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FINAL REPORT
OF
***Data Analysis Of Tokamak Experiments With
Singular Value Decomposition***

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I. Project Summary

Under the grant, the applicant has developed a method of identifying poloidal and toroidal modes active in tokamak plasmas. Except complicated situations the method has shown to work well. Even with the limited applications, the advantage from the method is significant and even crucial. The method can be used to identify

- (1) responsible coherent modes such as MHD or Resistive modes activity in plasma
- (2) onset of instabilities

and can be used to

- (3) plasma controls.

The method has been applied to the DIII-D tokamak experimental data, and some results are presented in this report. We also present how the method can be used to plasma controls.

II. Identification and Significance of the Problem

Characterization of coherent MHD waves in the plasma is crucial to understand MHD activity and to prevent undesired events. For this reason mode characterization of a tokamak plasma has long been pursued but the complexity of the toroidicity and plasma shaping makes it difficult. Toroidal mode identification[1] is relatively simple since tokamak equilibria have toroidal symmetry. However, toroidicity and shaping distorts the poloidal mode structure such that the usual Fourier Decomposition will generate unnecessarily many components. This is especially true for strongly shaped as well as high beta discharges. The applicant has developed a method[2] of identifying poloidal, as well as toroidal, modes in this complex situation utilizing the singular value decomposition (SVD) technique in conjunction with various numerical codes, in particular the VACUUM code[3]. This method has been applied to a number of tokamak discharges and found to work reasonably well.

The SVD method[4] has been commonly used in signal processing in engineering. For a series of signals it provides a way of reducing noise by maximizing covariance of the signals over a time window[5]. With synchronized signals at various locations one can obtain both the coherent spatial structure and time evolution in the signal. Greene and Kim[6,7] have also utilized the SVD technique to examine the dynamics governed by ordinary differential equations. Several years ago Nardon applied the method to tearing modes and Sawtooth mode analysis[8]. The SVD analysis represents a set of space and time data, or toroidal and poloidal magnetic fluctuation data, in terms of the smallest possible time- and space- bases vectors. Therefore we can store only the coherent modes that are significant. This can reduce the storage

data by an order of magnitude easily. The *space-vectors* are often referred as *eigenvectors* and *time-vectors* as *principal components*.

An SVD eigenvector may consist of many poloidal modes, especially in high beta plasmas. There has been an increasing need of identifying poloidal modes of the SVD eigenvectors, and the energy distribution in the poloidal modes. For instance, the understanding and control of neoclassical tearing mode depends on the modal components of 2/1 or 3/2 [9]. Identification of the onset of 3/2 mode is crucial for the control of the modes.

We have developed a method to identify poloidal modes from a SVD eigenvector utilizing various numerical codes. Following a brief description of the SVD method in the next section, we present our poloidal analysis method, examples applied to DIII-D tokamak experimental data, followed by future application of the method.

III. The Singular Value Decomposition Method

A rectangular matrix, or a singular matrix, M does not have a proper inverse. However, a matrix Q exists such that MQ and QM are both symmetric with $QM^T=Q$ and $MQM=M$. The matrix Q is called the pseudo-inverse of M . Here we consider a real matrix M . Then the matrices MM^T and $M^T M$ are real and symmetric, their eigenvalues form a complete orthonormal basis. Furthermore, their eigenvalues are the same. The square root of these eigenvalues are called the singular values of M . If U is the matrix whose column vectors are the normalized eigenvectors of MM^T , and V is the matrix whose column vectors are the normalized eigenvectors of $M^T M$, then the factorization of

$$M = U S V^T \quad (1)$$

is called the singular value decomposition, while the diagonal elements of matrix S

$$S_{ij} = s_i \delta_{ij} \quad (2)$$

with $s_i \geq 0$ are called the singular values of M . Conventionally the singular values are stored in descending order. The eigenvectors of MM^T which are the column vectors of U , are called the Principal Components(PCs) of M . The PCs are also proportional to the projection of M along V or the product of US . If M is a $p \times q$ matrix with ($p > q$), then U is a $p \times q$ matrix with $U^T U = I$ and $U U^T = I$, and V is a $q \times q$ unitary matrix.

