

SHELL-MODEL CALCULATIONS OF NUCLEAR-CHARGE RADII

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Abstract Shell-model calculations of charge radius differences in the Pb isotopes are discussed. Core quadrupole oscillations are found to be significant factors in the calculations. Existing data on the ^{210}Pb isotope shift and the $B(E2)$ strengths in ^{210}Pb are shown to be inconsistent. Ground-state correlation effects in light nuclei (i.e., O and Ca isotopes) introduce odd-even staggering effects and other qualitative features in agreement with existing data.

INTRODUCTION

At this conference we have heard about a number of beautiful experiments which produce extremely precise measurements of nuclear sizes, as extracted from isotope shift measurements. An obvious question is what can we learn from such measurements. Historically, there are two distinct classes of nuclear models. One is the collective model, wherein the nucleus is treated as some sort of liquid which can assume a shape, rotate, vibrate, etc. Bill Myers has discussed here the application of such

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models to calculations of nuclear radii. The other class of nuclear models is the microscopic model, where the nucleus is treated as an assembly of discrete particles, neutrons and protons, interacting through two-body forces. I would like to review here how calculations of nuclear sizes in a microscopic picture have been carried out. The liquid drop models have been rather successful as a means of calculating nuclear radii, so one can ask whether there is anything to be learned from microscopic calculations. Most microscopic calculations of nuclear sizes have been made in the framework of either spherical or deformed state Hartree-Fock theory. Such calculations yield relatively accurate single-particle wave functions which result from admixing orbits with the same orbital angular momentum but differing numbers of nodes. They do not treat effects of many particle-hole states at low energy, i.e., low-energy surface effects. The calculations are complicated by formal problems for non-closed-shell systems and for odd-mass nuclei. All these factors may be important when dealing with a series of isotopes over a rather narrow mass range. I will discuss primarily microscopic shell-model calculations which can, in principle, overcome the problems inherent in the Hartree-Fock approach. I hope to show that such calculations can teach us something about the residual shell-model interaction between nucleons, and they may also provide a sensitive test of nuclear wave functions. I will first discuss some recent shell-model calculations of isotope shifts in the Pb isotopes.¹ These calculations are probably the most "fundamental" microscopic calculations of a series of isotope shifts that have been made, in ways which I'll define below. They also serves to show how a straightforward

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calculation of isotope shifts proceeds. I'll review the microscopic calculations of Sorensen and collaborators,² in particular how they treated quadrupole effects, and I will discuss quadrupole effects in the Pb region. All these calculations are for relatively heavy nuclei, where the data are most accurate. The best shell-model calculations are for light nuclei (where the data are less accurate). I will next review some recent calculations of isotope shifts in the light nuclei by Brown, Massen, and Hodgson.³ These calculations suggest the isotope shift measurements may indeed provide tests of shell-model wave functions.

SHELL-MODEL CALCULATIONS OF ISOTOPE SHIFTS

In this section, I will describe, briefly, a straightforward mixed configuration shell-model calculation¹ of isotope shifts in the isotopes of $^{200-208}\text{Pb}$. The first step is to calculate eigenvectors for the ground states of the Pb isotopes. The shell-model space includes the lowest six single-hole states in the ^{208}Pb core (i.e., $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, $f_{7/2}$, $h_{9/2}$, $h_{13/2}$). For a given $^{208-n}\text{Pb}$ isotope, all n-hole configurations allowed by the Pauli principle are included. The space is truncated in that only states where the sum of the seniorities of the particles in each orbit is ≤ 2 . This truncation has been checked for ^{204}Pb and is found to be extremely accurate, particularly for the ground states which concern us here. In this model space we diagonalize an effective one-body plus two-body interaction. The interaction is a modification of the Kuo-Herling "realistic" interaction⁴ constructed for this space from the Hamada-Johnston nucleon-nucleon

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potential. Many properties of the Pb isotopes are successfully described⁵ by this calculation. Given the eigenvectors for each ground state, we want to calculate the mean square charge radius of the ground state, i.e.

$$\begin{aligned} \langle r^2 \rangle &= \frac{1}{Z} \left\langle \psi_{gs} \left| \sum_{i=1}^{n_{\pi}} r^2(i) \right| \psi_{gs} \right\rangle = \frac{1}{Z} \int \rho_{\pi}(r) r^2 d^3r_{\pi} \\ &= \frac{1}{Z} \sum_j \langle j || r^2 || j \rangle n_j^{\pi} \end{aligned} \quad (2.1)$$

where i sums over all proton coordinates, j sums over all proton-active orbits, and n_j^{π} is the proton occupation probability of the orbit j . The numbers we actually calculate are

$$\delta \langle r^2 \rangle = \langle r^2 \rangle_{208\text{Pb}} - \langle r^2 \rangle_{208-n\text{pb}}$$

