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Simplified Analytical Solution for Free Drops during NCT for Radioactive Materials Packagings (U)

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SIMPLIFIED ANALYTICAL SOLUTIONS FOR FREE DROPS DURING NCT FOR RADIOACTIVE MATERIAL PACKAGINGS

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ABSTRACT

To ensure structural integrity during normal conditions of transport (NCT), Federal regulations in 10CFR71.71 require that the nuclear material package designs be evaluated for the effects of free drops. The vessel stress acceptance criteria for these drops are given in Regulatory Guide 7.6 and ASME Section III Code. During initial phases of the package design, the effects of the NCT free drops can be evaluated by simplified analytical solutions which will ensure that the safety margins specified in R. G. 7.6 are met. These safety margins can be verified during the final stages of the package design with dynamic analyses using finite element methods. This paper calculates the maximum impact "g" loading on the vessels using single degree of freedom models for different drop orientations. Only end, bottom, and corner drops are analyzed for cylindrical packages or packages with cylindrical ends.

INTRODUCTION

Packages used in the transportation of radioactive material must demonstrate the ability to withstand severe impacts such as those prescribed by the 10CFR71 and IAEA regulations. Design for impact loading involves both tests and analyses. Some impact loading is involved in both normal conditions of transport (NCT) and hypothetical accident conditions (HAC). These loadings are described in 10CFR71 and 10CFR71.73. Regulations for NCT require that free drops in the most damaging orientations be conducted from 1-ft or 4-ft height depending on the weight of the package. The HAC loadings require a 30-ft drop followed by a puncture test and an engulfing fire. If the package meets the containment and stress acceptance criteria for a 30-ft drop, the package is considered to be structurally robust for transportation. Drop test observations are then used to benchmark non-linear dynamic analysis. Once the dynamic model is benchmarked, various drop orientations can be studied to minimize the number of actual drop tests. In this scenario, it could be argued that since the package was able to sustain a 30-ft drop, 1 to 4-ft drops are much less damaging and the package is considered acceptable for NCT drops. This approach has the advantage that NCT drops need not be analyzed by dynamic methods which are cumbersome and time consuming. Also, since only elastic analyses are recognized for NCT loadings, the acceptability of non-linear[1] dynamic analysis by the regulators is not certain. For packages designed for on-site transfer, certain accident loadings could be eliminated by demonstrating "equivalent safety" and extremely low probabilities of accidents by risk analysis. In such a scenario, the free drop loadings during NCT become important and must be analyzed to meet the acceptance criteria.

In this paper, simplified methods based on single degree of freedom spring mass model are used to calculate peak accelerations for the NCT drops for a cylindrical shaped package (LR-56S) at Savannah River Site. These methods are judged to be conservative since energy dissipation due to friction between contacting surfaces, material damping, and plastic deformation is ignored. In addition, for compact packages, the ratio of duration of impact force to the natural period of the package is large (>3) and therefore, dynamic amplification is not a concern. Such problems can be treated by quasi-static methods. This methodology is acceptable in NUREG/CR-3966 [Nelson & Chun] and is widely used in electronic packaging industry to evaluate packages [Mindlin].

METHODOLOGY

The analytical formulation is based on idealization of the cask with a single degree of freedom system shown in Figure 1. This simplification is possible when the shipping package can be separated into a containment part and an impact limiter part. If these two parts are structurally dissimilar (i.e., when the impact material is less stiff and less dense than the containments), then many analytical techniques are valid and this variety can be exploited to control the cost of analysis. If parts of a shipping package are structurally dissimilar, coupling between the parts is

[1] It should be recognized that a non-linear analysis is necessary due to the presence of impact limiter.

simplified and the parts can be analyzed separately and recombined only through interface forces. If the shear stress is much smaller than the crush strength, a one dimensional analysis can be performed for simple configurations. This is the case for a real package analyzed in this paper.

