

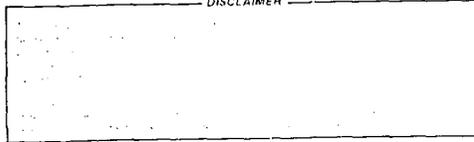
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RELATIONS BETWEEN THE SIMULTANEOUS AND SEQUENTIAL  
TRANSFER OF TWO NUCLEONS\*

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# RELATIONS BETWEEN THE SIMULTANEOUS AND SEQUENTIAL TRANSFER OF TWO NUCLEONS<sup>\*,†</sup>

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## 1. Introduction

The transfer of two nucleons between projectile and target in a direct or peripheral reaction such as  $(p,t)$  or  $(^{16}\text{O},^{14}\text{C})$  may occur in one-step or two-steps. These we refer to as 'simultaneous' and 'sequential' transfers, respectively. In the former, the interaction acts once and both nucleons are transferred. In the latter, the interaction acts once to transfer one nucleon, the system then propagates in one or more intermediate states and is followed by a second action of the interaction to transfer the second nucleon. This process may be symbolized for the above examples as  $(p,d;d,t)$  and  $(^{16}\text{O},^{15}\text{N};^{15}\text{N},^{14}\text{C})$ , implying the intermediate formation of a deuteron or the nucleus  $^{15}\text{N}$ . (Of course, the intermediate system may exist in more than one state of excitation.)

In terms of a perturbation theory expansion, such as the distorted-wave Born series, simultaneous transfer is possible in first order while sequential transfer requires second order. This is illustrated in Fig. 1. We are accustomed, perhaps, to thinking that a first-order process is more likely than a second-order one. However, a closer examination can prepare us for the possibility that this may not be so. The nuclear forces are predominantly two-body in

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character; hence, in first-order (Fig. 1a) only one of the two nucleons experiences an interaction. The possibility of finding that the other nucleon has also transferred arises only because its state within the projectile is not orthogonal to the state in the target into which it transfers. This process corresponds to a kind of quantum-mechanical tunneling from the projectile to the target.<sup>1</sup> However, in the two-step process (Fig. 1b) each nucleon is transferred under the direct influence of an interaction with the target; intuitively, this might seem more plausible. It requires an explicit calculation to determine which process is most likely in a given case, and such calculations are often beset with uncertainties. Nonetheless, it seems clear that the one-step and two-step amplitudes are frequently comparable in magnitude for light-ion reactions<sup>2</sup> while the two-step may dominate in reactions with heavy ions.<sup>3</sup> (The existence of strong  $Q$ -window effects, especially with heavy ions,<sup>4</sup> may enhance the sequential process when there is a large mismatch between the entrance and exit channels. The gap may be bridged more easily in two steps, with the interaction acting twice.) Consequently, it is not safe to ignore the existence of sequential transfer. However, one of the main reasons for studying two-nucleon transfers is to learn about the two-nucleon overlaps (existence and extent of pairing correlations, etc.). So we wish to know if this information is still available when the reaction proceeds by two, sequential, one-nucleon transfers. Our purpose here is to gain some insight into the relationship between the two amplitudes by using a simple approximate form of the theory. For simplicity, we shall discuss a light-ion reaction and, to be specific, we choose the  $(t,p)$  reaction (or the inverse  $(p,t)$  reaction). Similar considerations apply to other reactions.

## 2. The (t,p) or (p,t) Reaction

Apparently contradictory evidence exists for the relative importance of the two-step or sequential-transfer mechanism in (t,p) or (p,t) reactions. Indeed, the results of a confusing variety of sophisticated calculations are available, each of which treats properly some, but not all, aspects of these transitions. Among these aspects are: (i) the use of a realistic triton wavefunction and the associated realistic interaction;<sup>5-8</sup> (ii) an exact treatment of the finite-range of the interactions (as opposed to use of a zero-range approximation), in both the one-step<sup>5-9</sup> and the two-step<sup>8-11</sup> amplitudes; (iii) accounting for the non-orthogonality correction to the two-step term,<sup>2,9,11,12</sup> which tends to cancel the one-step amplitude;<sup>12-14</sup> (iv) allowing the intermediate n-p system of the two-step process to exist in continuum states, both spin-singlet and triplet, as well as in the bound, triplet deuteron ground state<sup>12,15</sup> (sometimes this is called the 'deuteron break-up effect'). In addition, there is a sensitivity of the results to the optical potentials employed, and a particular sensitivity of the cross section magnitudes to the nuclear wavefunctions used to construct the two-nucleon overlaps.<sup>6,7</sup>

In view of the complexity of the theoretical description of this transfer process, the agreement with measured cross sections obtained by calculations which only include some of these aspects must be viewed with some caution. In this situation it is valuable to have an understanding of any general, albeit approximate, features that the amplitudes may possess, independent of these details, which may illuminate the results of more detailed calculations.

The point of the present paper is to use a closure approximation to the second-order distorted-wave-Born expression for the two-step amplitude in order to exhibit the similarities and differences in comparison to the one-step

amplitude, especially with regard to the selection rules for spin-transfer.

This is by no means a new approach.<sup>3,16</sup> It has been used to explain why simultaneous and sequential processes tend to yield similar angular distributions and to depend upon the nuclear structure involved in the same way. Schaeffer and Bertsch<sup>16</sup> indicated schematically that, if the important intermediate states had similar energies, their Green functions could be replaced by an average one and closure applied to the nuclear states. This results in the nuclear overlaps for sequential transfer being the same as those for simultaneous transfer, with the same selection rules. A closer examination of the spin angular momenta involved modifies the conclusion about the selection rules a little. For example, in the two-nucleon transfer (t,p) reaction, if the n-p system or 'deuteron' associated with the intermediate states of the two-step amplitude appears in spin-singlet or spin-triplet states with equal weight, the selection rules for one- and two-step are identical. However, constraining it to be in a triplet state only (such as the common assumption that it is the physical deuteron ground state) acts as a spin-filter and determines that the two neutrons are transferred with a particular mixture of total spin  $S = 0$  or  $1$ . This mixture will differ from that for the one-step term. In this way, inclusion of singlet deuterons could completely change the results for vector analyzing powers and might modify conclusions that have been drawn from measurements of these quantities. It is sufficient, for our purpose, to consider the first and second-order distorted-wave Born amplitudes and to make various simplifying assumptions such as zero-range. While these may have large effects upon the magnitude of the amplitudes, they do not affect the underlying structure.