(For linguistic simplicity, we often refer to $\delta \langle r^2 \rangle$ as "isotope shift" in this talk.) In our model, the Pb isotopes differ from each other only in the number of neutron holes. There are no proton holes, and the removal of a neutron particle does not change the charge radius. Therefore, one must resort to an effective operator theory. There is a long history⁶ of the use of effective quadrupole charges in shell model calculations. The argument is that a neutron (or proton) in nature polarizes the shell model closed core, or gives the core a quadrupole shape. This deformed core can then radiate by E2 decay. This core contribution is included by treating the neutron as a "proton" with charge e_n . With this effective charge, one can calculate E2 observables with amazing accuracy. One must do the analogous thing for isotope shifts; i.e., one assumes in calculating isotope shifts that a given

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neutron orbit has an effective charge radius $\delta \langle r^2 \rangle_{\text{eff}} = \langle j \| r^2 \| j \rangle_{\text{eff}}$, and then calculates

$$\delta \langle r^2 \rangle = \frac{1}{Z} \sum_{j=1}^{n_v} \langle j \| r^2 \| j \rangle_{\text{eff}} n_j. \quad (2.2)$$

Because the mean-square radius operator is a one-body scalar operator, the only way the shell model many-particle wave function enters the calculation is through the occupation number, n_j , of a given j orbit. If we use harmonic oscillator single-particle wave functions and restrict the model space to one complete oscillator shell, all single-particle orbits have the same value of $\langle r^2 \rangle$. Then Eq. (2.1) becomes $\mathcal{N} \langle r^2 \rangle / Z$, where \mathcal{N} is the number of active holes or particles, i.e. the change in the charge radius is simply linear in the total number of holes (particles). A particularly interesting facet of existing data on isotope shifts for a series of isotopes is the so-called odd-even staggering effect, i.e.

$$(\delta \langle r^2 \rangle^{N+1} - \delta \langle r^2 \rangle^N) \neq \frac{1}{2} [\delta \langle r^2 \rangle^{N+2} - \delta \langle r^2 \rangle^N].$$

In shell models which involve only one active oscillator shell, there is no possibility of any odd-even staggering effect in the calculated mean square charge radii. The only source of a non-linear effect in this simple one-body picture is through differences in the values of $\langle j \| r^2 \| j \rangle_{\text{eff}}$. It is unlikely that such effects will emerge from any simple shell model involving only one type of particle. The odd-even staggering effect will be discussed at more length below.

To return to the Pb calculation, in order to proceed, we need effective radial moments for the single-hole

states in ^{207}Pb . This problem was studied by Speth, Ring, and Zamick.⁷ They calculated the "effective-neutron-charge" radius in first-order perturbation theory. They considered the change in the charge radius due to the excitation of a proton particle hole pair in the ^{208}Pb core, or diagrammatically

$$\langle j || r^2 || j \rangle_{\text{eff}} = \text{Diagram} \quad (2.3)$$

In (2.3) the bubble represents a proton particle-hole state in ^{208}Pb . Ring and Speth⁸ had reported on RPA calculations of the state of ^{208}Pb . In the calculation of the isotope shift, i.e. in Eq. (2.3), they couple hole states to eigenstates of ^{208}Pb , as opposed to a simple hole-particle perturbation theory calculation. Ring and Speth had studied the states of ^{208}Pb in a RPA calculation using the Migdal parameterization of the hole particle interaction. The particle-hole interaction was a density-dependent δ -force interaction, with both spin and isospin dependences. With each spin dependence, the force has the form

$$V(r_1, r_2) = \delta(r_1 - r_2) f(r) \quad (2.5)$$

where

$$f(r) = f_{\text{ex}} + (f_{\text{in}} - f_{\text{ex}}) \rho(R) \quad (2.6)$$

$$\rho(R) = \frac{1}{(1 + \epsilon^{(R-R_0)/\alpha})} \quad (2.7)$$

The parameters of the force were originally selected to

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give a good description of the $J = 2^+, 3^-, 4^+, 5^-$ states with no consideration of monopole properties. In Speth, et al.,⁷ the parameters were readjusted to fit the monopole properties (i.e., E0 transitions, nuclear radii, etc.). It was found that the density dependence of the strength of the (p,p) and (p,n) interactions were altered. As a result, the previous good results were retained, but results for monopole properties were significantly improved. For instance, the change in the mean square radius for the ground state of ^{207}Pb changed from $.012 \text{ fm}^2$ to 0.061 fm^2 , as compared with the observed value of $.076 \text{ fm}^2$. Similar changes were found for all the neutron hole states. The calculations also show that there are significant variations of the effective charge radii from one orbit to the next. These effective neutron mean-square charge radii have been used to calculate the change of the charge radii for the Pb isotopes. The results are compared with the experimental values¹ for $^{200}\text{Pb} \rightarrow ^{208}\text{Pb}$ in Fig. 1. The experimental data clearly exhibit odd-even staggering.

