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INVERSE PROBLEM FOR BREMSSTRAHLUNG RADIATION

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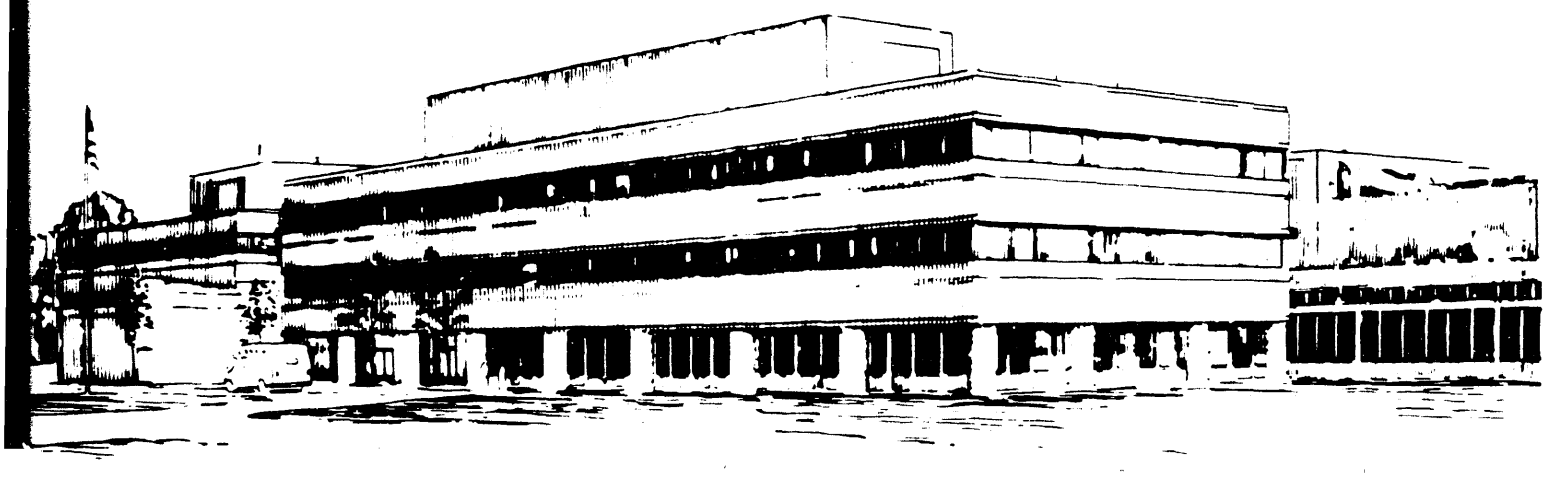

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Inverse Problem for Bremsstrahlung Radiation

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Abstract

For certain predominantly one-dimensional distribution functions, an analytic inversion has been found which yields the velocity distribution of superthermal electrons given their Bremsstrahlung radiation.

Paper

In general, it is not possible to write an analytic expression for the distribution function of electrons simply on the basis of observing their Bremsstrahlung radiation in a plasma. There is a set of circumstances, however, in which just such a data inversion is possible: Consider a homogeneous plasma of electrons and ions of charge Z_i. The Bremsstrahlung emission at energy $\hbar\omega$ is given by

$$R(\hbar\omega,\theta) \equiv \frac{dP}{d(\hbar\omega)d\theta} = \int_{v_{min}}^{\infty} d^3\mathbf{v} f(\mathbf{v}) n_e n_i v \hbar\omega \frac{d\sigma}{d(\hbar\omega)d\theta}, \tag{1}$$

where f(v) is the electron distribution function, and $d\sigma/d(\hbar\omega)d\theta$ is the Bremsstrahlung radiation cross-section. If the electrons are non-relativistic, yet are of large enough energy

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for the Born approximation to be valid ($v \gg c \cdot 2\pi Z_i/137$), then the cross-section is given by Koch and Motz (1959) as:

$$\frac{d\sigma}{d(\hbar\omega)} = \frac{16 Z_i^2 r_0^2}{137} \frac{1}{\hbar\omega} \frac{c^2}{v^2} \ln \left[\frac{(\sqrt{\frac{1}{2}mv^2} + \sqrt{\frac{1}{2}mv^2 - \hbar\omega})^2}{\hbar\omega} \right]. \tag{2}$$

In certain cases, the distribution of the most energetic electrons is primarily one dimensional, i.e. $f(\mathbf{v}) = f(v_{\|})$, where $v_{\|}$ might be the dimension along a strong imposed magnetic field. This is the case in tokamak plasmas for a slide-away discharge (Pieroni and Segre, 1975) or for discharges sustained by rf-driven currents (Fisch, 1987). Using Eq. (2) and $f(\mathbf{v}) = f(v_{\|})$ in Eq. (1), we have

$$R(s) = R_0 \int_s^\infty dx f(x) \frac{1}{x} \ln \left[\frac{x}{s} + \sqrt{\frac{x^2}{s^2} - 1} \right] = R_0 \int_s^\infty dx f(x) \frac{1}{x} \cosh^{-1}\left(\frac{x}{s}\right), \tag{3}$$

where energies are normalized such that: $x^2 \equiv \frac{1}{2}mv_{\|}^2$, $s^2 \equiv \hbar\omega$. Note that the lower bound takes into account the cutoff of electrons below the photon energy.

If we now take the derivative of Eq. (3), we get

$$\frac{dR}{ds} = R_0 \int_s^\infty dx f(x) \left[-\frac{1}{s} \frac{1}{(x^2 - s^2)^{1/2}} \right] = \frac{R_0}{s} \int_s^\infty dx \left[\frac{f(x)}{2x} \right] \frac{2x}{(x^2 - s^2)^{1/2}}, \tag{4}$$

which is the familiar form of an Abel invertible equation (Bronshtein and Semendyayev, 1985), so we can invert to yield:

$$f(x) = \frac{2x}{R_0 \pi} \int_x^\infty \frac{d}{ds} \left(s \frac{dR}{ds} \right) \frac{ds}{(s^2 - x^2)^{1/2}}. \tag{5}$$

A similar inversion can be made for a distribution which is primarily in the perpendicular direction, e.g. a distribution of highly energetic electrons created by electron cyclotron resonant heating. In this case (where we label the energy as $y^2 \equiv \frac{1}{2}mv_{\perp}^2$), the resulting inversion is:

$$f(y) = \frac{2}{R_0 \pi} \int_y^\infty \frac{d}{ds} \left(s \frac{dR}{ds} \right) \frac{ds}{(s^2 - y^2)^{1/2}}. \tag{6}$$

The restrictions on this sucessful inversion of Bremsstrahlung data are that of high energy (Born approximation), yet non-relativistic electrons, and a primarily one-dimensional distribution. All three of these conditions can be met by the electrons in an extended tail distribution of a typical tokamak discharge (1-10 keV), where the tail extends to about 5 thermal speeds. Thus it should be possible to apply experimentally this inversion technique, after first subtracting off the background emission from thermal electrons, to determine, for example, the very important features of rf-driven tokamak discharges.

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