## 2.1. One-Step Amplitudes

The triton wavefunction can be decomposed into components in which the two neutrons have a definite total spin  $S$  (where  $S = 0$  or  $1$ ),

$$\psi_{\frac{1}{2} m_t} = \sum_S \psi_{\frac{1}{2} m_t}^S(\text{spins}) \psi_t^S(\text{space}). \quad (1)$$

Then, aside from uninteresting multiplicative constants, the first-order DWBA or one-step amplitude for the  $A(t,p)B$  reaction is a sum of components,

$$T_{BA}^{(1)}(S) = \int d\mathbf{r}_d d\mathbf{r}_p d\mathbf{r}_n x_p^{(-)*}(\mathbf{R}_p) \langle \frac{1}{2} m_p; B | (\frac{1}{2}, S) \frac{1}{2} m_t; A \rangle V(\mathbf{r}, \mathbf{r}_2) \psi_t^S(\mathbf{r}, \mathbf{r}_2) x_t^{(+)}(\mathbf{R}_t), \quad (2)$$

for each value of the total spin  $S$  of the two transferred neutrons. The spatial coordinates are defined in Fig. 2. The  $x_p$  and  $x_t$  are the usual distorted waves. The round brackets denote integration over the internal coordinates of nucleus  $A$  and the spin coordinates of the proton and two neutrons; this factor remains a function of the spatial coordinates  $\mathbf{r}_1$  and  $\mathbf{r}_2$  of the two neutrons. Also,  $\psi_t^S$  is the spatial part of the triton internal state as defined in Eq. (1). The transfer interaction  $V$  is taken to be the post form,  $V_{pB} - U_{pB} = V_{p,2n} + (V_{pA} - U_{pB})$ , with  $U_{pB}$  being the optical potential for the outgoing proton, and with the usual assumption that  $(V_{pA} - U_{pB})$  may be neglected. Then

$$V = V_{p,2n} = V_{pn_1} + V_{pn_2} = v(|\mathbf{r}_2 - \frac{1}{2} \mathbf{r}|) + v(|\mathbf{r}_2 + \frac{1}{2} \mathbf{r}|). \quad (3)$$

In general,  $V$  depends on the spins, but since it cannot change the spin  $S$  of the transferred neutrons,<sup>17</sup> it is sufficient for our purpose to treat it as spin-independent.

The spin matrix element gives a Clebsch-Gordan coefficient expressing the angular momentum coupling of the spins and Eq. (2) can be rewritten

$$T_{BA}^{(1)}(S) = \langle S \frac{1}{2} M_{S m_p} | \frac{1}{2} m_t \rangle \int x_p^{(-)*}(\underline{R}_p) \phi_{SM_S}^{BA}(\underline{r}_1, \underline{r}_2) \times V(\underline{r}, \rho) \psi_t^S(\underline{r}, \rho) x_t^{(+)}(\underline{R}_t) d\underline{R} d\underline{r} d\rho, \quad (4)$$

where the function  $\phi_S$  is the spatial part of the overlap of the two nuclei, i.e. the two-neutron form factor, corresponding to spin  $S$ ,

$$\phi_{SM_S}^{BA}(\underline{r}_1, \underline{r}_2) = (B | SM_S; A) \equiv \int \psi_B^*(\xi_A, \underline{r}_1 \sigma_1, \underline{r}_2 \sigma_2) \psi_A(\xi_A) \psi_{SM_S}(\sigma_1, \sigma_2). \quad (5)$$

It is this two-nucleon overlap which embodies the nuclear structure information about pairing, etc., that we wish to extract from measurements on two-nucleon transfer reactions.

The predominant part of the triton wavefunction is the symmetric  $S$ -state. Within it the neutrons have  $S = 0$ , and the space function  $\psi_t^0$  is a function of the single symmetric variable,

$$u^2 = r_{12}^2 + r_{23}^2 + r_{13}^2 = \frac{3}{2} r^2 + 2\rho^2. \quad (6)$$

Frequently, a zero-range approximation is used in which one sets

$$V \psi_t^0 \approx D(r) \delta(\rho), \quad (7)$$

so that the proton is assumed to be at the centre of mass of the two neutrons and

$$T_{BA}^{(1)}(S=0) \approx \delta_{m_t m_p} \int x_p^{(-)}(\underline{R}') \phi_{00}^{BA}(\underline{r}_1, \underline{r}_2) D(r) x_t^{(+)}(\underline{R}) d\underline{R} d\underline{r}, \quad (8)$$

where  $\underline{R}' = \underline{R}A/(A+2)$ .

The importance of the mixed symmetry  $S'$ -state of the triton and of the  $D$ -state have been stressed by Nagarajan, et al.<sup>17</sup> We consider here the  $S'$ -state

only. It is a sum of two terms, one with  $S = 1$ , one with  $S = 0$ . We ignore the latter component, since it results in the same selection rules as the symmetric  $S$ -state. The term with  $S = 1$  has a spatial part of the form  $\underline{r} \cdot \underline{\rho} g(u)$ , so that there is a contribution to the amplitude

$$T_{BA}^{(1)}(S=1) = \langle 1 \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle \int \chi_p^{(-)}(R_p)^* \phi_{1M_S}^{BA}(\underline{r}_1, \underline{r}_2) V_{\underline{r} \cdot \underline{\rho}} g(u) \chi_t^{(+)}(R_t) d\underline{R} d\underline{r} d\underline{\rho}. \quad (9)$$

One cannot make the zero range ( $\rho = 0$ ) approximation on this term because of the factor  $\underline{r} \cdot \underline{\rho}$ . However, it is useful to consider the lowest-order non-vanishing terms in an expansion of the distorted waves in powers of  $\rho$ ;

$$\chi_t^{(+)}(R_t) = \chi_t^{(+)}(R + \frac{1}{3} \underline{\rho}) = \chi_t^{(+)}(R) + \frac{1}{3} \underline{\rho} \cdot \underline{\nabla}_R \chi_t^{(+)}(R) + \dots, \quad (10)$$

with a corresponding expansion for  $\chi_p^{(-)}(R' + \underline{\rho})$ . Since from Eqs. (2) and (4),  $V$  and  $g$  are even functions of  $\rho$ , we have

$$\int d\underline{\rho} g(u) V_{\underline{r} \cdot \underline{\rho}} = 0, \quad (11)$$