In the shell-model calculations, there is no evidence for odd-even effects even though there are differences in the radii of different single-particle orbits. The largest change occurs near the closed shell. The ground state of ^{207}Pb in the shell model is a pure $\nu p_{1/2}^{-1}$ state. The ground state of ^{206}Pb is an admixed function of states of the form $(\nu j^{-2})_{J=0}$. All orbits have smaller values of $\delta(r^2)$ than does the $p_{1/2}$ orbit, so the average $\delta(r^2)$ of ^{206}Pb must be smaller than for $(\nu p_{1/2}^{-2})$. This effect is seen in both theory and experiment, but the magnitude of the effect is not in good agreement. The theory-experiment agreement is quite good for the even isotopes, but this may be only coincidental, as discussed below.

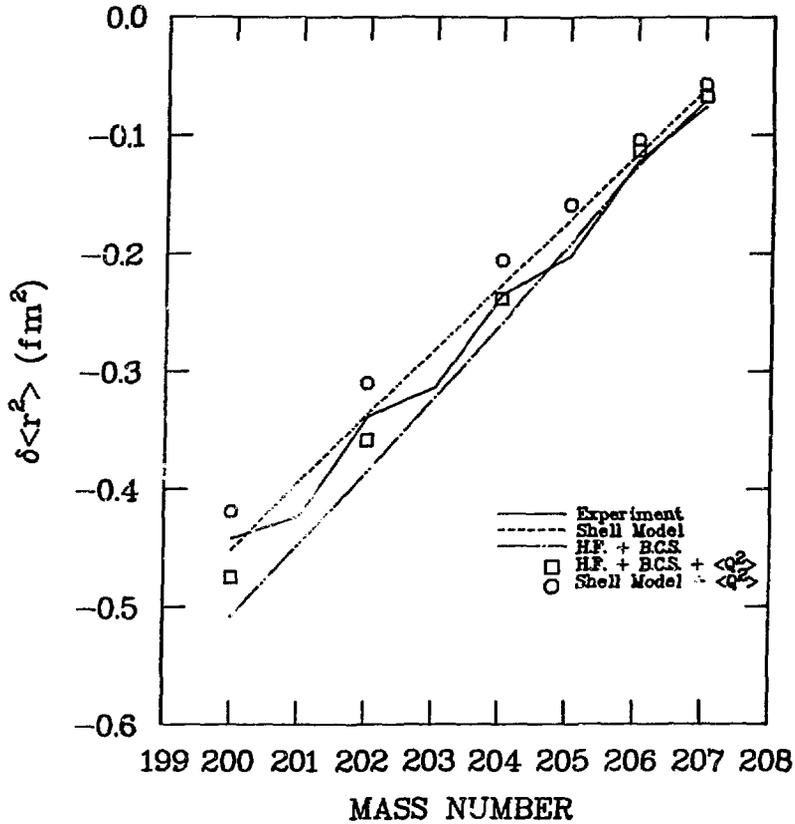


FIGURE 1. Isotope shifts, $\delta \langle r^2 \rangle$ $^{A}_{\text{Pb}}\text{-}^{208}\text{Pb}$, for the lead isotopes. The points with squares and circles are the calculated shifts which include contributions from ground-state quadrupole fluctuations.

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These calculations represent a particularly simple illustrative example of a microscopic calculation of the mean square charge radius. They introduce the way in which an effective charge radius is used in the shell-model calculation. These calculations include only the results of coupling in $J=0^+$ core states, so they include only the "monopole" effects. They are the closest thing to a first-principles microscopic calculation that is available. The results of Speth, et al.⁷ show that such calculations are very sensitive to components of the residual nuclear-nuclear force; in particular, the density dependence. The inclusion of the monopole effects are crucial to pinning down the nucleon-nucleon parameters.

Thompson⁹ has also calculated the isotope shift in the even-A Pb isotopes in a slightly different microscopic approach. He performs a Hartree-Fock calculation using the Skyrme VI force with a constraint. He uses the ^{207}Pb single-hole energies and performs a BCS calculation of the ground states of $^{198-206}\text{Pb}$. This gives a set of occupation probabilities for the valence hole orbits. He then defines the density of the given Pb isotope in terms of these occupation numbers. The Hartree-Fock calculations implicitly include, to all orders, the monopole effects that are treated in perturbation theory of Speth, and no "effective" neutron radii are needed. The BCS occupation numbers are an approximation to the exact occupations which result from the shell-model diagonalizations. Thompson's results are also shown in Fig. 1. His results are again roughly linear with particle number, with a distinctly different slope from the results using Speth, et al.'s matrix elements. Generally, Thompson's results underestimate the shifts consistently. As discussed below, this may be a positive result. In any

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case, we will see that the slope of this calculated monopole result is very important.

It is obvious that the exact structure of the shell-model wave functions in a given model space is not critical in calculations of charge radii. The bulk of the physics is in the determination of the effective single-particle charge radii to be used in the calculation, or equivalently how to estimate core-polarization effects for calculations involving nuclei away from closed shells.