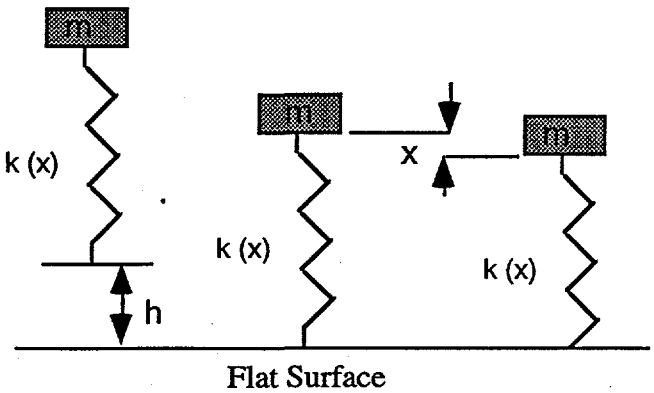

Figure 1 - Math Model

In Figure 1, k is the impact limiter stiffness and m is the combined mass of the containment vessel, the shielding and the payload. Figure 1 shows the 3 stages of the drop sequence, i.e., moment before the drop from height h, the moment (t = 0) impact limiter touches the rigid surface, and the moment after some deformation x of the impact limiter.

The free body diagram at the time of deformation x is shown in Figure 2. In Figure 2, P(x) is a generalized impact force.

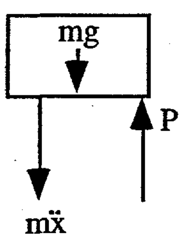

Figure 2 - Free Body Diagram

Applying Newton's law to the free body diagram gives

$$m\ddot{X} - mg = -P(x) \qquad \text{Eq. 1}$$

In this equation P(x) depends upon the spring rate characteristics of the impact limiter. R.D. Mindlin has shown that for small package weight (mg) as compared to the impact force P, this equation can be approximated to

$$m\ddot{X} + P(x) = 0 \qquad \text{Eq. 2}$$

This equation can be integrated with respect to x to give energy equation.

$$\int_0^x m\,\ddot{x}\,dx + \int_0^x P(x)\,dx = c$$

$$m\int_0^x \frac{d}{dt}\left(\frac{dx}{dt}\right)dx + \int_0^x P(x)\,dx = c$$

$$\frac{m}{2}\int_0^v d\left(\frac{dx}{dt}\right)^2 + \int_0^x P(x)\,dx = c$$

$$\frac{m}{2}v^2 + \int_0^x P(x)\,dx = c$$

At the instant of impact, x = 0, and $v = \sqrt{2gh}$. This gives c = mgh. The maximum force P is exerted when the deformation x of the impact limiter is maximum, $x = d_m$. This gives the following general equation of interest.

$$\int_0^{d_m} P(x)\,dx = mgh \qquad \text{Eq. 3}$$

If P(x) is known, maximum deflection dm can be calculated from Eq. 3.

CONFIGURATIONS CONSIDERED FOR ANALYSIS

Only simple geometries are amenable to analytical solutions. Packages with circular or rectangular cross sections can be analyzed. These packages when dropped in basic orientations, i.e., end drop, bottom drop, and corner drop, have foot prints which can be analytically analyzed. The foot prints for a cylindrical shaped package in different drop orientations are shown in Figure 3.

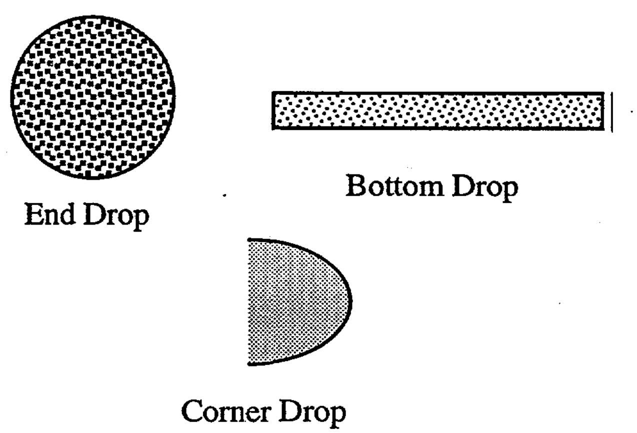

Figure 3 - Foot Prints

MATERIALS CONSIDERED FOR ANALYSIS

Since, the methodology is a zero or one dimensional analysis, only impact limiters having low shear stress as compared to crush strength can be analyzed. Some commonly used impact limiters such as redwood, balsa wood, fiberboard, and precrushed aluminum or steel honeycombs are typical candidate materials for such an analysis. P(x) depends upon the elastic properties of the impact limiter and some of the analytical approximations [Mindlin] that have been mathematically analyzed are:

- elastic-perfectly plastic
- linear elasticity
- cubic elasticity
- tangent elasticity

The force deflection curves for the above mentioned elastic behavior are schematically shown in Figure 4.