so that the lowest-order term appearing in the expansion of  $T_{BA}^{(1)}(S=1)$  contains one factor of  $\underline{\rho} \cdot \underline{\nabla}_R \chi$ . If we define a function  $E$  according to

$$\int d\underline{\rho} g(u) V_{\underline{r} \cdot \underline{\rho}} (\underline{\rho} \cdot \underline{\nabla}_R \chi) = E(r) \underline{r} \cdot \underline{\nabla}_R \chi, \quad (12)$$

then we can write

$$T_{BA}^{(1)}(1) \approx \langle 1 \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle \int d\underline{R} d\underline{r} E(r) \phi_{1M_S}^{BA}(\underline{r}_1, \underline{r}_2) \times \left\{ \frac{1}{3} \chi_p^{(-)}(R')^* \underline{r} \cdot \underline{\nabla}_R \chi_t^{(+)}(R) + \frac{A+2}{A} \underline{r} \cdot \underline{\nabla}_R \chi_p^{(-)}(R')^* \chi_t^{(+)}(R) \right\}. \quad (13)$$

Later we will show that the sequential transfer process generates amplitudes very similar to Eqs. (8) and (13).

## 2.2. Two-Step Amplitudes and the Closure Approximation

In second-order DWBA, the transition amplitude  $T^{(2)}$  for the two-step process  $A(t,d;d,p)B$ , with intermediate nuclear states  $C$ , has a part  $\hat{T}^{(2)}$  which is second order in the interactions and a non-orthogonality correction term  $T^{(NO)}$ . The first part has the form<sup>18</sup>

$$\hat{T}_{BA}^{(2)} = \sum_C \int \dots \int \chi_p^{(-)*}(\underline{R}_p)(p,B|V_{pn_1}|d,C)G_C^{(+)}(\underline{R}_d,\underline{R}'_d)(d,C|V_{dn_2}|t,A)\chi^{(+)}(\underline{R}_t) d\underline{R}_p d\underline{R}_d d\underline{R}'_d d\underline{R}_t, \quad (14)$$

where we ignore spins for the moment. Here  $G_C$  is the Green function for the propagation of the intermediate deuteron  $d$  (relative to the intermediate nucleus  $C$ ) under the influence of an optical potential. If this optical potential were neglected so that the propagation was in plane waves rather than distorted waves,  $G_C$  would reduce to

$$G_C^{(+)}(\underline{R},\underline{R}') \sim \exp(ik_d s)/s, \quad s = |\underline{R}-\underline{R}'|. \quad (15)$$

(The wavenumber  $k_d$  could be reinterpreted as the local value  $k_d(\bar{R})$  at the position  $\bar{R} = \frac{1}{2}|\underline{R}+\underline{R}'|$ , thus introducing a 'local-energy' approximation<sup>3</sup> for the effects of the optical potential and the Coulomb field.) The quantities with round brackets are the usual nuclear overlap functions, or 'form factors', for one-nucleon transfer. In the post-post interaction form that we have adopted, the non-orthogonality term  $T^{(NO)}$  has a similar form except for having the opposite sign and for the omission of two scalar factors, the Green function  $G_C$  and the interaction  $V_{dn_2}$  for the  $(t,d)$  step. It tends to cancel the one-step amplitude.

The closure approximation consists of assuming that the intermediate states  $\psi_C$  that contribute importantly in the expression for  $\hat{T}^{(2)}$  are sufficiently close

together in excitation energy that the associated Green functions  $G_C$  can be replaced by an average  $\bar{G}_C$ . We then assume that we have a complete set of states  $\psi_C$  and use the completeness relation for the nuclear overlaps,

$$\sum_C (B|C)G_C(C|A) \approx \bar{G}_C \sum_C (B|C)(C|A) \quad (16a)$$

$$= \bar{G}_C (B|A), \quad (16b)$$

because of the closure relation

$$\sum_C |C\rangle\langle C| = 1. \quad (16c)$$

Despite the somewhat condensed notation, comparison with Eq. (5) shows that this procedure results in the appearance of the same two-neutron overlap functions or form factors  $\phi^{BA}(\underline{r}_1, \underline{r}_2)$  as those which appear in the one-step amplitude (4).

Consequently, provided the important intermediate states are sufficiently close for the approximation (16a) to be valid, nuclear structure effects such as pairing correlations will manifest themselves in much the same way in one-step or two-step processes. In this way, one can understand how analyses of measured cross sections using the theory for a one-step transfer alone may yield useful information on the relative behavior of spectroscopic factors for different states and different nuclei,<sup>19</sup> even when the theory fails to explain the absolute magnitudes of the cross sections.

### 2.3. Further Comments on the Closure Approximation

One can easily construct model situations in which the procedure (16) becomes exact. For example, consider two-neutron transfer to a closed-shell target in which the intermediate states  $\psi_C$  are pure single-particle  $j$ -states and the residual state  $\psi_B$  has a pure  $j^2$  configuration. Only one intermediate state can contribute, namely the corresponding single-particle state in nucleus  $A+1$  with the same  $j$ , and Eq. (16b) becomes exact.

The expression (14) assumes a single intermediate state for the intermediate particle  $c = a-1$ , in this case a deuteron,  $c = d$ . More generally, both intermediate nuclei may be in more than one state and expressions like (14) need to be supplemented by an additional sum over the states of  $c$ . In a heavy-ion reaction, both  $c$  and  $C$  may be comparable and hence should be treated on the same footing. This was done by Feng, et al.<sup>3</sup> when they applied the closure approximation. The tendency has been to treat light-ion reactions differently, by assuming that the intermediate light particle remains in its ground state. This has been a matter of expediency; excited states of light ions are unbound and thus introduce a continuum of intermediate  $> 3$ -body states which greatly increases the complexity of the calculation. This has been done<sup>12,15</sup> for the intermediate 'deuteron' (i.e.,  $n$ - $p$  system) in  $(p,t)$  reactions.