PHENOMENOLOGICAL DETERMINATION OF CORE-POLARIZATION EFFECTS

The most extensive microscopic calculations of charge radii were made some years ago by Uher and Sorensen.² They treated a large number of approximately spherical nuclei in a model which combined phenomenology with microscopy. The model starts in the same fashion as a conventional shell-model calculation. An inert core is assumed and protons and neutrons are distributed in a finite set of single-particle orbit. A pairing plus quadrupole-quadrupole residual interaction is used. Instead of an exact diagonalization of an effective residual two-body interaction, they apply the techniques of BCS theory to treat the pairing interaction, and they include effects of a quadrupole-quadrupole interaction in the quasi-particle random phase approximation. They include core-polarization effects due to monopole and quadrupole deformations by procedures described below.

The monopole core-polarization effects are introduced in terms of "effective" radii for neutrons and protons. The core is treated as a compressible fluid. The potential

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seen by valence particles due to the core particles is approximated by a Woods-Saxon potential. A first-order theory of the change in the valence wave function due to a compressive change of the core is constructed, and the change in the wave function is parametrized in terms of derivatives of the assumed Woods-Saxon parameters. The parameters are chosen to fit the gross trends in the charge radii and then used to calculate changes between isotopes of nearly equal mass. It is not my intention to describe the method here. The monopole corrections are significant, but they are not the major contribution to the qualitative variations of the charge radii. The monopole corrections included by Sorensen and Uher² phenomenologically are comparable to the renormalization calculated by Speth, et al.⁷ microscopically.

The contributions due to quadrupole core deformation play a more important role in the calculation of overall qualitative features of the isotope shifts. To calculate the quadrupole polarization, Uher and Sorensen assumed the core to be an incompressible fluid which undergoes small quadrupole oscillations around a spherical shape. As a result of the zero-point motion of this oscillation, the radius of the core is altered. The mean square radius of the core is expressed in terms of a deformation parameter β , i.e.

$$\begin{aligned} \langle r^2 \rangle &= \frac{3}{5} R_{co}^2 \left[1 + \frac{5}{4\pi} \beta^2 \right] = \langle R_{co}^2 \rangle \left[1 + \frac{5}{4\pi} \beta^2 \right] \\ &\equiv \langle R_{co}^2 \rangle + \delta \langle R^2 \rangle \end{aligned} \tag{3.1}$$

where R_{co} is the radius of the liquid-drop core. In this same model, there exists a sum rule for the $B(E2)$ -values from the ground state to all excited core states, i.e.

$$\sum_i B(E2)_{gs+i} = \left[\frac{5 Z_0 e \langle R_{co}^2 \rangle^2}{4\pi} \right]^2 \beta^2 \quad (3.2)$$

where i runs over all excited core states. Then the change in the mean square radius of the core due to zero-point quadrupole fluctuations can be written in terms of the $B(E2)$ -values between the ground state and all excited states.

$$\delta \langle r^2 \rangle = \frac{1}{(Ze)^2} \frac{4\pi}{5 \langle R_{co}^2 \rangle} \sum_i B(E2)_{gs+i} \quad (3.3)$$

In order to determine the $\delta \langle r^2 \rangle$ due to zero-point fluctuations, one must know the $B(E2)$ -values in the core. The prescription of Uher and Sorensen for this is as follows. The total $B(E2)$ values of the nucleus in question are calculated in the valence space in the pairing plus quadrupole model used to calculate the mean square radius. Effective $E2$ operators for the neutrons and protons are found to reproduce the observed $B(E2)$ values. The $B(E2)$ strength due to the added effective charges is ascribed to the $E2$ transition of the core state, i.e. all the $E2$ strength not explicitly present in the shell-model calculation is assumed to arise from the decay of the first 2^+ state of the core (it is assumed all the ground state $E2$ strength goes to the first $J=2^+$ state). The deduced core $B(E2)$ -value is used to calculate $\delta \langle r^2 \rangle$ from Eq. (3.3).

Thus, Uher and Sorensen break up the calculation in two parts -- one part is the contribution of the valence particles where effective single-particle operators introduce monopole core deformation effects, and a second part which explicitly introduces a change in radius due to a change in

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quadrupole deformation in the core, as reflected in observed $B(E2)$ strengths. There is reasonable agreement between theory and experiment. For those cases where there are significant deviations from simple uniformly charged spheres, most of the deviation is accounted for by the zero-point quadrupole motion of the core. For somewhat deformed nuclei, there are significant changes in $\delta \langle r^2 \rangle$ due to properties of the valence particles, but still the largest contribution is from the core.