IV. The Application of SVD to tokamak data analysis.

A pertinent question is why is this SVD method relevant to identifying modes in experimental data? First consider a set of p synchronized signals from q Mirnov coils on a poloidal plane. One can then construct a matrix M as follows, with $f_j(t_i)$ being the magnetic amplitude at time t_i from j -th coil,

$$M = \frac{1}{\sqrt{pq}} \begin{pmatrix} f_1(t_1) & \cdots & f_q(t_1) \\ \vdots & \ddots & \vdots \\ f_1(t_p) & \cdots & f_q(t_p) \end{pmatrix} \quad (3)$$

where \sqrt{pq} is a normalization introduced for convenience.

As is indicated earlier the SVD, Eq.1, represents a rectangular matrix M with the eigenvectors and eigenvalues of $M^T M$ and MM^T . First consider $M^T M$. The elements of the matrix $M^T M$ are the time average of the product of signals at different positions with an additional factor of $1/q$.

$$(M^T M)_{ij} = \frac{1}{q} \left(\frac{1}{p} \sum_{k=1}^p f_i(t_k) f_j(t_k) \right) \quad \text{for } i, j = 1, \dots, q \quad (4)$$

is a covariance of signals at the i -th and j -th coils. Thus, the matrix $M^T M$ is a space-covariance matrix of M . The eigenvector corresponding to the maximum eigenvalue of the covariance matrix, the first PC of M , is the vector along which the space-covariance is maximized. Similarly MM^T represents the spatial average of the product of signals at different times, with an additional factor of $1/p$. The covariance of the signals at i -th and j -th time is:

$$(MM^T)_{ij} = \sum_k M_{ik} M_{jk} = \frac{1}{p} \left(\frac{1}{q} \sum_{k=1}^q f_k(t_i) f_k(t_j) \right) \quad \text{for } i, j = 1, \dots, p \quad (5)$$

Again the eigenvector of MM^T corresponding to the maximum eigenvalue represents the vector along which the time-covariance is maximized. Utilizing Eq.1 one can prove that

$$M^T M V = (V S U^T)(U S V^T)V = S^2 V \quad \text{and} \quad M M^T U = (U S V^T)(V S U^T)U = U S^2 \quad (6)$$

Thus the singular values of M are the squares of the eigenvalues of $M^T M$ or $M M^T$.

The beauty of the SVD is that the method separates spatial structure from the temporal evolution, and thus simplifying the analysis. It should be clear by now that U is the matrix that controls the time evolution and V is the matrix for spatial structure. The square of singular values is a measure of energy content of the mode.

When a mode is purely sinusoidal, the SVD and the Fourier Decomposition are essentially the same. Often, however, experimentally one observes coherent structures, but not necessarily purely sinusoidal. For instance sawtooth modes, or short pulses would require many Fourier modes to describe the structures. The SVD works with the amplitude of the mode at each assigned space directly, and gives us coherent structures as a whole. Furthermore toroidicity and shaping of the tokamak plasma experiments require new ways of looking at the basis functions of the modes.

V. Identification of poloidal modes

The SVD application to a spatial-time series data will separate time and spatial eigenvectors in the order of coherency. In general symmetric and anti-symmetric pair of eigenvectors are obtained since the plasma is rotating.

For each Principal Component, the experimental eigenvectors are decomposed into those of numerically generated eigenvectors for various poloidal modes. Here experimental vectors mean the spatial eigenvectors of the data matrix obtained via Singular Value Decomposition. Numerically the eigenvectors for the signals at the wall location corresponding to the (m,n) mode are obtained from the numerical code VACUUM. We introduce an (m,n) mode perturbation on the $q=m/n$ surface, and obtain magnetic fluctuations at Mirnov coil locations at DIII-D, and obtain spatial eigenvectors. The following block diagram shows the procedure of the analysis.