The closure approximation (16) may be applied to the states of  $c$ , as well as those of  $C$ , if the important contributions still come from a sufficiently narrow band of energies (i.e. narrow enough that the Green function  $G_{CC}$  does not vary much). This results in the appearance of a second two-nucleon overlap or form factor  $\langle b|a \rangle$ , which again is the same as that which occurs in the one-step amplitude  $T^{(1)}$ . Then the nature of the intermediate nucleus  $c$  plays no rôle and, for example, angular momentum selection rules are the same for one-step and two-step transfer. It is plausible that this is approximately valid for heavy-ion reactions<sup>3</sup> where the important intermediate states are likely to be bound states of moderate excitation energy. It is not so obvious for light-ion reactions. The explicit calculations<sup>12,15</sup> for  $^{48}\text{Ca}(p,t)$  at 20 and 40 MeV included the  $n$ - $p$  scattering states, both singlet and triplet. Important contributions came from the continuum states at 40 MeV, but the effect (or the cross section) was small at 20 MeV. The intermediate spin-singlet state is always associated

with continuum states of relative motion. Hence, the transition is dominated by triplet intermediate states if the contribution of the continuum is negligible. This constitutes a spin-filter through which the two-step process must pass and introduces differences in the spin selection rules compared to the one-step process. We return to this later. It is not clear from the results<sup>12,15</sup> for 40 MeV, where the continuum is important, whether this kind of spin selectivity is still active.

#### 2.4. Non-Orthogonality and the Second-Order Amplitude

If one introduces a truly complete set of intermediate states (for both intermediate nuclei), it is easy to show<sup>12,14</sup> that, in the post-post (or prior-prior) interaction form of the amplitude, the non-orthogonality correction term  $T^{(NO)}$  becomes equal and opposite to the one-step amplitude  $T^{(1)}$ . The two-step term  $\hat{T}^{(2)}$  is then left as the lowest-order contribution to the transition. In practice, however, it is not feasible to use a complete set. Formally, it implies non-convergence of the Born series because of the mathematical difficulties associated with the 3- and more-body states that appear when the intermediate states are unbound. One standard approach is to regard our calculations as being done within a truncated model space, limited to 2-body channels, and with effective interactions. Thus, a complete set is ruled out from the start. Nonetheless, the tendency remains for  $T^{(NO)}$  and  $T^{(1)}$  to interfere destructively, sometimes quite strongly (see Refs. 9 and 12-15 for examples), and this enhances the importance of the two-step process described by  $\hat{T}^{(2)}$ .

One final note: the separation of the second-order amplitude  $T^{(2)}$  into  $\hat{T}^{(2)}$  and  $T^{(NO)}$  is artificial. Despite the intuitive appeal of interpreting  $\hat{T}^{(2)}$  as "the two-step process", one cannot physically distinguish these two terms from each other (nor they from the first-order term  $T^{(1)}$ ). This is

emphasized by the lack of uniqueness in separating  $T^{(2)}$  into two parts; for example, if we had chosen the prior-post interaction form for  $T^{(2)}$ , there would be no  $T^{(NO)}$  term and hence no possibility of the cancellation of  $T^{(1)}$ . This lack of uniqueness also renders ambiguous even the conceptual division into one-step and two-step. Only the total amplitude has direct physical meaning; its breakdown into pieces is merely a consequence of the way we do calculations. Nonetheless, the discussion of this paper can be valuable as an aid to understanding the results of those calculations.

### 2.5. Approximate Two-Step Amplitude for (t,p) Reactions

We now consider the amplitude (14) in more detail. At first, we only allow ground-state (triplet) deuterons in the intermediate state. Further, for simplicity, we adopt the zero-range approximation for each of the single-nucleon transfers and only consider the dominant symmetric S-state for the triton.

Consequently, the spins of the two neutrons within the triton are coupled to  $S = 0$ . Then Eq. (14) becomes

$$\begin{aligned} \hat{T}_{BA}^{(2)} = & \sum_{C, m_d} D_0(t,d) D_0(d,p) \int d\mathbf{r}_1 \chi_p^{(-)*} \left( \frac{A+1}{A+2} \mathbf{r}_1 \right) \left( \frac{1}{2} m_p; B | s_d m_d; C \right) \\ & \times \int d\mathbf{r}_2 G_C^{(+)} \left( \mathbf{r}_1, \frac{A}{A+1} \mathbf{r}_2 \right) (s_d m_d; C | \frac{1}{2} m_t; A) \chi_t^{(+)}(\mathbf{r}_2), \end{aligned} \quad (17)$$

where  $D_0(a,b)$  is the usual zero-range normalization constant for one-nucleon transfer. The coordinates used are indicated in Fig. 2(b). Also,  $s_d = 1$  is the spin of the deuteron.

The spin overlaps are evaluated by first recoupling the spins in the triton in terms of the resultants  $\hat{\mathfrak{S}}_d$  of the spins of the proton and the first neutron. Our assumption of a particular deuteron state (triplet,  $s_d = 1$ ) then selects the

$\tilde{s}_d = 1$  part of the triton. We then get

$$\begin{aligned} \hat{T}_{BA}^{(2)} = & \sum_{S=0}^1 (2s_d+1)^{\frac{1}{2}} (2S+1)^{\frac{1}{2}} W\left(\frac{1}{2} s_d S \frac{1}{2}; \frac{1}{2} \frac{1}{2}\right) \langle S \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle D_0(t,d) D_0(d,p) \\ & \times \sum_C \int d\mathbf{r}_1 d\mathbf{r}_2 x_p^{(-)*} \left(\frac{A+1}{A+2} \mathbf{r}_1\right) (B|C) G_C^{(+)}(C|A) x_t^{(+)}(\mathbf{r}_2) \end{aligned} \quad (18)$$

where  $S$  is the angular momentum transferred due to the spins of the two neutrons. Although these spins were assumed to be originally coupled to zero within the triton, the 'spin-filter' introduced by interposing a deuteron with  $s_d = 1$  has resulted in the appearance of transfers with both  $S = 0$  and 1.

In order to simplify Eq. (18) further, we now make the closure approximation (16). Substituting the numerical values of the Racah coefficients in Eq. (18), we finally arrive at

$$\hat{T}_{BA}^{(2)} \approx \frac{\sqrt{3}}{2} \hat{T}_{BA}^{(2)}(S=0) - \frac{1}{2} \hat{T}_{BA}^{(2)}(S=1), \quad (19)$$

with

$$\hat{T}_{BA}^{(2)}(0) = \delta_{m_p m_t} D_0(t,d) D_0(d,p) \int d\mathbf{r}_1 d\mathbf{r}_2 x_p^{(-)}(\mathbf{r}_1)^* \phi_{00}^{BA}(\mathbf{r}_1, \mathbf{r}_2) \bar{G}_C x_t^{(+)}(\mathbf{r}_2), \quad (20a)$$

and

$$\hat{T}_{BA}^{(2)}(1) = \langle 1 \frac{1}{2} M_S m_p | \frac{1}{2} m_t \rangle D_0(t,d) D_0(d,p) \int d\mathbf{r}_1 d\mathbf{r}_2 x_p^{(-)}(\mathbf{r}_1)^* \phi_{1M_S}^{BA}(\mathbf{r}_1, \mathbf{r}_2) \bar{G}_C x_t^{(+)}(\mathbf{r}_2), \quad (20b)$$

where

$$\mathbf{r}_1 = \frac{A+1}{A+2} \mathbf{r}_1.$$

Next, introduce the coordinates of the separation of the two neutrons, and of their CM relative to A,

$$\underline{r} = \underline{r}_1 - \underline{r}_2, \quad 2\underline{R} = \underline{r}_1 + \frac{A+2}{A+1} \underline{r}_2$$

or

$$\underline{r}_1 = \underline{R} + \frac{1}{2} \underline{r}, \quad \underline{r}_2 = \underline{R} - \frac{1}{2} \underline{r}.$$