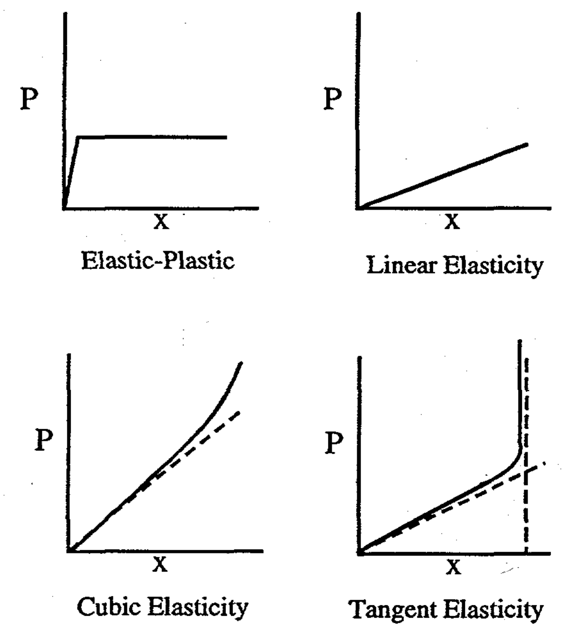

Figure 4 - Force Deflection Curves

ANALYTICAL FORMULATION FOR LR-56S

The analytical derivations are for a cylindrical shaped package LR-56S. This package is to be used for transporting liquid waste onsite.

The package uses 10 cm x 10 cm redwood and balsa wood blocks glued together as the impact limiter. The blocks are oriented so as to crush parallel to the grains. This is shown schematically in Figure 5.

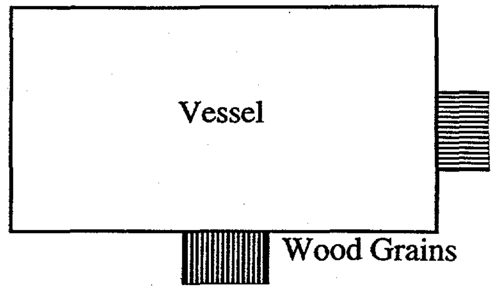

Figure 5 - Wood Grain Orientation

The load deflection curves for redwood and balsa wood [Ried et.al.] in different orientation are given in Figure 6.

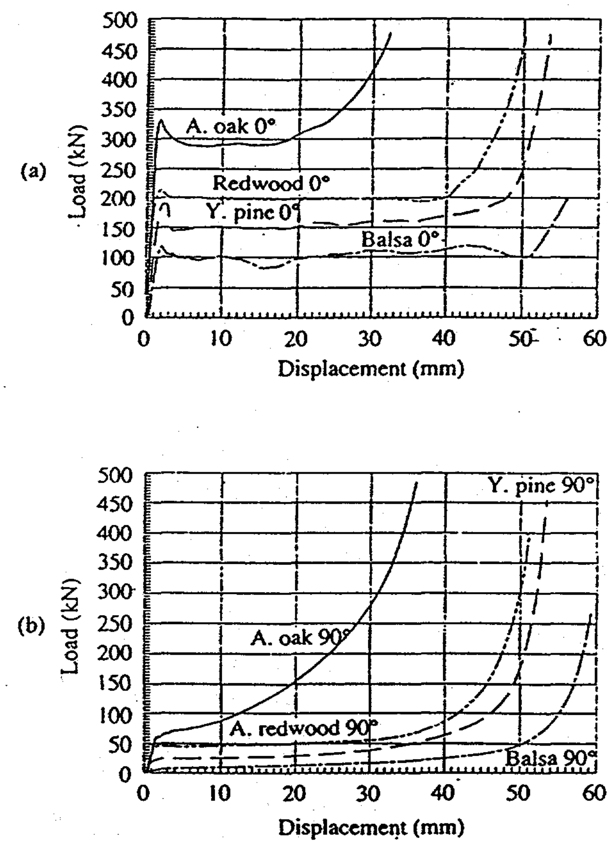

Figure 6 - Quasi-static Uniaxial Load-displacement Curves (a) along and (b) across the Grains

The curves show a well defined crush strength and negligible elastic region. The crush strength for these materials is given in Table 1.

