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Fast Monte Carlo for Radiation Therapy: the PEREGRINE Project•

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ABSTRACT

The purpose of the PEREGRINE program is to bring high-speed, high-accuracy, high-resolution Monte Carlo dose calculations to the desktop in the radiation therapy clinic. PEREGRINE is a three-dimensional Monte Carlo dose calculation system designed specifically for radiation therapy planning. It provides dose distributions from external beams of photons, electrons, neutrons, and protons as well as from brachytherapy sources. Each external radiation source particle passes through collimator jaws and beam modifiers such as blocks, compensators, and wedges that are used to customize the treatment to maximize the dose to the tumor. Absorbed dose is tallied in the patient or phantom as Monte Carlo simulation particles are followed through a Cartesian transport mesh that has been manually specified or determined from a CT scan of the patient.

This paper describes PEREGRINE capabilities, results of benchmark comparisons, calculation times and performance, and the significance of Monte Carlo calculations for photon teletherapy. PEREGRINE results show excellent agreement with a comprehensive set of measurements for a wide variety of clinical photon beam geometries, on both homogeneous and heterogeneous test samples or phantoms. Operating on low-cost, multi-CPU, SMP Pentium Pro-based systems, PEREGRINE is capable of calculating >350 million histories per hour for a standard clinical treatment plan. This results in a dose distribution with voxel standard deviations of <2% of the maximum dose on 4 million voxels with 1 mm resolution in the CT-slice plane in under 20 minutes. Calculation times include tracking particles through all patient-specific beam delivery components as well as the patient.

Most importantly, comparison of Monte Carlo dose calculations with currently-used algorithms reveal significantly different dose distributions for a wide variety of treatment sites, due to the complex 3-D effects of missing tissue, tissue heterogeneities, and accurate modeling of the radiation source.

1. INTRODUCTION

The success of radiation therapy requires three critically linked components: patient evaluation, treatment planning, and treatment delivery. In the last several decades, technology for patient evaluation (CT and MRI scanning) and treatment delivery (accelerators) has improved dramatically, providing an array of imaging and treatment delivery devices that has so far outmatched the radiation therapy field's ability to fully utilize them. Full realization of the benefits of these advances requires sophisticated, accurate treatment planning.

A critical part of treatment planning is the accurate determination of the dose distribution in the patient. Even though ICRU Report 42[1] recommends a dose calculation accuracy goal of 2% in low dose gradient regions, currently-used methods can result in errors of 3% to more than 10% for heterogeneous media[2 3 4 5]. Convolution/superposition algorithms [6 7 8 9 10 11 12 13] can be significantly more accurate that older methods, but often at the expense of considerable computation time.

Monte Carlo dose calculations are the gold standard, being capable, in principle, of accurately computing dose under almost all circumstances.[14 15 16] However, until recently, a Monte Carlo calculation for a single treatment plan could take days or weeks even on supercomputers. By taking advantage of recent advances in computer technology, combined with state-of-the-art Monte Carlo transport algorithms, PEREGRINE performs high-resolution Monte Carlo radiation treatment planning calculations in times compatible with use in a radiation therapy clinic, using low-cost commodity hardware. Because of its speed and ease of use, PEREGRINE has the capacity to bring Monte Carlo radiation transport calculations to the clinical RTP environment.

This work describes the transport data and algorithms used in PEREGRINE, results of benchmark comparisons, and implementation and timing for a symmetric multiprocessor-based system. Although

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PEREGRINE is designed to calculate dose distributions for photon, electron, neutron, and proton therapy, this work focuses on photon beam therapy.

2. IMPLEMENTATION OF MONTE CARLO FOR PHOTON TELETHERAPY DOSE CALCULATIONS

The PEREGRINE Monte Carlo dose calculation process depends on four key elements: complete material composition description of the patient as a transport mesh, accurate characterization of the radiation source, and first-principles particle transport algorithms combined with reliable, self-consistent particle-interaction databases. PEREGRINE uses these elements to provide efficient, accurate Monte Carlo transport calculation for radiation therapy planning.