Further, assume that  $\bar{G}$  has a short range in the variable  $r$ , so that we may approximate it by  $\bar{G}_0$ , the monopole part of  $\bar{G}$ , where

$$\bar{G} = \sum_{\lambda} \bar{G}_{\lambda}(r, R) P_{\lambda}(\cos\theta). \quad (21)$$

Here  $\theta$  is the angle between  $\underline{r}$  and  $\underline{R}$ . Then the lowest-order term arising from an expansion of the distorted waves in this range, like Eq. (10), should be a reasonable approximation. It is

$$\hat{T}_{BA}^{(2)}(0) \approx \delta_{m_p m_t} D_0(t, d) D_0(d, p) \int d\underline{R} d\underline{r} x_p^{(-)*}(\underline{R}') \phi_{00}^{BA}(\underline{r}_1, \underline{r}_2) \bar{G}_0(r, R) x_t^{(+)}(\underline{R}). \quad (22)$$

Equation (22) is identical in form to Eq. (8), with  $D(r)$  there replaced by  $\bar{G}_0(r, R)$ . (Note, however, that  $D(r)$  is usually assumed real, while  $\bar{G}_0$  is complex.)

Similar approximations can be made for  $\hat{T}_{BA}^{(2)}(1)$ . Because of antisymmetry, the overlap function  $\phi_{1M_S}$  must be an odd function of the variable  $r$ ; therefore the non-vanishing term of lowest-order in an expansion of the  $x$ 's in powers of  $r$  is

$$\begin{aligned} \hat{T}_{BA}^{(2)}(1) \approx \langle 1 \frac{1}{2} M_S m_p \frac{1}{2} m_t \rangle D_0(t, d) D_0(d, p) \int d\underline{R} d\underline{r} \phi_{1M_S}^{BA}(\underline{r}_1, \underline{r}_2) \bar{G}_0(r, R) \\ \times \left\{ -\frac{1}{2} x_p^{(-)}(\underline{R}')^* \underline{r} \cdot \underline{v}_R x_t^{(+)}(\underline{R}) + \frac{A+2}{A} \underline{r} \cdot \underline{v}_R x_p^{(-)}(\underline{R}')^* x_t^{(+)}(\underline{R}) \right\}. \end{aligned} \quad (23)$$

Equation (23), although not exactly the same as Eq. (13), is remarkably similar in form.

Clearly, a number of severe approximations have been made, so that the results obtained are qualitative and suggestive, not quantitative and rigorous. Nevertheless, they are useful aids in understanding the roles played by the simultaneous and sequential transfer amplitudes.

The remarkable similarity of Eqs. (8) and (22) for the  $S=0$  amplitudes confirms a result frequently observed in numerical calculations, that sequential transfer amplitudes tend to have the same dependence on angle as simultaneous transfer amplitudes. They also tend to depend in the same way on nuclear bound-state properties, so that relative spectroscopic factors obtained from analysis of data based on the one-step DWBA are expected to be quite reliable, even when sequential transfer amplitudes are significantly large. These points have also been made earlier.<sup>7,16</sup>

The similarity of Eqs. (13) and (23) for the  $S=1$  amplitudes is also interesting. These amplitudes have the same selection rules, the same dependence on bound-state properties and, although not of exactly the same mathematical form, appear to be very much alike. As a result, including sequential transfer amplitudes can be expected to have effects comparable to including the mixed-symmetry  $S'$  state in an exact finite-range treatment of the first-order amplitude. Both effects should influence the sensitive vector analyzing power. This injects some uncertainty into analyses based on the zero-range approximation, such as Ref. 20, although the calculations of the vector analyzing power presented there are quite suggestive of the importance of sequential transfer. It would be very interesting to compare the results of Ref. 20 with finite-range calculations made, for example, with the computer programs used in Refs. 7 or 9.

## 2.6. Importance of the Deuteron Singlet State

Only one author has reported<sup>12,15</sup> including the singlet state ( $s_d = 0$  in Eq.

(18)) of the deuteron in the calculations of sequential transfer. A proper treatment of the singlet-state is non-trivial since it lies in the continuum. If assumed to be quasibound, it could be included without difficulty in the same way as the bound triplet-state; however, it seems that this would be a poor approximation.<sup>15</sup> If the singlet continuum is treated properly, it is inconsistent to ignore the triplet continuum; indeed, the only reported calculations of the 'break-up' effect<sup>12,15</sup> do include both singlet and triplet contributions. Granted that the problem is complicated, it is still illuminating to discuss the consequences of the deuteron singlet states using a simplified approach.

Assume that the singlet and triplet intermediate n-p states contribute equally and that they propagate in the same way (i.e. that  $G_C$  is the same for both). For example, one might assume that the triplet deuteron ground-state and the zero-energy  $^1S$  resonance dominated, and neglect the differences between their energies and wavefunctions.

With this assumption, the total amplitude for sequential transfer is the same as Eq. (17) except that there should also be a summation over  $s_d = 0$  and 1. Since the Green functions for singlet and triplet deuteron propagation are assumed to be the same, summation of the projector  $|s_d M_d\rangle\langle s_d M_d|$  over  $s_d, M_d$  gives unity. As a result, the complete amplitude is equal to  $\hat{T}_{BA}^{(2)}(S=0)$  of Eq. (20a). The coefficient of  $\hat{T}_{BA}^{(2)}(S=1)$  is identically zero. Similar results are obtained for the non-orthogonality terms. The spin-transfer selection rules are then exactly the same for zero-range sequential transfer as for simultaneous transfer with a totally symmetric S-state triton. If this model were correct, including the sequential transfer amplitude could be expected to have little or no effect on the vector analyzing power. The model is oversimplified; for example, Hashimoto<sup>15</sup> finds the triplet states to be more important than the

singlet ones. Nonetheless, it serves as a warning that it may be impossible to completely decipher the reaction mechanism problem unless the deuteron continuum is properly included.