Table 1 - Crush Strength of Redwood and Balsa Wood

Wood and Grain Orientation	Initial Crush Stress (N/mm^2)	Locking Strain
Balsa 0°	27.0	0.68
Balsa 90°	1.6	0.65
Redwood 0°	43.0	0.65
Redwood 90°	10.0	0.58

Crush to shear strength ratio along the grains for these woods is about 6.5 [Mark's]. This ratio is large enough that the effect of shear strength in the analysis can be neglected.

End Drop

The end drop is the simplest to analyze since the area of the foot print does not vary as the impact limiter is crushed. Eq. 3 can be broken into elastic and plastic parts. If X_y is the deflection at yield, the equation then becomes,

$$\int_0^{x_y} P_e(x)\,dx + \int_{x_y}^{d_m} P_p(x)\,dx = mgh \qquad \text{Eq. 4}$$

where $P_e(x)$ is the elastic part and $P_p(x)$ is the plastic part of the load-displacement curve. A review of the load-displacement curve shows that the energy absorbed in the elastic region is much smaller than in the plastic region. In addition the plastic behavior is close to perfectly plastic in nature. This is true for both the woods. If σ_c is the crush strength of combined redwood and balsa wood blocks and A is the crushed area, the Eq. 4 can be written as follows:

$$\int_0^{d_m} \sigma_c A(x)\,dx = mgh \qquad \text{Eq. 5}$$

For the end drop foot print area A is constant, therefore, Eq. 5 becomes

$$\sigma_c A\, dm = mgh$$

$$G \text{ loading} = \frac{\sigma_c A}{mg} \qquad \text{Eq. 6}$$

Bottom Drop

In the bottom drop, the foot print increases as the crush depth increases. This can be evaluated using the geometry in Figure 6.

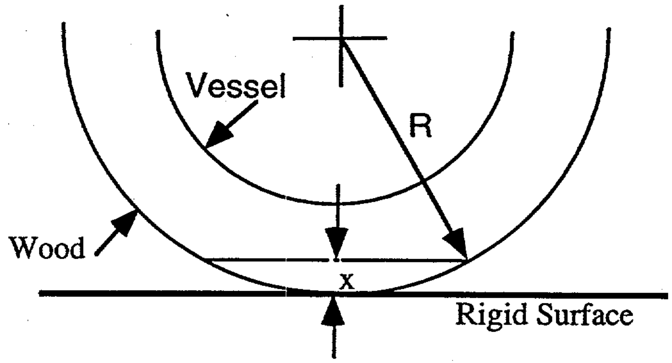

Figure 6 - Bottom Drop

The area A(x) in Eq. 5 is rectangular. Its area is given by,

$$A(x) = 2 * L * \sqrt{R^2 - (R-x)^2}$$

where L is the length of the package and x is the crush depth. The expression for A(x) can be simplified since $x << R$ and the term x^2 can be neglected. The expression for A(x) becomes,

$$A(x) = 2 * L * \sqrt{2Rx}$$

Eq. 5 can now be integrated. This gives

$$1.886 * \sigma_c L \sqrt{R}\, d_m^{1.5} = mgh$$

The above expression gives the maximum crush depth d_m. Knowing d_m, the crushed area A at $x = d_m$ can be calculated. The impact force is then given by

$$\text{Impact force} = \sigma_c A$$

$$G \text{ loading} = \frac{\sigma_c A}{mg} \qquad \text{Eq. 7}$$

Corner Drop

The corner drop in the case of cylindrical package is the drop through its center of gravity (CG) on its rim. This is the hardest configuration to analyze but it results in a softer impact due to greater depth of crushing. The corner drop schematic in Figure 7 will help in visualizing the affected crushed area.

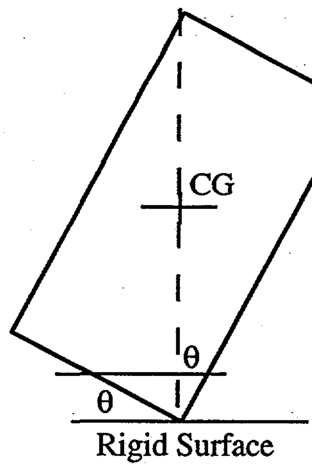

Figure 7 - Corner Drop

An exploded view of the crushed corner is given in Figure 8. The cask end is the circular end.