The Uher-Sorensen calculations did not reproduce the odd-even staggering. In two subsequent papers,² it was shown that there can be a blocking of ground-state quadrupole correlations by an odd particle quite analogous to the blocking effect of the pairing model, and this blocking was not adequately treated in the original work of Uher and Sorensen. Sorensen showed, in particular, that this effect emerged from an evaluation of $\langle Q_e^2 \rangle$ in a simple j^n model (where $Q_e = e r^2 Y_e^2$). Sorensen and Reehal² found that by performing non-perturbative calculations of $\langle Q_e^2 \rangle$, the ground-state expectation value of the square of the electric quadrupole operator, and relating the change in $\langle Q_e^2 \rangle$ between the even and odd system to a change in β_o^2 , the ground-state deformation, and thence to a change in $\delta \langle r^2 \rangle$, they could qualitatively reproduce odd-even staggering. The calculation of charge radii by Sorensen and co-workers are the most extensive microscopic calculations in the literature. They qualitatively reproduce the data and, in some cases, achieve almost quantitative agreement. Many of the largest effects arose from the inclusion of core effects in a phenomenological way and reflect little of the microscopic structure of the wave functions (with perhaps the exception of the odd-even staggering).

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Let us return to the lead isotope shifts. The shell-model calculations did not reproduce the odd-even staggering, and apparently fit the even-even shifts. The BCS-Hartree-Fock calculations were closer to the shifts for the odd systems. Following the ideas of Sorenson, the contributions of quadrupole fluctuations to the change in the charge radius between ^{208}Pb and the lead isotopes with $A = 200, 202, 204, 205, 206,$ and 207 have been calculated in a rather simple prescription. (Similar calculations for $^{201,203}\text{Pb}$ are not economically feasible.) $B(E2)$ -values for all transitions from the ground states to all excited states in the shell-model space described have been carried out. Woods-Saxon single-particle wave functions with the correct binding energies were used in the calculation. An effective neutron charge of $e_n = 1.11$ was found to reproduce the $B(E2)$ values for the 2_1^-0 transitions in $^{204,206}\text{Pb}$. This same effective charge was used in all $B(E2)$ calculations. With this effective charge, the sum of all $B(E2)$'s from the ground state to all excited states was evaluated for $^{200,202,204,205,206,207}\text{Pb}$, to obtain a value of $\langle Q \rangle^2$ for all the lead hole ground states. For the even systems, $\sim 75-80\%$ of the calculated $E2$ strength from the ground state is in the first $J=2^+$ state. For the odd nuclei, there is significant splitting of strength for low-spin final states. For high-spin final states which dominate the sum rule, again $75-80\%$ of the $E2$ strength is in the lowest state. From the calculated $B(E2)$ sums, values of $\langle \beta^2 \rangle$ and $\delta \langle r^2 \rangle$ were evaluated. The results are summarized in Table 1. There are several features evident in this table. The ground-state fluctuations increase with particle number for the even nuclei, although the increase is not linear with the number of particles. The calculations for $^{204,202,200}\text{Pb}$ are done

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TABLE I Values of $\Sigma B(E2)_{o+f}$, $\langle\beta^2\rangle$, and $\delta \langle r^2 \rangle$ for the Pb-Hole Isotopes.

	$\Sigma B(E2) \text{ e}^2\text{fm}^4$	β^2	$\delta \langle r^2 \rangle \text{ fm}^2$
A = 200	2658	.0027	0.033
202	2204	.0023	0.027
204	1730	.0018	0.021
205	1046	.0011	0.013
206	1013	.0010	0.012
207	328	.0003	0.004

in a truncation scheme described above. Thus, there is some uncertainty in the calculations due to this truncation. In the case of ^{204}Pb , where both the exact and the truncated calculations are possible, there is a negligible difference in the calculated $B(E2)$ -values. It would be surprising if this truncation effect is large in $^{202,200}\text{Pb}$. There is definite staggering in the $^{206,205,204}\text{Pb}$ calculations for β^2 and so there is some staggering in the calculated $\delta \langle r^2 \rangle$ values. If one adds the $\delta \langle r^2 \rangle$ due to deformation to the monopole shifts as calculated by Speth, et al., the deformation contributions destroy the agreement for the even systems and worsen the agreement for the odds. This is shown in Fig. 1. It is possible that a readjustment of the density dependence of the hole-particle forces could correct this deficiency. This has not been done. The Thompson Hartree-Fock result shows the theory-experiment deviation increases with decreasing mass. If the difference is due to quadrupole fluctuations, then these fluctuations must increase with decreasing mass. This trend is reproduced by the shell-model calculations in Table I. The magnitude of

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the shift is too small for the lightest isotopes. This may reflect a breakdown of the spherical shell model for the Pb hole isotopes near $A = 200$. But the difference is very sensitive to the slope of the monopole line. An improved calculation may decrease the required correction.

There are measurements of the isotope shift in ^{210}Pb , and the results are puzzling. At this meeting, Prof. Rebel has reported a value of $\delta \langle r^2 \rangle_{210-208} = 0.218(0.15) \text{ fm}^2$. If $\delta \langle r^2 \rangle_{210-208}$ is calculated using single-particle matrix elements of Speth, et al. and Kuo-Herling matrix elements, the resulting value is 0.160 fm^2 . In ^{206}Pb , the total shift is -0.122 fm^2 , and the comparable calculated value is -0.116 fm^2 . The theory does give a significantly larger shift in ^{210}Pb than in ^{206}Pb , but there remains a large difference between theory and experiment for ^{210}Pb , $\sim 0.05 \text{ fm}^2$.