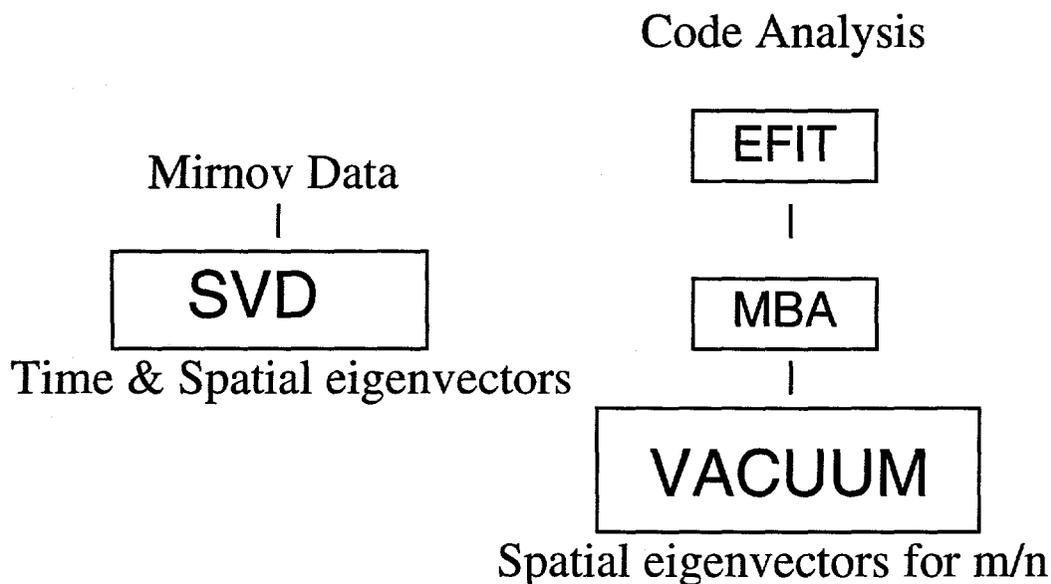


Figure 1. Block diagram of mode analysis

1. From Mirnov data, calibrate data, then apply SVD as shown in the left hand side block diagram.
2. Obtain spatial eigenvectors at Mirnov coil locations for each (m,n) mode using the VACUUM code as shown on the right hand side block diagram. The VACUUM code requires information of the equilibrium at the $q=m/n$ surface and Pest coordinates on that surface. Equilibrium information from the EFIT contains the q-surfaces, and the MBA code will generate the Pest coordinate on each q surface.
3. We then compare the experimental eigenvectors to the VACUUM code generated vectors. Each SVD analyzed spatial eigenvector can be represented in terms of the VACUUM code generated spatial eigenvectors of various poloidal modes.

Here we present an example poloidal mode analysis[2,10] applied to DIII-D plasma discharge. The DIII-D tokamak has 31 poloidal probes on one toroidal plane for poloidal mode structure analysis. For the DIII-D shot #86145 during 2300-2310 msec discharge, the first two pairs of principal components and the corresponding spatial eigenvectors are shown in figure 2a and 2b respectively. SVD calculation of the Mirnov fluctuation data show that the first pair of principal components contains 87% of the total fluctuation energy and with the second pair of components included the energy was 99%. Note that there are a pair of principal components for a rotation mode.

Meanwhile we can evaluate the expected eigenvectors of the Mirnov coil signals for each (m,n) mode, which are obtained from the VACUUM code. The VACUUM code generates vectors for $n=1$ as shown in figure 3.

