system when the original equations are written in adimensional fashion. When the system is not in equilibrium, J is greater than zero and dJ/dto is less than zero; thus, J can be considered as a Liapunov function. This fact guaranties that the equilibrium reached using this control will be stable.

Let's define a modified cost function as

$$\underline{J} = J - \int_{to}^{T} p^{+} (dx/dt - f(x,t)) \quad dt \quad , \tag{4}$$

where the vector expression $dx/dt = f(x,t)$ represents the system of equations, (1) and (2), and p is a two dimensional adjoint. Then, taking variations in Eq. (4), a set of four first-order differential equations is obtained for $x = (n, c)$ and $p = (p_1, p_2)$ as well as a condition that relates the control variable, $\rho(t)$, to the state variables and adjoints

$$dn/dt = p_1 n^2/R\Lambda^2 + \alpha n_0 n(d-n)/\Lambda - \beta n/\Lambda + \lambda c \tag{5}$$

$$dc/dt = \beta n/\Lambda - \lambda c \tag{6}$$

$$dp_1/dt = np_1^2/R\Lambda^2 - (d-2n)\alpha n_0 p_1/\Lambda + (p_2-p_1)\beta/\Lambda - (d-n)Q \tag{7}$$

$$dp_2/dt = \lambda(p_2-p_1) \tag{8}$$

where the control has been substituted for

$$\rho = p_1 n/R\Lambda + \alpha n_0 d \tag{9}$$

The variational calculation yields as boundary condition that the adjoint, p, at final time be zero. This boundary condition makes the solution rather difficult. Thus, the approach taken in this work has been to realize that if, at the beginning of the transient, steady state conditions for demand and state variables were satisfied, then p must be zero at the initial time. In this way, the equations can be solved as an initial

value problem. These extra constraints allow to solve the problem easily making "on-line" solutions possible.

Figure 1. shows the result of a computer simulation of this technique. The solid line represents the demand and the circles correspond to the actual neutron density.

In summary, optimal control techniques can be applied to nuclear reactors. Control constraints, nonlinearities and multivariate control problems can be handled by using variational techniques. The results presented in this paper show that depending on the formulation of the cost function, the solution of the system of equations car be clearly simplified.

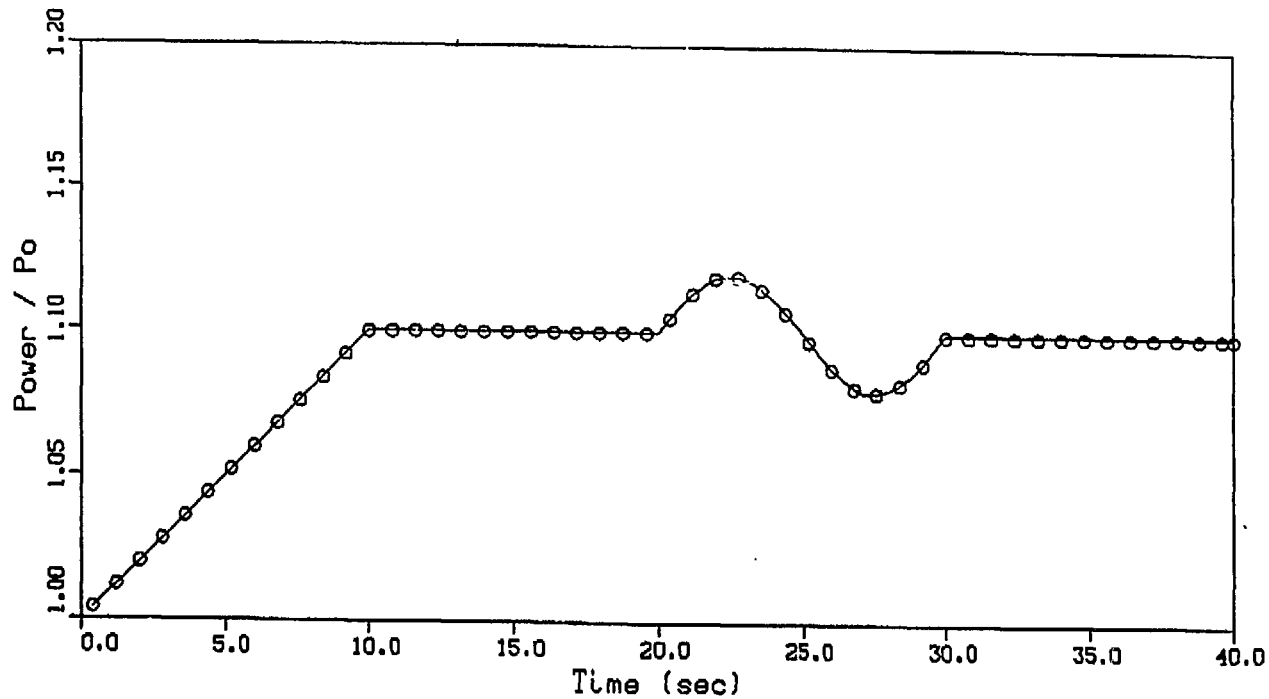
References

1. Pontryagin, "The Mathematical Theory of Optimal Processes," Trans. by K. N. Trirogoff, ed. by L. W. Neustadt, Wiley-Interscience, New York, 1962

2. Rosztoczy et al., "Application of the Pontryagin's Maximum Principle to Flux-State Changes in Nuclear reactors," Proc. Reactor Kinetics and Control, University of Arizona, March 25-27, 1963

3. D. G. Luenberger, " Introduction to Dynamic Systems," Wiley & Sons, 1979

Figure 1. Response of the reactor to the optimal control. The solid line represents the demand and the circles the normalized neutron density.


Power / Po
1.00
1.05
1.10
1.15
1.20
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
Time (sec)