The magnitude of this discrepancy depends on the accuracy with which the monopole contribution is calculated. If the discrepancy is ascribed to quadrupole fluctuations, and we use the same prescription to relate these fluctuations with $B(E2)$ values, then the isotope shift discrepancy implies a $B(E2)$ for the 2-0 in $^{210}\text{Pb} \approx 840$. If we use the Kuo-Herling wave functions for ^{210}Pb , and if we use harmonic oscillator single-particle wave functions, the implied neutron effective charge is 2.36e. If we had used harmonic oscillator wave functions to calculate ^{206}Pb $B(E2)$ values, the effective charge for the 2-0 transition in ^{206}Pb is 0.79e. (Note that in the previous calculations we used Woods-Saxon single-particle wave functions. Relative $B(E2)$ values are not sensitive to the difference between oscillator and Woods-Saxon wave functions.) The isotope shift measurement implies a much larger quadrupole fluctuation for ^{210}Pb than for ^{206}Pb . There are other measurements of $B(E2)$ values in

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^{210}Pb . The measured values are summarized in Table II. The same $B(E2)$ values have been calculated using Kuo-Herling wave functions and oscillator wave functions. The neutron effective charge needed to reproduce experiment is also shown here.

TABLE II Measured $B(E2)_{J_1 \rightarrow J_f}$ values in ^{210}Pb . The effective charge is the value to reproduce the measured $B(E2)$, using Kuo-Herling wave functions and oscillator single-particle wave functions.

J_1	J_f	$B(E2)_{i \rightarrow f}$ $e^2\text{fm}^4$	Effective Charge (e)
2	0	$103^a \pm$	0.79
4	2	$413^b \pm 66$	1.37 ± 0.14
6	4	$372^c \pm 187$	1.56 ± 0.24
8	6	$63^c \pm 7$	1.0 ± 0.06

^aRef. 10.

^bRef. 11.

^cRef. 12.

The $B(E2)$ value for the 2-0 transition is obtained from comparing cross sections for triton inelastic excitation of the first $J=2^+$ state in ^{206}Pb and ^{210}Pb . The remaining $B(E2)$ values are determined from decay widths. The experimental values for the 2-0 transition are much weaker than the value inferred by the isotope shift measurement. On the other hand, the effective charges needed to reproduce the $B(E2)$ values for the transitions from the $J=4^+$ and $J=6^+$ states are much larger than the 2-0 based effective charge. A direct measurement of the $B(E2)$ values in ^{210}Pb by Coulomb excitation would be very helpful.

We have considered only quadrupole fluctuations here,

and no consistent picture emerges. We have not considered fluctuations of other multipoles. Further study of this effect is called for.

SEMI-SELF-CONSISTENT CALCULATIONS OF CHARGE RADII

Much of the effort in the area of isotope shifts involve medium and heavy mass nuclei. More precise measurements are available there. There is interesting, albeit less precise data, on lighter systems. From a theory viewpoint, more extensive shell-model calculations are available on lighter systems. Recently, Brown, Massen, and Hodgson³ have studied nuclear charge densities of light systems in a model which is a hybrid of Hartree-Fock and shell-model calculations. As discussed briefly above, the Hartree-Fock calculations are the best microscopic calculations for determining accurate single-particle wave functions, particularly insofar as the shape of the wave function in the lead region is concerned, but they do not include effects of low-lying multi-particle, multihole excitations, and there are formal problems for non-closed-shell and odd nuclei. To determine the single-particle wave functions, Brown, et al. use the concept of a nuclear potential as did Sorensen and Uher,² but the potential is expressed as a function of the nuclear density, and self-consistency between the density and the potential is imposed. To expand, the nuclear potential is often expressed as

$$V = \left[V_0 - 2\tau_Z V_1 \frac{(N-Z)}{A} \right] f(r) + V_{ls}(r) + V_{Coul}^{direct} \quad (4.1)$$

where $f(r)$ is the usual Woods-Saxon form factor. Brown, et al. try to improve this by replacing N , Z , and A by density