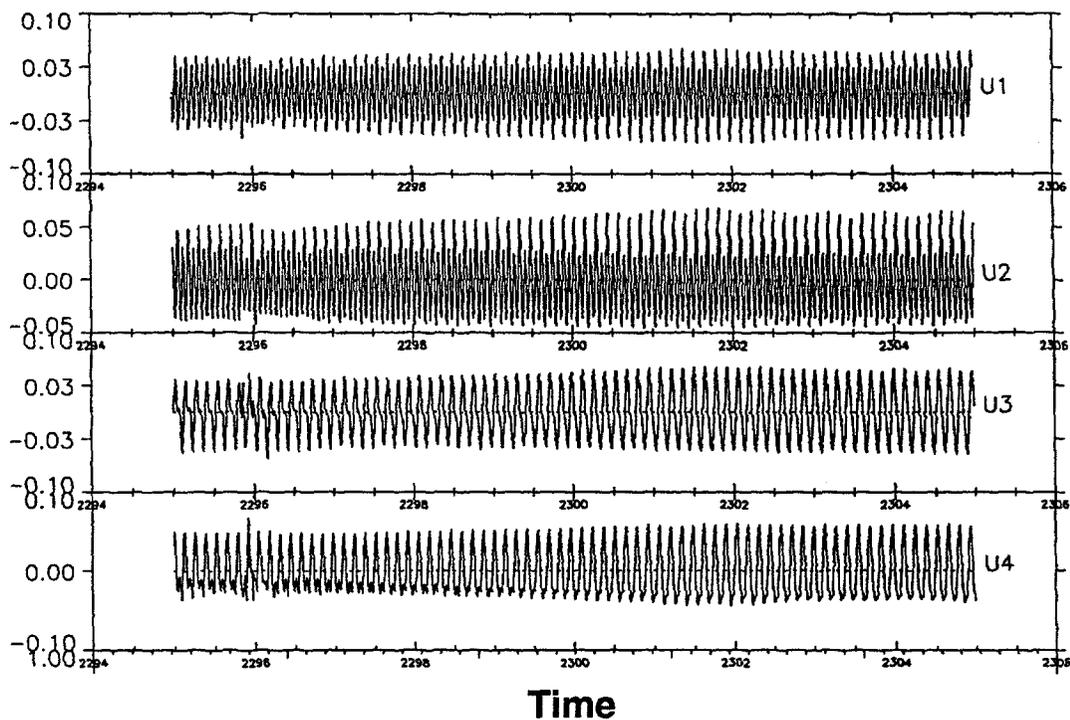


Figure 2a. First two pairs of Principal Components for the DIII-D discharge during 2300 msec-2310 msec.

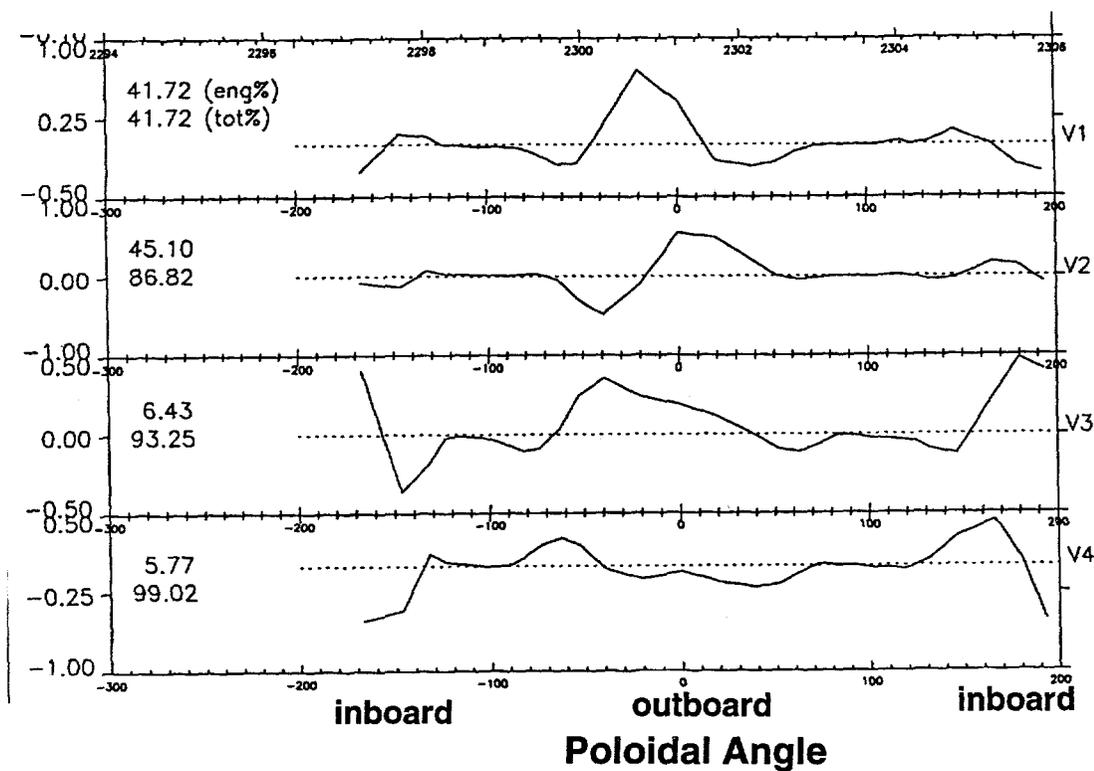


Figure 2b. First two pairs of Eigenvectors(b) for the DIII-D discharge during 2300 msec-2310 msec.

All we have to do now is to decompose each spatial eigenvector in terms of the VACUUM code generated vectors.

$$V^n(\text{experiment}) = \sum_m c_{m,n} V^{m,n}(\text{vacuum-code})$$

The coefficient $C_{m,n}$ can be obtained by inverting a rectangular matrix $V^{m,n}$. This inversion can be performed using singular value inversion technique. For the example case shown above, we have the following coefficients for the first principal components for $n=1$ mode.