2.1 PATIENT DESCRIPTION

The patient transport mesh is a Cartesian map of material composition and density determined from the patient's CT scan. Each CT scan pixel is used to identify the atomic composition and density of a corresponding transport mesh voxel. Atomic composition is determined from CT threshold values set by the user or by default values based on user-specified CT numbers for air and water. The user also assigns materials and densities to the interior of contoured structures.

If the user specifies a structure as the outer contour of the patient, PEREGRINE constructs a transport mesh that is limited to the maximum extent of that structure, and sets all voxels outside that structure to be air. This provides a simple method of subtracting the CT table from the calculation.

2.2 RADIATION SOURCE DESCRIPTION

The PEREGRINE source model,[17] [18] designed to provide an compact, accurate representation of the radiation source, divides the beam-delivery system into two parts: an accelerator-specific upper portion and a patient-treatment-specific lower part. Figure 1 shows a schematic diagram of a representative photon beam source.

The accelerator-specific upper portion, consisting of the tungsten electron-target, flattening filter, primary collimator and monitor chamber is precharacterized based on the machine vendor's model-specific information. These precharacterized sources are derived from Monte Carlo simulations from off-line Monte Carlo simulations using BEAM[19] and MCNP4A[20]. Particle histories from off-line simulations are cast into multidimensional probability distributions, which are sampled during the

PEREGRINE calculation. The photon beam is divided into three subsources: primary photons, scattered photons, and contaminant electrons. Separating the source into subsources facilitates investigation of the contributions of each individual component. To ensure site-specific model accuracy, the installation procedures consists of a limited number of beam description parameter adjustments, based on simple measurements.

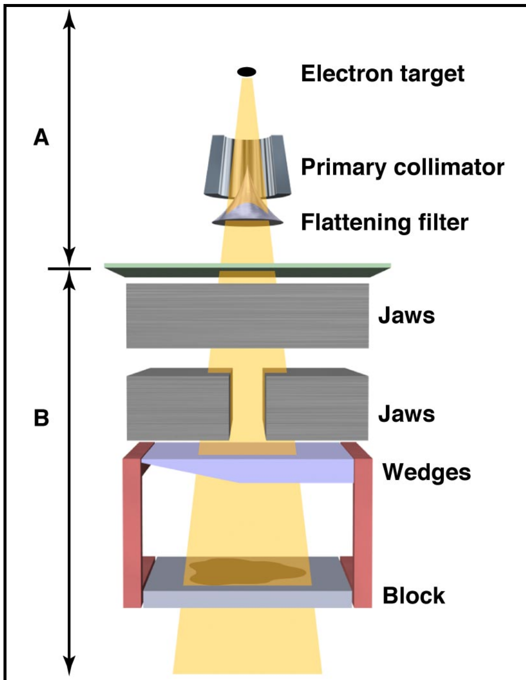

Figure 1. Schematic diagram of a photon beam radiation source. This figure illustrates how PEREGRINE divides the photon source into accelerator-specific (A) and patient-treatment-specific (B) components.

The lower portion of the radiation source consists of treatment-specific beam modifiers such as collimators, apertures, blocks, and wedges. This portion is modeled explicitly during each PEREGRINE calculation. Particles are transported through this portion of the source. Photons intersecting the collimator jaws are absorbed. Photons intersecting the block or wedge are tracked through the material using the same physical database and methods described below for patient transport. However, all electrons set in motion by photon interactions in the block or wedge are immediately absorbed.

2.3 MONTE CARLO TRANSPORT DATA AND METHODS

Using the Monte Carlo transport method, PEREGRINE tracks all photons, electrons, positrons and their daughter products through the transport mesh until they reach a specified minimum tracking energy or leave the patient transport mesh. A dose map is created by dividing the energy deposited in each voxel by the voxel's mass. For photon/electron/positron transport, PEREGRINE relies on the Lawrence Livermore National Laboratory Evaluated Physical Database, combined with stopping powers supplied by the National Institute of Standards and Technology.

2.3.1 Photons

PEREGRINE accounts for photon interactions via the photoelectric effect, incoherent/coherent photon scattering, and pair production. All photon cross sections used by PEREGRINE are derived from the Lawrence Livermore National Laboratory Evaluated Photon Data Library (EPDL).[21] EPDL data are taken from a variety of sources that have been selected for accuracy and consistency over a wide range of photon energies (10-eV–100-MeV) for all elements.