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dependences $\rho_1(r) = \rho_n(r) - \rho_p(r)$ and $\rho_0 = \rho_n(r) + \rho_p(r)$, thus making the potential depend linearly on the particle densities. The total nuclear densities are expressed as

$$\rho_n(r) \equiv \sum_{n\ell j} \frac{n_n(n\ell j)}{4\pi} \left| \frac{U_{n\ell j}^n(r)}{r} \right|^2 \quad (4.2)$$

$$\rho_p(r) \equiv \sum_{n\ell j} \frac{n_p(n\ell j)}{4\pi} \left| \frac{U_{n\ell j}^p(r)}{r} \right|^2$$

where n_n and n_p are the occupation numbers of the particular orbit and $U_{n\ell j}^\sigma(r)$, ($\sigma = p, n$) are the radial wave functions of single-particle eigenstates of the nuclear potential. Thus, finally

$$V(r) = [V_0^c \rho_0^c(r) + V_0^v \rho_0^v(r) - 2t_z (V_1^c \rho_1^c(r) + V_1^v \rho_1^v(r))] F^c(r) \quad (4.3)$$

where $F^c(r) = f(r)/\rho_0^c(r)$ and c and v refer to core and valence contributions. Equations (4.2) and (4.3) are treated to self-consistency between V and $\rho_{n\ell j}$. Brown, et al.³ deal with nuclei near 160 and ^{40}Ca . The core and valence parameters V_0 and V_1 are fixed by fitting the single-particle binding energies in 170 and 220 , and in ^{41}Ca and ^{48}Ca , respectively. The occupation numbers are determined from shell-model calculations for the valence systems. Thus, a shell-model calculation of 180 gives $n_{d_{5/2}}$ in 180 . Then

$$n_{d_{5/2}}^v \equiv n_{d_{5/2}} - n_{d_{5/2}}^c \quad (4.4)$$

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From this model follows a number of interesting properties of nuclear densities which are subject to experimental test, but which we will not discuss here. It has also been applied to calculate charge radii.

Brown, et al. tried a number of systems in the sd shell and the fp shell. The first case of interest is the series of $^{16},^{17},^{18}\text{O}$ isotopes. In Table III experimental values of $\sqrt{6} \langle r^2 \rangle$ for $^{17}\text{O}-^{16}\text{O}$ and $^{18}\text{O}-^{16}\text{O}$ are compared with calculation for three nuclear structure models.

$$\begin{aligned} \text{Model 1: } & ^{17}\text{O} = \nu d_{5/2} \times ^{16}\text{O} \text{ closed shell} \\ & ^{18}\text{O} = \nu(sd)^2 \times ^{16}\text{O} \text{ closed shell} \end{aligned}$$

$$\text{Model 2: } ^{18}\text{O} = \nu(sd)^2 \times ^{16}\text{O} + \beta^2 (p_{1/2}^{-2}(sd)^4)$$

$$\text{Model 3: } ^{17},^{18}\text{O} = ^{12}\text{C} \text{ closed core} + (psd)^{5,6}$$

TABLE III Change in RMS charge radii in $^{16},^{17},^{18}\text{O}$ for the structure models described above.

	$\sqrt{6} \langle r^2 \rangle ^{17}\text{O}-^{16}\text{O}$	$\sqrt{6} \langle r^2 \rangle ^{18}\text{O}-^{16}\text{O}$
Expt.	$-.008 \pm .007$ fm	$.074 \pm 0.005$ fm
Model 1	0.006	0.010
Model 3	0.003	0.073

Experimentally, there is a negligible change in the charge radius between ^{16}O or ^{17}O , but a large one between ^{18}O and ^{16}O . The simple shell-model calculation does not reproduce this. In Model 2, only $^{18}\text{O}-^{16}\text{O}$ has been calculated and β^2 varied to fit experiment. These result in

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$\beta^2 = 0.20$. However, this β^2 is twice as large as the value of β^2 needed to fit a number of other observables in 180 . In the Model 3 with multiparticle, multihole configurations, there is very good agreement. Thus, in this case, the behavior in the charge radii seems to put some constraints on the shell-model wave function.

Similar calculations have been performed on the Ca isotopes. There is rather extensive and accurate data on rms large radii in the Ca isotopes. The data indicate that the radii increase from ^{40}Ca to ^{44}Ca , and then decrease so that the radii of ^{48}Ca and ^{40}Ca are roughly equal. There is also odd-even staggering seen. Brown, *et al.* analyze the data in terms of a model similar to Model 2 above for 180 . They assume that the wave function for ^{40}Ca is

$$^{40}\text{Ca} = \sqrt{1-\beta^2}|0\rangle + \beta |d_{3/2}^{-2} (f_{7/2} p_{3/2})^2 \rangle \quad (4.5)$$

where $|0\rangle$ is the spherical closed shell and $d_{3/2}^{-2} (f_{7/2} p_{3/2})^2$ is a two-particle, two-hole admixture. For ^{40+n}Ca , the wave function is

$$^{40+n}\text{Ca} = \sqrt{1-\beta^2} (f_{7/2} p_{3/2})^n + \beta (d_{3/2}^{-2} (f_{7/2} p_{3/2})^{n+2}). \quad (4.6)$$