The following analogy is useful. The deuteron channel acts much like an optical polaroid filter. In the triton wave function the neutron spins are coupled to  $S = 0$ , which from recoupling we find is a linear combination of  $s_d = 1$  and  $0$ . The assumption of a triplet deuteron filters out the  $s_d = 0$  component, and the resulting triton function, after filtration, is no longer pure  $S = 0$ ; it also contains  $S = 1$  neutron components. If the singlet and triplet deuteron states are included with equal weights, the filter is removed and only  $S = 0$  neutron transfer is possible.

### 3. The $(\alpha, d)$ Reaction

A similar analysis has been made of the  $(\alpha, d)$  reaction.<sup>21</sup> Two sequential processes are available here,  $(\alpha, t; t, d)$  and  $(\alpha, h; h, d)$  (where  $h \equiv {}^3\text{He}$ ). Only  $s_d=1$  transfer is possible for the simultaneous transfer because of the zero spin of the alpha. If we assume an S-state of maximum symmetry for the alpha, only spin -  $\frac{1}{2}$  states are allowed for the triton and helium. Thus, although they act as spin -  $\frac{1}{2}$  'spin filters', they impose no further constraint on the spin transfer. The spin-transfer selection rules are the same for simultaneous and sequential transfer. There could be, however, an isospin filtering effect which would result from any difference in propagation of the t and h particles in unsymmetric, charged nuclear matter (i.e. if they had different Green functions).

By introducing similar zero-range and closure approximations, the one-step and two-step amplitudes are reduced to the same forms as Eqs. (8) and (22) for

the (t,p) reaction. Again, the two amplitudes are seen to depend upon nuclear structure in the same way. The same two-nucleon overlap, analogous to Eq. (5), appears in both. In particular, there does not appear any phase difference between them that depends upon the spin of the residual nuclear state.

The latter point is especially relevant because of some observations of the Pittsburgh group<sup>22</sup> on the  $^{208}\text{Pb}(\alpha,d)^{210}\text{Bi}$  reaction. Consider excitation of the  $(g_{9/2}h_{9/2})_J$  multiplet of states with  $J = 0,1,\dots,9$ . Simple angular momentum considerations, together with the tendency of  $(\alpha,d)$  reactions to favor large L-transfers, leads to the prediction that the one-step cross sections should show a regular stepped pattern when plotted against J, with the rise at each step increasing as J increases. DWBA calculations<sup>22</sup> confirm this (see Fig. 3 a). The discussion of the present paper suggests that a similar pattern should be seen for the two-step process by itself and again explicit calculations<sup>22</sup> confirm this (Fig. 3(a)).

The measured cross sections<sup>22</sup> show a rather different, saw-tooth pattern (Fig. 3); the predicted steady rise with L transfer is observed; however, the cross sections for the even-J members of the  $J=(L-1,L)$  pairs are larger than those for the odd-J members. Coupled-channels calculations<sup>22</sup> including direct and sequential transfers reproduced these trends and seemed to indicate that the saw-tooth pattern resulted from successively constructive and destructive interferences of direct and sequential amplitudes due to a J-dependence in their relative phase. No such J-dependent phase factor appears in the present treatment<sup>†</sup>, although it is able to account for the trends shown by the individual

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<sup>†</sup>Although a coupled reaction channels code was used, the calculations of Refs. 22 were made in such a way as to correspond exactly to the second-order distorted-wave Born approximation discussed here, including the use of the zero-range approximation. Further, the non-orthogonality correction  $T(\text{NO})$  was not included. However, it is easy to see that this term, which tends to cancel the one-step amplitude, has exactly the same structure. Consequently, no J-dependent interference is expected from non-orthogonality corrections, either.

one-step and two-step amplitudes in terms of the reaction dynamics and the LS-jj transformation coefficients involved in the two-nucleon overlap factor.<sup>21</sup> Indeed, within our model, the integrand of the two-step amplitude differs from that for the one-step only by the replacement of the light-ion overlap function  $D(r)$  by  $\bar{G}_0(r,R)$ , the average of the triton and helion Green functions. The symmetric part of the two-nucleon overlap  $\phi(r_2, r_1)$ , from which any J-dependence must arise, is common to both amplitudes.

Our analysis makes it difficult for us to understand either the calculations presented in Refs. 22 or the experimental data, which seem to be nicely explained by the coupled-channels calculations. Cross sections for  $^{208}\text{Pb}(^3\text{He},p)^{210}\text{Bi}$  show a similar behavior.<sup>23</sup> The  $(^3\text{He},p)$  reaction differs from  $(\alpha,d)$  in that  $S=1$  and  $S=0$  transfer are both possible. However, nuclear structure factors make the  $S=0$  contribution much smaller than that for  $S=1$ , so that, except for kinematic effects, one should expect the two reactions to behave in a very similar way.

Our approach differs from that using coupled channels in that we do not consider intermediate states explicitly but make the closure approximation. However, with the assumption of a single  $(g_{9/2}h_{9/2})$  configuration for the final state in the  $^{208}\text{Pb}(\alpha,d)^{210}\text{Bi}$  case, the result of making the closure approximation yields the same result as restricting the intermediate states to be the  $^{209}\text{Pb}$  and  $^{209}\text{Bi}$  ground states, as was done in the coupled-channels calculations.<sup>22</sup>