At low incident photon energies (< 0.1 MeV for tissue components, < 1 MeV for high-Z materials such as lead and tungsten), the photoelectric effect is the dominant absorption mechanism. The cross sections contained in PEREGRINE were obtained by direct evaluation of the relativistic S-matrix in a screened central potential .[22] These cross sections accurately describe ionization from electrons bound in isolated atoms and provide predictions at the percent level for compounds where the K and L shells are well-represented by atomic orbitals.

For most elements, at energies typical of those encountered for clinical photon beams, Compton scattering is the most important process in the photon-atom interaction. The Compton scattering cross sections used in PEREGRINE are obtained in the incoherent scattering factor approximation.[23] This approximation includes screening effects. Relativistic effects enter through use of the Klein-Nishina cross section.

Coherent scattering does not contribute significantly to the total photon-atom interaction cross section for most radiation therapy applications. However, these cross sections are still modeled, and were obtained under similar assumptions to those for incoherent scattering.

At very high incident photon energies (> 30 MeV for tissue components, > 5 MeV for high-Z materials such as lead and tungsten), the dominant photon interaction mechanism is pair production. The cross sections for pair and triplet production used by PEREGRINE include Coulomb and screening effects and radiative corrections.[24]

PEREGRINE transports photons through the body using the standard analog method.[25] Woodcock or delta-scattering[26] is used to efficiently track particles through the transport mesh. All photons below 0.1-keV energy are absorbed locally.

2.3.2 Electrons/Positrons

Electrons and positrons are transported using a Class II condensed history scheme[27] whereby secondary electrons created through the Møller[28], Bhabha[29] or radiative processes are transported if their kinetic energies are greater than the threshold of 200 keV. The sampling routines for the Møller and Bhabha interactions are similar to those used in the EGS4 code[30] [31] with the incorporation of an improvement in the energy sampling of large-energy knock-ons.[32] Bremsstrahlung cross sections used by PEREGRINE are from the LLNL Evaluated Electron Data Library (EEDL).[33] These cross sections were determined by Seltzer and Berger[34] by interpolating between the relativistic S matrix data from the code of Tseng and Pratt[35] available up to 2 MeV and the Bethe-Heitler result, expected to be valid to above 50 MeV.

Sub-threshold inelastic processes are treated in the continuous slowing down approximation (CSDA). The CSDA radiative and collisional stopping powers used in PEREGRINE are obtained from the data of Berger and Seltzer. These values are the recommended values of the International Commission on Radiation Units and Measurements.

Electron/positron stopping powers and cross sections are tabulated at points that are separated in energy by 8–20% of the electron's energy. Because of this energy point spacing, electron/positron steps are limited allow a to a maximum of energy loss of 8–20%.

Elastic scattering is simulated using the Goudsmit-Saunderson formalism.[36] The screened Rutherford cross section is employed with a high-energy form of the Moliére screening factor[37] as employed by EGS4. Traditionally, the Goudsmit-Saunderson method has been very difficult to employ in a Class II scheme because the pathlength can assume almost any value unlike Class I scheme where t follows a pre-determined grid. Recent work has surmounted this difficulty.[38]

Detour, or pathlength corrections, which account for the difference between actual pathlength and geometric transport distances between multiple-scattering vertices have been implemented.

The minimum electron/positron tracking energy is 10 keV. In addition, electron tracks are also stopped using a range rejection algorithm that takes into account both the medium surrounding the voxel of interest and the direction of travel. Two 511-keV photons are created at the end of each positron range. The direction of the first photon is chosen randomly, while the second is set to 180° opposed to the first.

3. RESULTS

We demonstrate the accuracy of PEREGRINE transport calculations by benchmarking PEREGRINE against a wide range of measurements and well-established Monte Carlo codes such as EGS4 and MCNP. PEREGRINE benchmarks can be divided into two classes: the first set of comparisons validates the transport algorithms used to calculate dose in the patient; the second set of comparisons tests the accuracy of radiation source characterization and implementation, as well as transport through the patient. The second set is also useful for assessing the benefits of Monte Carlo calculations over current dose calculation algorithms.