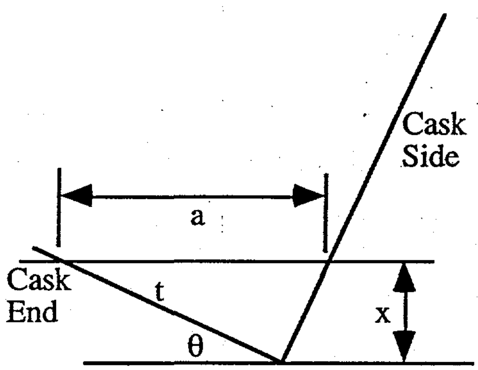

Figure 8 - Exploded View of Corner

As indicated before, the foot print in the corner drop is a half ellipse as shown in Figure 9. The dimension a in Figure 8 is the semiaxis as shown in Figure 9. 2b is the other axis of the ellipse on the circular end of the cask and can be calculated as shown in Figure 10.

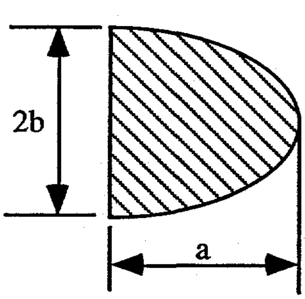

Figure 9 - Corner Drop Foot Print

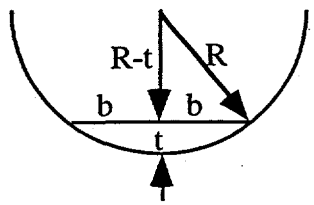

Figure 10

From Figure 8,

$$t = \frac{x}{\sin\theta}$$

$$a = x^*\tan\theta + t^*\cos\theta$$

or

$$a = x\,(\tan\theta + \cot\theta)$$

From Figure 10, b can be calculated. This gives,

$$b = \sqrt{R^2 - (R-t)^2}$$

Since t << R for small deformations for NCT drops, t^2 can be neglected. This gives

$$b = \sqrt{2Rt} = \sqrt{2R\frac{x}{\sin\theta}}$$

Area of the foot print which is a half ellipse is given by

$$A(x) = \frac{\pi ab}{2} = K\,x^{1.5}$$

where

$$K = \frac{\pi\,(\tan\theta + \cot\theta)}{2}\sqrt{\frac{2R}{\sin\theta}}$$

Eq. 5 can now be integrated. This gives,

$$2\,\sigma_c\,K\frac{d_m^{2.5}}{5} = mgh$$

This equation gives the maximum deformation dm. Once dm is known, crushed area A can be determined from the expressions given above. The impact force is then given by

$$\text{Impact force} = \sigma_c A$$

$$\text{G loading} = \frac{\sigma_c A}{mg} \qquad \text{Eq. 8}$$

MAXIMUM G LOADING FOR PACKAGE LR-56S

Maximum G loading for package LR-56S were calculated using Eqs. 6, 7, and 8. The package is designed as type B package and has double containment. Since the package is to be used for onsite transfer only, the design requirements for HAC have been reduced or eliminated by showing equivalent safety and extremely low probabilities of accidents by risk analysis. Therefore, for structural evaluation, only NCT design criteria based on 10CFR71.71 requirements are addressed. Package LR-56S weighs over 19000 kg (excluding wood), and therefore only 1-ft drops are analyzed for NCT loadings. Using the crush strength values from Table 1, maximum G values can be calculated for the different drop orientations. These results are summarized in Table 2.

Table 2 - G Loadings for LR-56S

Drop Orientation	G Loading (g)
End drop	184
Bottom drop	114
Corner drop	16

EFFECT OF DAMPING

The foregoing analytical formulation neglects structural and material damping effects on the package response. The effects of damping have been analyzed by Mindlin where it is shown that if the damping is less than 50% of the critical damping, the maximum accelerations are equal to or less than the accelerations obtained by elastic methods. The main effect of damping is to shift the acceleration peak during the impact. If the damping factor is ≤ 0.5, the peak acceleration occurs after $t = 0$, however, if the damping factor is > 0.5, the peak acceleration occurs at $t = 0$.

CONCLUSIONS

The paper presents simplified analytical solutions for basic drop orientations. Such analyses can be used for small deflections and during the initial phases of the package design. Since the analyses give conservative peak acceleration values, acceptable safety margins can be maintained when the detailed analyses are performed. The designer can use these simple methods to save time and cost.

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