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LIMIT CYCLES AND BIFURCATIONS IN NUCLEAR SYSTEMS*

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Recent stability tests[1-3] have shown that Boiling Water Reactors (BWRs) are susceptible to reactivity instabilities when operated at low-flow conditions. When such instabilities occur, limit cycles are observed in the measured process signals, indicating that these instabilities cause a transition between a linear regime (normal operation) and a nonlinear regime (unstable operation in the linear sense). The purpose of this paper is to describe the phenomenological dynamical behavior of BWRs in the non-linear regime under deterministic and stochastic excitations.

It has been shown in Refs. 4-6 that the following equations underly the simplest phenomenological model that retains the essential physical processes controlling the dynamic behavior of a BWR:

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda}n(t) + \lambda c + \frac{\rho}{\Lambda} , \tag{1}$$

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda}n(t) - \lambda c , \tag{2}$$

$$\frac{dT(t)}{dt} = a_1 n(t) - a_2 T(t) , \tag{3}$$

$$\frac{d^2\rho_\alpha(t)}{dt^2} + a_3\frac{d\rho_\alpha(t)}{dt} + a_4\rho_\alpha = kT(t) , \tag{4}$$

$$\rho(t) = \rho_\alpha(t) + DT(t) , \tag{5}$$

where n(t) is the excess neutron density normalized to the steady state neutron density; c(t) is the excess delayed neutron precursors concentration, also normalized to the steady state neutron density; T(t) is the excess average fuel temperature; and $\rho_\alpha(t)$ is the excess void reactivity feedback.

As detailed in Ref. 4, the parameters in Eqs. (1)-(5) were obtained by functionally fitting the transfer function of the Vermont Yankee reactor with operating conditions equivalent to those of the stability test 7N (when a limit cycle was experimentally observed)[2] to obtain:

$a_1 = 25.04\ Ks^{-1}$, $a_2 = 0.23\ s^{-1}$, $a_3 = 2.25\ s^{-1}$, $a_4 = 6.82\ s^{-2}$,

$k_0 = -3.70 \times 10^{-3}\ K^{-1}s^{-2}$, $D = -2.52 \times 10^{-5}\ K^{-1}$, $\beta = 0.0056$,

$\Lambda = 4.00 \times 10^{-5}\ s^{-1}$, $\lambda = 0.08\ s^{-1}$.

The parameter k, which is proportional to the void reactivity coefficient and the fuel heat transfer coefficient, controls the gain of the feedback and, thus, defines the linear stability of this reactor model. This can be demonstrated by applying, at $t=0^+$, a 10% step-perturbation to n(t) in Eq. (3) and allowing the solution to converge freely to its final state. The numerical solution of the model shows that limit cycles appear when the feedback gain k is increased past a critical value k_0.

In the linear region the stability of the nuclear system is quantified in terms of an *asymptotic decay ratio* (DR)[7]; however, in the nonlinear regime, the asymptotic DR is always equal to 1.0 due to the appearance of limit cycles. Therefore, the DR is not a good descriptor of the reactor's dynamic state in this regime. As shown in Refs. 5 and 6, a better dynamic descriptor in the nonlinear regime is the amplitude of the limit cycle oscillations.

Thus, the stability of the amplitude of the oscillations becomes the main concern in the nonlinear regime, a fact that is highlighted in Fig. 1. This figure shows the development of the limit cycle for three different values of the feedback gain: (a) k = 1.2, (b) k = 1.4, and (c) k = 1.5. Note that in both cases (a) and (b) the amplitude eventually converges to a final value, but in case (c) the amplitude describes an undamped periodic oscillation. Thus the amplitude of the limit cycle in case (c) has become unstable and is following a new limit cycle of its own with twice the original period. This fact causes the original signal to periodically exhibit two pulses of different magnitude. This process is customarily called a "period-doubling bifurcation."

Further increasing the feedback gain produces a cascade of period-doubling bifurcations which leads to an aperiodic regime. As has been detailed in Ref. 8, the transition to aperiodicity is governed by a set of universal constants with values as predicted by Feigenbaum's universality theory.

To analyze the effects of nonlinearities on the BWR behavior under stochastic (random) excitations (sources), the phenomenological model [cf., Eqs. (1)-(5)] was externally driven with a band-limited Gaussian white noise, and the equations were solved numerically in the time domain. The traces generated for n(t) were Fast-Fourier transformed to obtain power spectral densities (PSDs).