Comparisons of PEREGRINE calculations with conventional clinical algorithms for actual patient treatment plans have revealed significant dose distribution differences for a variety of cancer sites. These differences are illustrated by a comparison for a lung cancer patient.

3.1 COMPARISONS USING A SIMPLE RADIATION SOURCE

We have compared PEREGRINE results with independent measurements and calculations for simple electron and photon sources. Figure 2 compares PEREGRINE results with EGS4 calculations and calorimeter measurements.[39] Absorbed dose as a function of depth is compared for 1- and 20-MeV electron pencil beams incident normal to semi-infinite water, carbon, and iron slab phantoms. EGS4 calculations were completed with bremsstrahlung photon (AP) and delta-ray production (AE) cutoff energies of 10 keV and 521 keV, respectively [22]. Results are expressed in absolute dose per incident electron. PEREGRINE agrees well with both independent calculations and measurements.

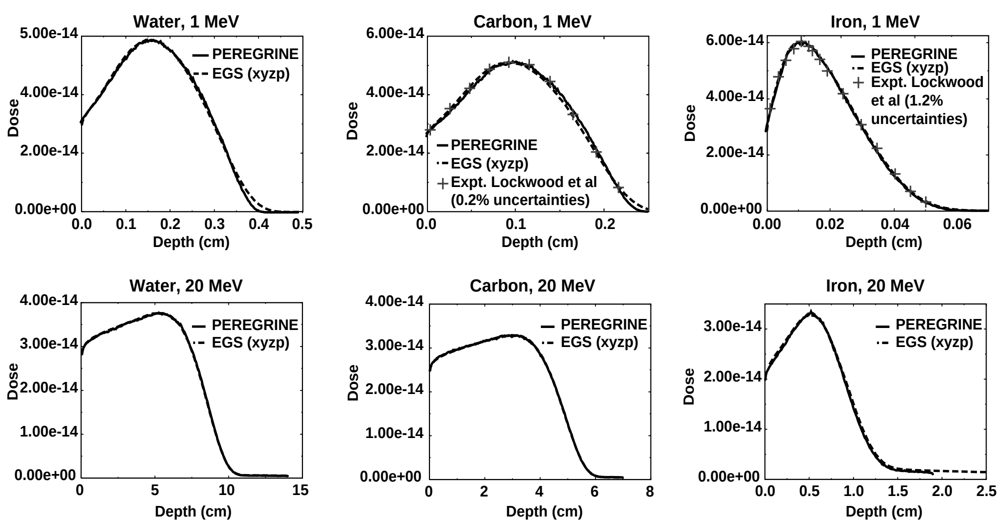

Figure 2. 1 MeV, and 20 MeV electron pencil beams incident on a water phantom.

Figure 3 illustrates the results of comparing PEREGRINE calculations with EGS4 [23] for 10 keV, 1 MeV, and 50 MeV photon pencil beams incident normal to the center of a broad water-aluminum slab phantom. EGS4 calculations were completed with bremsstrahlung photon (AP) and delta-ray production (AE) cutoff energies of 10 keV and 521 keV, respectively. Results for both codes are in excellent agreement.

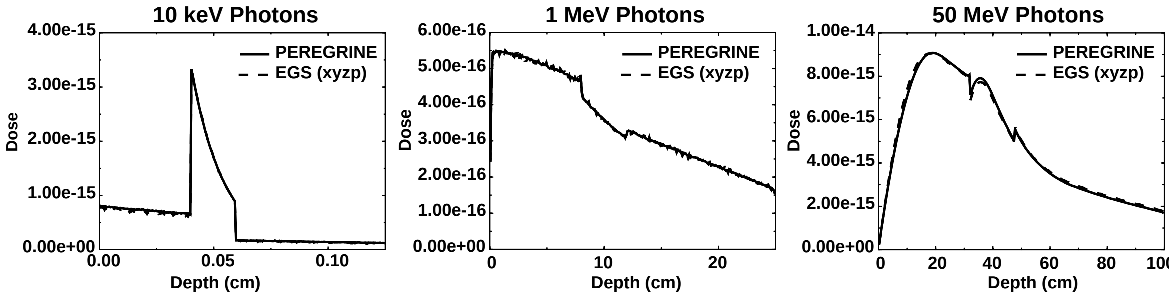

Figure 3. 10 keV, 1 MeV, and 50 MeV photon pencil beams incident on a water-iron-water phantom.