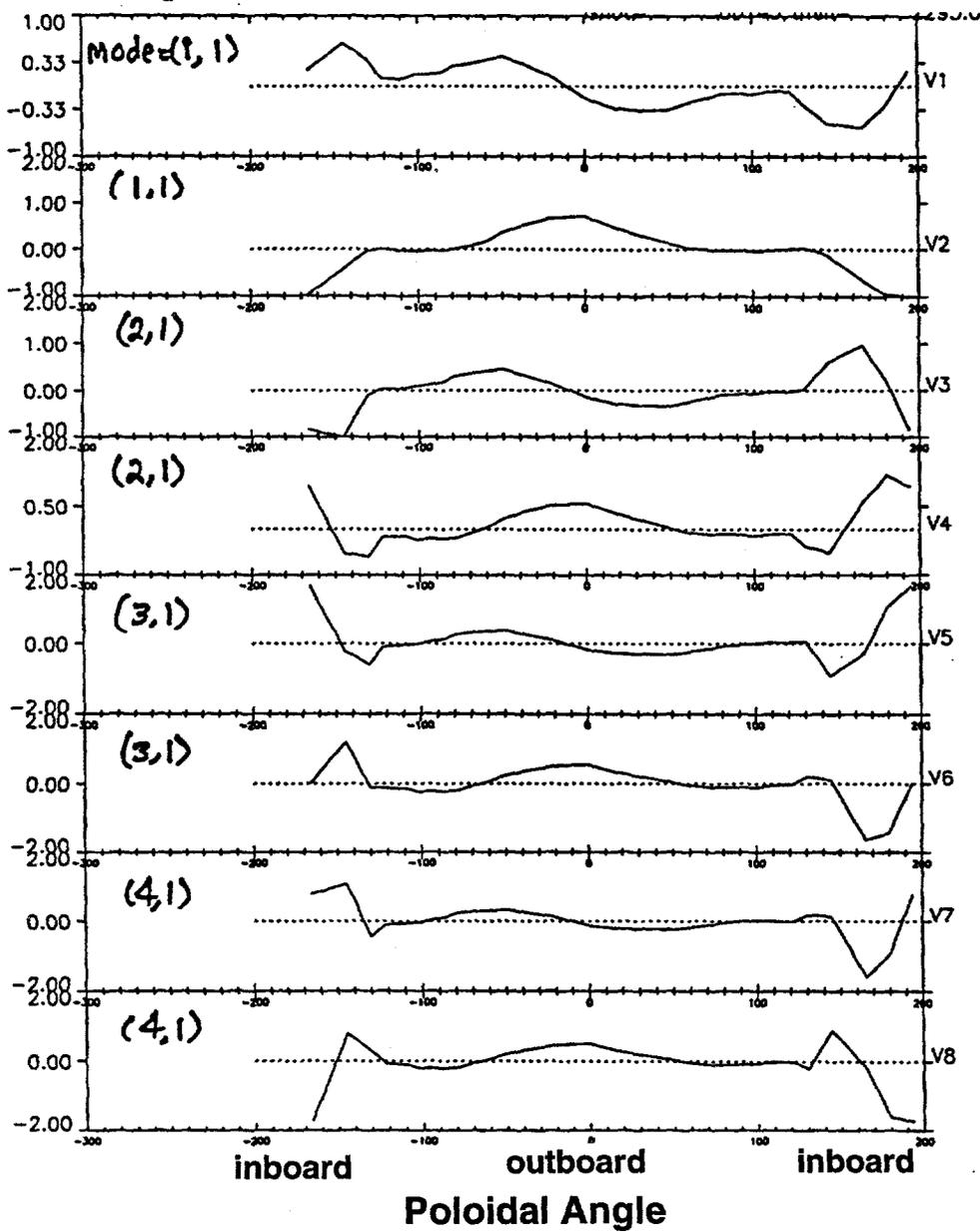


Figure 3. Numerical vectors $V^{m,n}$ for $n=1$ for the equilibria of the DIII-D discharge during 2300 msec-2310 msec.

The poloidal mode energy contents is proportional to the sum of the squares of symmetric and anti-symmetric components. The reconstructed experimental eigenvectors from these poloidal components are shown in figure 4 for comparison.