They calculate the self-consistent densities as a function of $\Delta n = 2\beta^2 =$ number of $d_{3/2}$ holes. For comparison, they calculate the radii in a spherical two-orbit model $^{40+n}\text{Ca} = (f_{7/2} p_{3/2})^n$. For the simple $(f_{7/2} p_{3/2})^n$ model, the radii monotonically increase from ^{40}Ca to ^{48}Ca . Excellent agreement with experiment is obtained with the values $\beta^2 = 0.7, 0.85, 0.9, 0.53,$ and 0.15 for $^{40,42,44,46,48}\text{Ca}$, respectively. It is known that there are light nuclei near closed shells that exhibit low-lying states that do not

naturally fit in spherical shell-model pictures. These states are assumed to be based on deformed orbitals which coexist with spherical orbits. This is true, for instance, near ^{16}O and ^{40}Ca . There have been extended spherical shell-model calculations of some of these coexistence states, where the deformed states are dominated by two-particle and four-particle excitations from the closed core. One can interpret the existence of large multi-particle, multi-hole components in a given state as a manifestation of deformation and consequently an increased radius. Large multi-particle-hole admixtures imply large radii. Zuker, Buck, and McGrory¹³ and Reehal and Wildenthal¹⁴ calculated the low-lying states of the oxygen isotopes in a model with a ^{12}C core, where the $p_{1/2}$, $d_{5/2}$, and $s_{1/2}$ orbits were active. Zuker¹⁵ carried out some calculations of the Ca isotopes in a model with a ^{32}S core and particles in the $d_{3/2}$ and $f_{7/2}$ shell. It is amusing to look at the probability that the core is in the conventional shell-model configuration in the wave functions of the ground states of the O and Ca isotopes in these models. Thus, for $^{16-22}\text{O}$, what percentage of the wave function has a filled $p_{1/2}$ shell, and in $^{40-48}\text{Ca}$ what percentage of the wave function has a $d_{3/2}$ ⁸ configuration? This is done in Table IV for the ZBM wave functions¹³ of O and for the Zuker wave functions for the Ca isotopes. Also included here are some results for the hole nuclei ^{15}O , ^{38}Ca , ^{39}Ca . There are persistent features for the two sets of isotopes. If we assume our picture above that a larger closed shell component implies a smaller radius, we see a distinct size increase at the closed shell. For the even isotopes, the radius increases to the middle of the shell and then decreases towards the end of the shell. This feature is exactly what is observed in the Ca isotopes,

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TABLE IV Probability (in %) for a spherical core in the ground states of O and Ca nuclei.

150	77 ($p_{1/2}^3$)	^{38}Ca	70 ($d_{3/2}^6$)
160	66 ($p_{1/2}^4$)	^{39}Ca	73 ($d_{3/2}^7$)
170	66 "	^{40}Ca	60 ($d_{3/2}^8$)
180	47 "	^{41}Ca	72 "
190	60 "	^{42}Ca	53 "
200	55 "	^{44}Ca	52 "
210	80 "	^{46}Ca	58 "
220	85 "	^{48}Ca	70 "

and the pattern is similar to that for the β^2 extracted by Brown, *et al.*³ There is also evidence here for an odd-even staggering effect. ^{41}Ca is distinctly less deformed than are $^{40},^{42}\text{Ca}$. Results for $^{43},^{45},^{47}\text{Ca}$ are not available. For the O isotopes, 170 is the same "size" as 160 , and then there is clearly an odd-even staggering effect through the rest of the shell. The odd-even staggering here clearly reflects some combination of Pauli-blocking effects and strong pairwise correlations. It certainly reflects some of the quadrupole deformation effect discussed by Sorensen. We show the O results because the results for all the isotopes are available. More complete results for the Ca isotopes would surely reveal the same effects.

Zuker, Caurier, and Poves¹⁶ have also studied the isotope shifts in the calcium isotopes. They use an isospin-projected Hartree-Fock technique and show that by introducing an effective three-body residual force, they can reproduce the parabolic behavior of the Ca isotope shifts.

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SUMMARY

We have attempted to show some examples where microscopic shell-model calculations provide some insight into the features of data on charge radii. By combining an incompressible fluid model of ground-state quadrupole fluctuations with shell-model calculations of $B(E2)$ values, it is possible to account for some distinctive qualitative features of lead isotope shift data. It is critical to have an accurate calculation of monopole effects, and more effort is needed in this area. We have also suggested that shell-model calculations with "open cores" give qualitative insight into the data on charge radii near closed shells. Core-excitation effects could account for odd-even staggering effects in light nuclei, as well as distinctive "parabolic" features of the Ca isotope shift data. Our calculations all deal with near-closed-shell nuclei. Exact diagonalizations still cannot be extended very far from closed shells. The interacting boson model (IBM), an algebraic approximation to the exact shell model, can be extended to deformed nuclei. The IBM provides a particularly simple parameterization¹⁷ of the isotope shift data. The model has been applied to nuclei in the Ba, Ce, and Cs mass region. The only isotope shift data available on these isotopes has a particularly simple structure which does not provide a stringent test of the model. The predictions in the middle of the deformed region for these isotopes have very distinctive structures, and data in these regions could provide a severe test of the model.

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