3.2 COMPARISONS WITH CLINICAL MEASUREMENTS

In addition to validating the accuracy of PEREGRINE Monte Carlo transport algorithms, we have also compared PEREGRINE calculations with a wide variety of clinical measurements for homogeneous and heterogeneous phantoms. Benchmarking against clinical measurements verifies the accuracy of radiation source characterization and implementation, as well as transport through the patient.

Figure 4 demonstrates the accuracy of PEREGRINE calculations for water phantoms measurements made with Varian 2100C 6- and 18-MV photon beams. We compare PEREGRINE dose calculations with ion chamber measurements [24] in a water phantom for 5×5, 10×10, and 20×20-cm fields. Profile and depth dose comparisons are shown. PEREGRINE calculations were completed using energy and angular distributions derived from a manufacturer-specified description of the Varian 2100C accelerator head. The only free parameters in the accelerator source characterization was the electron spot size, plus a <2% field-size-dependent factor accounts for radiation that is backscattered into the beam monitor chamber. Measurements were made using a Scanditronix photon diode (p-type silicon, chip thickness of 0.45 mm, 2.5 mm diameter).

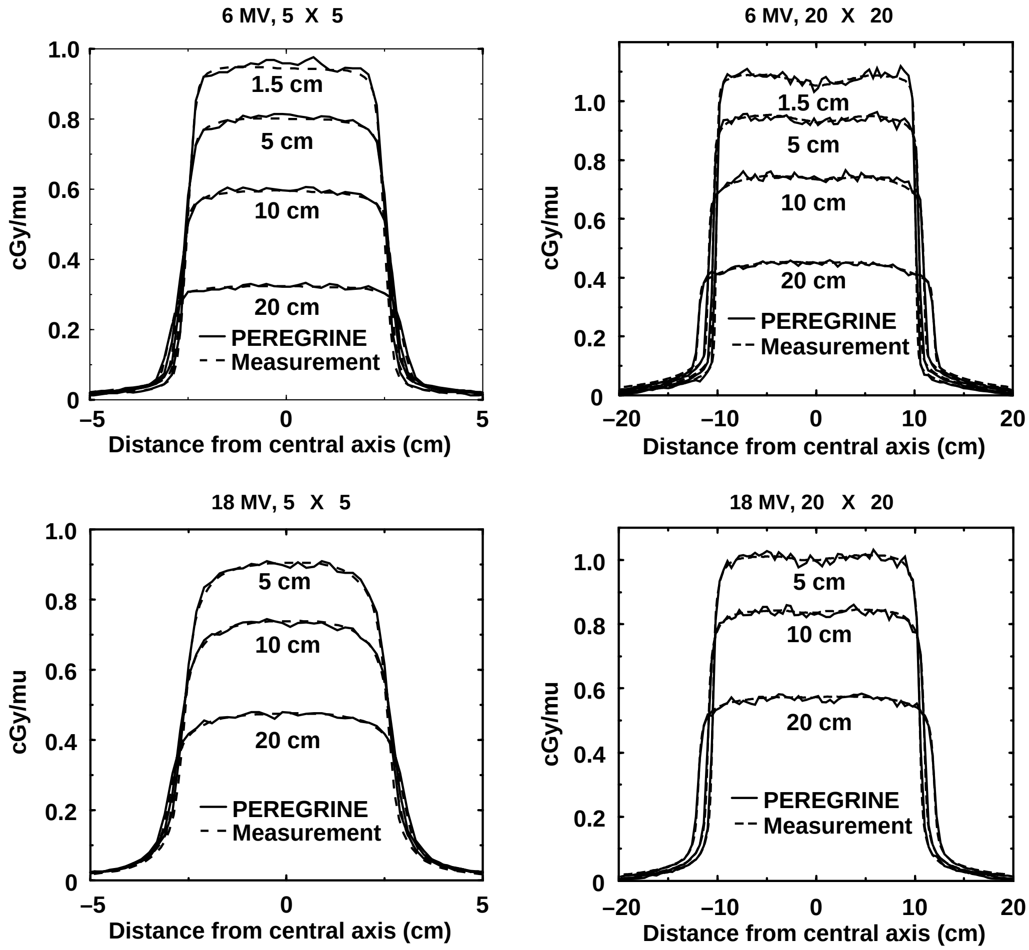

Figure 4. Comparison of PEREGRINE calculations with beam profile measurements made with a photon-sensitive diode in a water phantom for Varian Clinac 2100C 6-MV and 18-MV photon beams.

Figure 5 illustrates the excellent agreement between PEREGRINE calculations and preliminary radiochromic film measurements for heterogeneous phantoms .The phantom was irradiated with a 10 × 10-cm cross-section Varian 2100C 6 MV beam at 100-cm source-to-surface distance. The solid water phantom had a 3-cm-wide-by-3-cm-thick square air heterogeneity (infinitely long on the gantry-target axis) centered along the central axis of the beam, with its top surface at 1.5-cm depth. The film was positioned perpendicular to the beam at 6-cm depth in the solid water phantom.