References and Notes

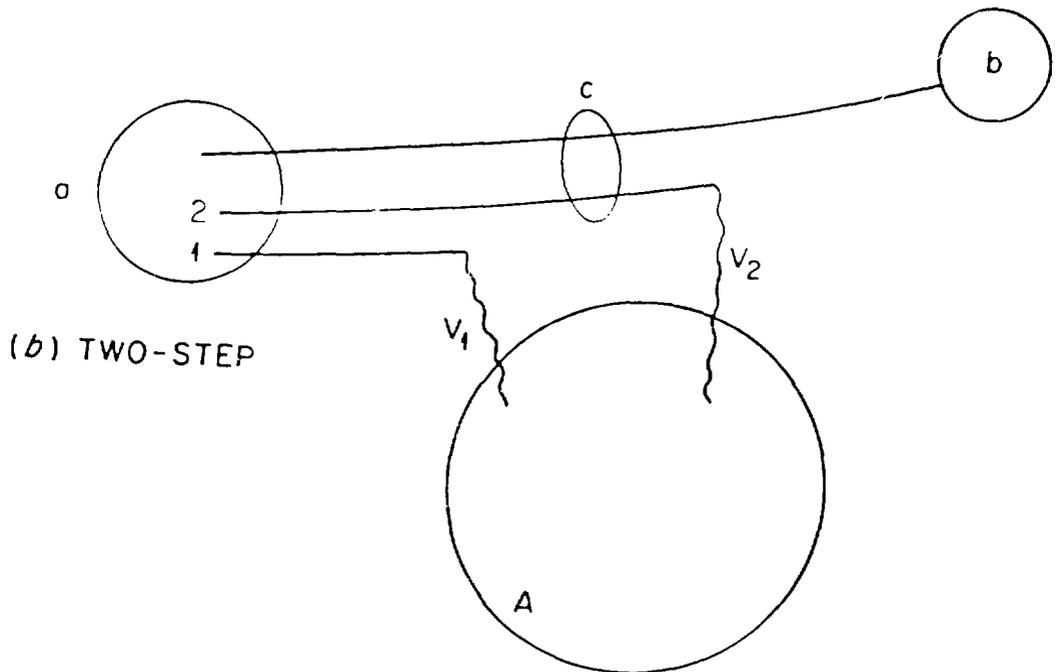
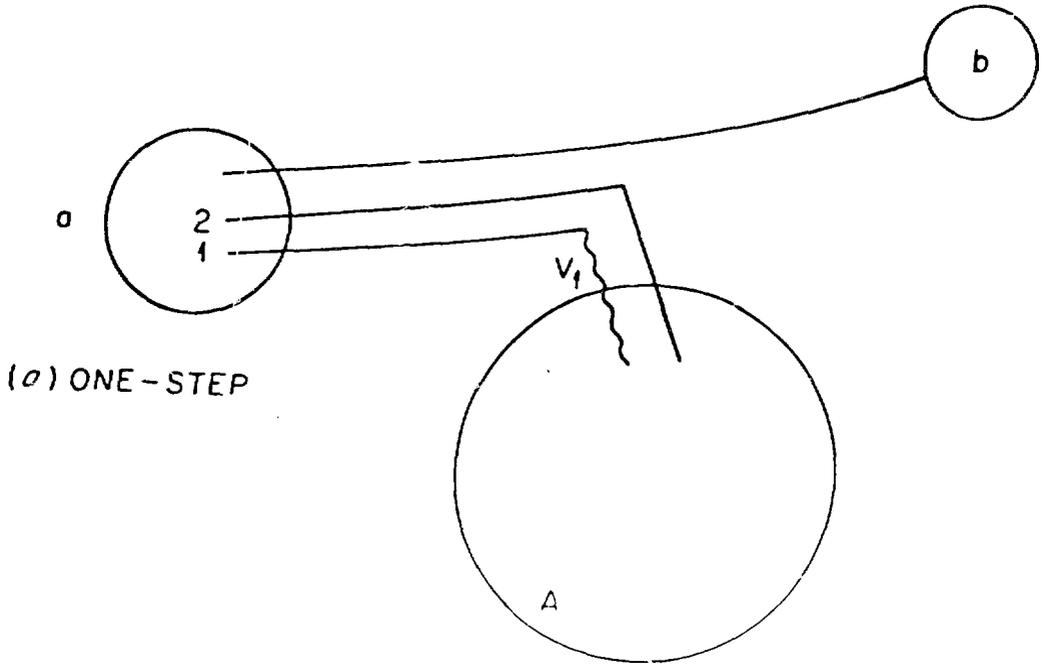
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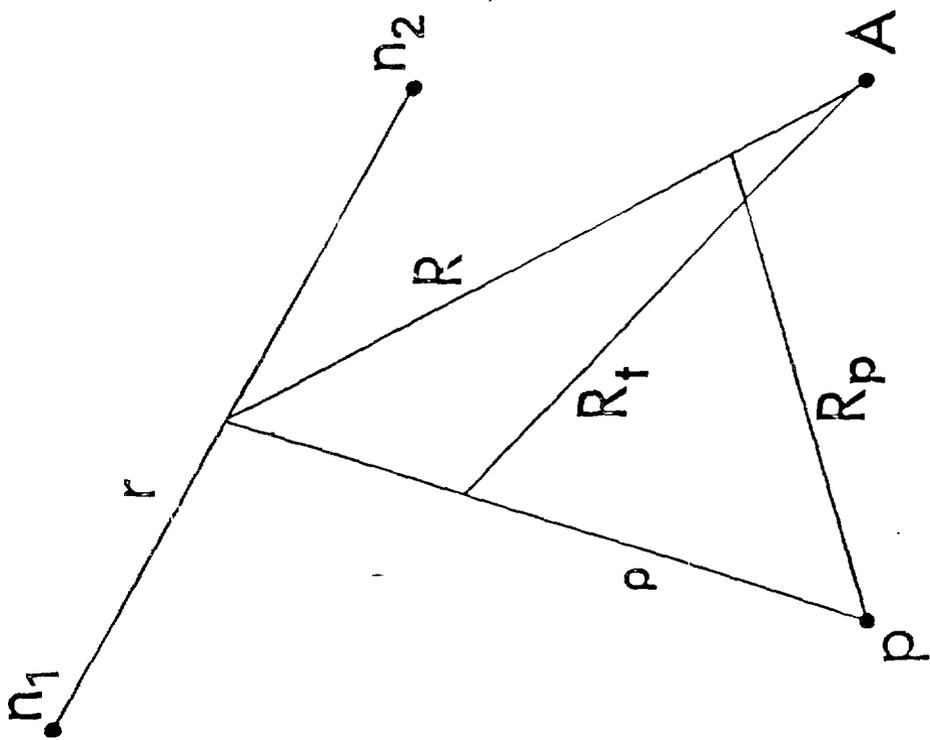
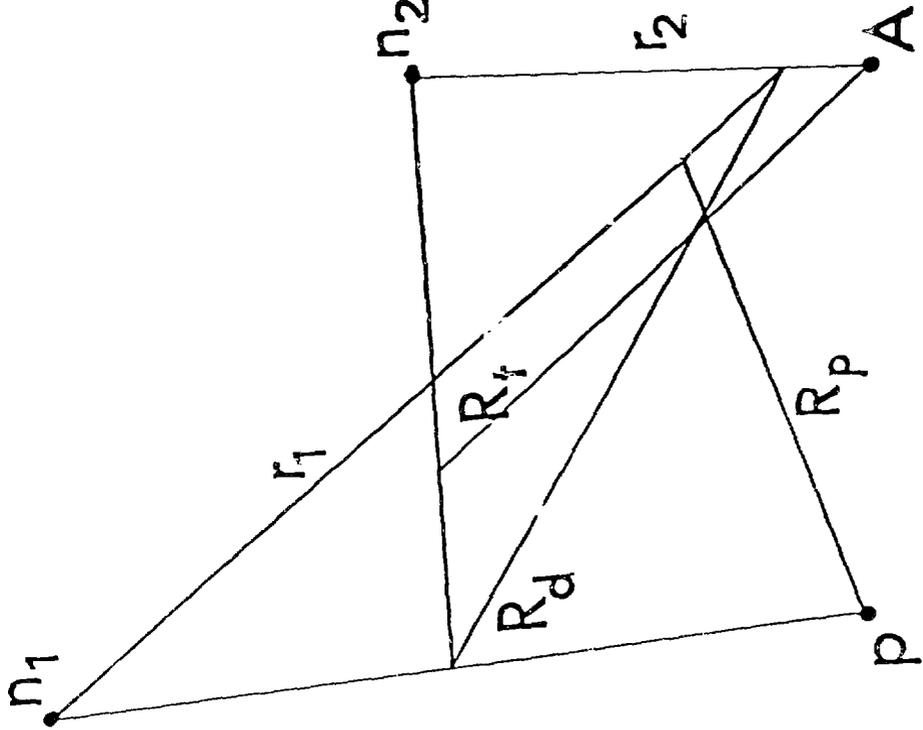
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Figure Captions