Table I. The coefficient $C_{m,n}$ with $n=1$.

Mode m/n	1/1	2/1	3/1	4/1
symmetric	0.083	0.39	-0.035	-0.035
anti-symm.	0.061	0.41	-0.073	-0.050
ENG(%)	3%	94%	2%	1%

There are many calculations involved in this analysis. Most of the analysis, though can be performed in advance if we know the equilibrium we will run. It is possible to reproduce a similar equilibrium discharge and in such a case we only need to use SVD on the Mirnov data in real time followed by a decomposition in terms of prepared numerical vectors. Today equilibria can be obtained in real time. With a faster dedicated computer processor, and parallel processing, it may be possible to compute all in real time in the near future. This will be part of the Phase II work.

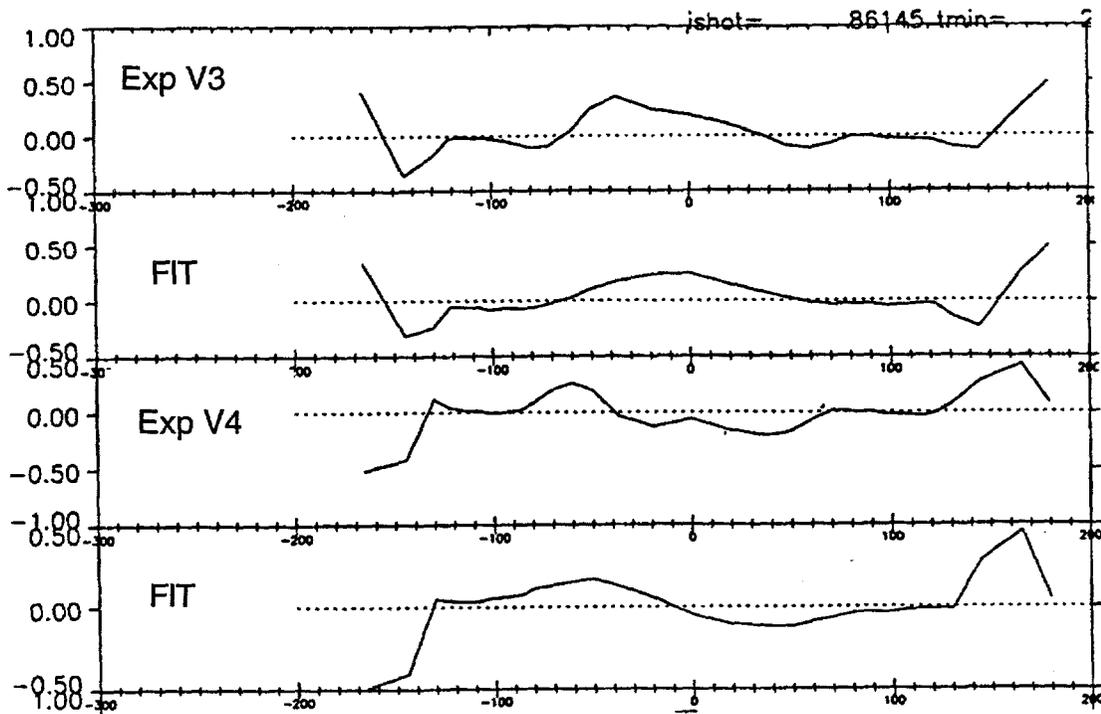


Figure 4. Experimental eigenvectors and fitted vectors by summing over the poloidal components in table I.

VI. Conclusions

The applicant has developed a method that can identify poloidal and toroidal modes from a series of synchronized Mirnov data on a poloidal plane of a tokamak. Utilizing the SVD method, and various codes, in particular, the VACUUM code, we can evaluate energy contents of poloidal and toroidal modes active in the plasma, as evidenced from the examples given in the text.

The developed method may play a significant role to the fusion community. Coherent MHD instabilities in tokamak plasmas can lead to reduced performance or possible termination of the plasma discharge. This becomes an even larger problem for future power plant-sized tokamaks such as ITER. Identification and characterization of these instabilities in real time offers the potential of reducing or eliminating their effects through an appropriate control mechanism. Success of this proposal will provide a path to realize this control. The mode identification method presented here will play a critical role in current plasma research and development. The method for identifying coherent modes and the control algorithm can be used at other plasma facilities nationwide and worldwide. The success of fusion energy will be of great benefit to the nation and the world.

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