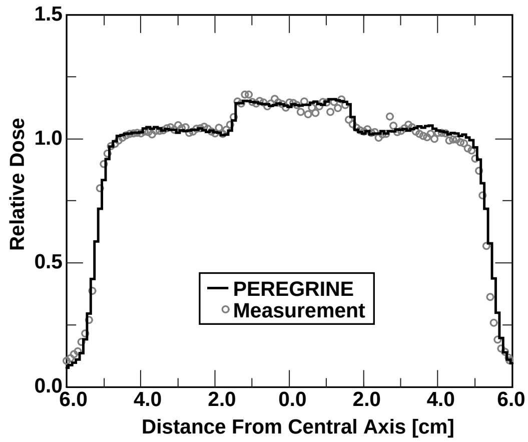

Figure 5. Comparison of PEREGRINE calculations with a radiochromic film profile measurement made at 6-cm depth in a solid water phantom with a 3 × 3-cm square air heterogeneity with its top surface at 1.5-cm depth. The radiation source was a Varian Clinac 2100C 6-MV 10 × 10-cm photon beam.

3.3 COMPARISONS WITH CONVENTIONAL CLINICAL CALCULATIONS

Preliminary studies indicate that the enhanced capability in dose calculation accuracy provided by PEREGRINE will have significant impact on patient treatment planning. As an example, Figure 6 shows a comparison of PEREGRINE results a conventional dose calculation method[10] for a five-field treatment of a lung cancer patient. Two fields included wedges, and all fields were treated with 6 MV Varian Clinac 2100C beams. The treatment was planned so that the 70 Gy isodose line encompassed the entire target volume. Compared to the conventional dose calculation method, PEREGRINE Monte Carlo calculations reveal underdosing of the tumor target volume by 5-30%. PEREGRINE Monte Carlo calculations show that significant portions of the target volume are outside the 70 Gy isodose line. Discrepancies are highest near beam edges and tissue interfaces, due to the effects heterogeneities on of photon scattering and electron transport in the body.

Effects should be even more dramatic for smaller tumors. Work is under way to conduct a detailed examination of the clinical significance of PEREGRINE for a variety of patients and tumor types, including lung, head and neck, larynx, breast, and prostate tumors.