Increasing the feedback gain k shows that as long as $k<k_0$ the power oscillations increase in time and eventually reach a limit cycle, with an enhancement of the harmonic components of the PSD as seen in Fig. 2. This figure shows three PSDs for different values of k. In case (a) the model is barely stable and only the fundamental peak is clearly discernible at about 0.5 Hz. Case (b) represents a small amplitude limit cycle, for which the value of k is only slightly above the critical value k_0. Case (c) corresponds to a fully developed large-amplitude limit cycle. The main difference between the stable and the unstable PSDs is the appearance of higher harmonics. These harmonics have a large magnitude, so they should be measurable in real-life experiments even in the presence of measurement

and process noise.

In summary, this work provides a basis for scoping calculations to determine the dynamic behavior - both linear and nonlinear - of BWRs. Additional work is now underway to establish the feasibility of routine operation of nuclear systems in the nonlinear (limit-cycle) regime.

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FIGURE CAPTIONS

Fig. 1. Development of an instability of the limit cycle amplitude.

Fig. 2. Power spectral densities before and after the development of a limit cycle: (a) slightly stable, (b) slightly unstable, (c) fully developed limit cycle.

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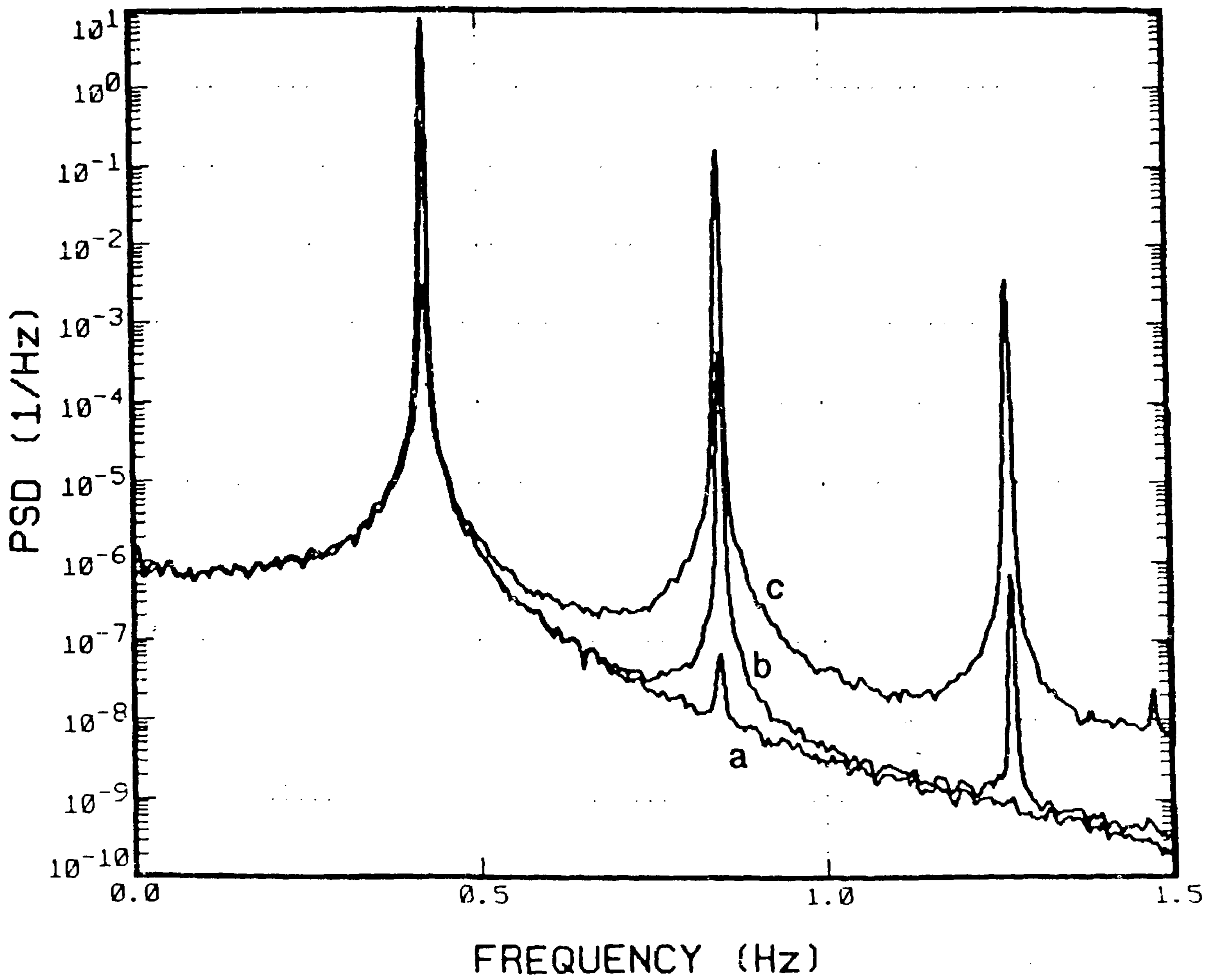

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