- Fig. 1. (a) Simultaneous transfer, (b) sequential transfer. The wavy line corresponds to one action of the projectile-target interaction.
- Fig. 2. Coordinates used for (a) simultaneous, and (b) sequential transfer.
- Fig. 3. Integrated cross sections, versus  $J$ , for the  $^{208}\text{Pb}(\alpha, d)^{210}\text{Bi}$  reaction exciting the  $(h_{9/2}g_{9/2})_J$  multiplet. (a) Individual one-step and two-step cross section; (b) coherent sum of one-step and two-step; as reported in Ref. 22.

$A(a,b)B$





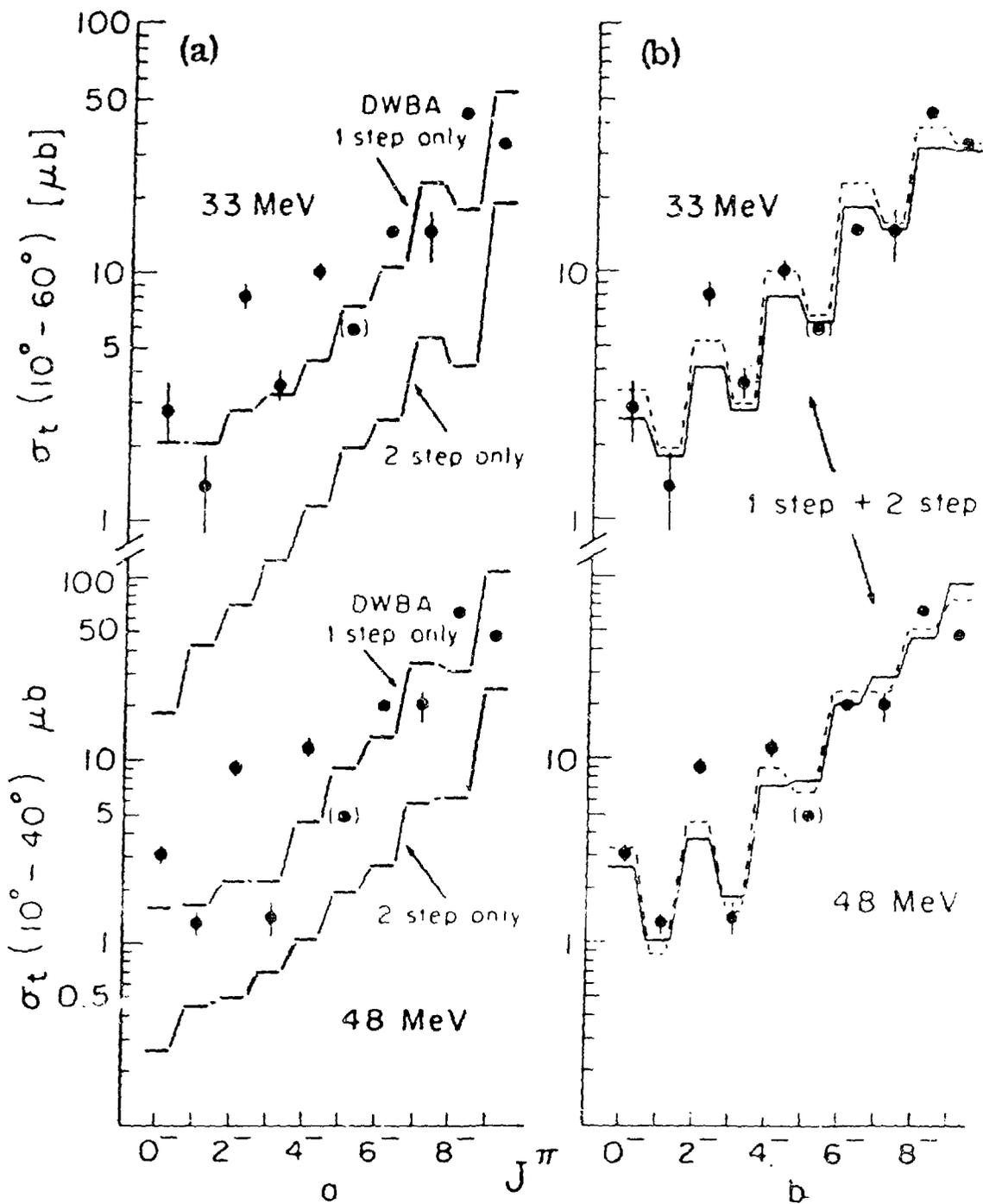


Fig. 3