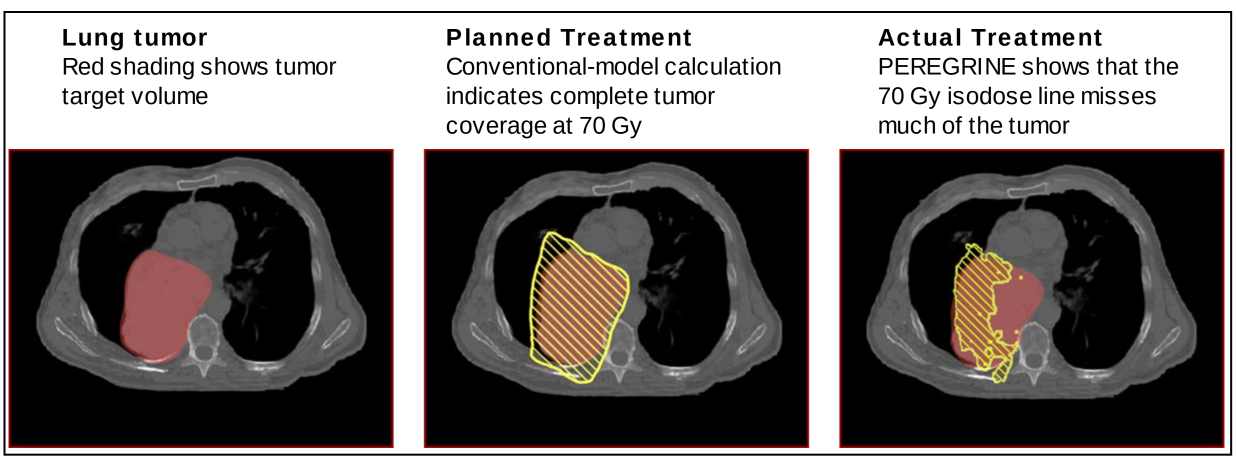

Figure 6. Comparison of PEREGRINE calculations with a conventional dose calculation method for a lung cancer patient. This figure shows the position of the prescription isodose level (70 Gy) overlaid on an axial CT-scan slice through the patient's chest. The shaded area in the center of the scan shows the tumor volume, while the contour enclosing hash marks shows the 70 Gy isodose line.

4. CLINICAL IMPLEMENTATION

The PEREGRINE Dose Calculation Engine (PDCE) combines the hardware, software, and networking components required to add this single-unit desktop design to any radiation treatment planning (RTP) system via a simple network connection, much like a file server.

The PDCE implements the PEREGRINE physics software under control of a modern, multithreaded operating system that supports the use of server-class, symmetric multiprocessing (SMP) microprocessors. The software design efficiently distributes the calculations for the problem so that dose is calculated by many microprocessors in parallel. The design is scaleable in a master-slave configuration such that a number of slave main-boards can be configured to compute in parallel, while result accumulation and communications are controlled by a single master board.

The PDCE is constructed from off-the-shelf components originally developed for file- and Internet-server applications. The design combines a configurable number of motherboards interconnected by an internal high-speed network. Each main board supports up to four PentiumPro CPUs with supporting memory and peripherals. The use of commodity hardware and a flexible hardware architecture allows the engine to be configured at a number of cost/performance points. The design supports hardware upgrades for increased capability in microprocessor hardware technology.

The PDCE can be optionally equipped with a display and supporting software to provide real-time visualization of the PEREGRINE Monte Carlo dose calculation while it computes. The display illustrates the effects of multiple beams and facilitates an early assessment of the plan's efficacy. Additional display capabilities are being developed to quantitatively monitor the progress of the dose calculation.

Once installed, PEREGRINE will operate as a dose calculation engine for a radiation therapy planning (RTP) system, calculating dose distributions for individual patient treatment plans. The RTP communicates with PEREGRINE via the an industry standard patient data exchange format, augmented by a small number of extensions, which provide additional beam- and patient-description data needed for Monte Carlo calculations. Once the treatment plan description is sent, the PDCE authenticates and checks files, computes the dose, and returns the results in new files for display and manipulation on the RTP system.

5. SUMMARY

The PEREGRINE dose calculation system is designed to provide high resolution, high-accuracy dose calculations for clinical radiation therapy planning. PEREGRINE can be economically integrated into existing RTP systems as an "invisible" dose computer, providing state-of-the-art capability to all clinics. The availability of such calculations could improve effectiveness of radiation therapy by providing accurate radiation treatment planning for every patient, facilitating accurate clinical trials and reliable implementation of these results throughout the medical community, providing accurate estimates of required doses for tumor control and normal tissue tolerance, and aiding in advancing the field of radiation oncology.

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