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# A GENERAL FORMALISM FOR THE TRANSITION MATRIX OF NUCLEAR REACTION THEORY[1]

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In this past decade there has been an increased activity in the development of new methods to compute resonant cross sections.[3-6] The usual approach deals with the solution of a set of coupled radial Schrödinger equations (the multichannel equations), involving matrix elements of the nuclear Hamiltonian in the channel eigenfunction representation.[7] The multichannel equations are solved by expansion in either R-matrix states or shell-model eigenfunctions.[8-10] In the first instance, the series obtained shows a poor rate of convergence and several prescriptions have to be introduced to compensate for truncation effects.[8,9] Once the pertinent R-matrix is obtained, one has to perform a matrix diagonalization to obtain the collision matrix. The results obtained depend strongly on the values chosen for the channel radii and the boundary condition numbers.[5] The second method of solution suffers from the difficulty that the shell model eigenfunctions do not satisfy the appropriate boundary conditions.[10]

To obviate these problems, we have developed a general formalism for the transition T-matrix based on the use of Bloch's L-operator[11] and invariant imbedding techniques.[12] The result is a matrix-integro-differential equation for the T-matrix as a function of $\tau$, where $\tau$ is any of the parameters entering in the nuclear reaction formalism.

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We have

$$\frac{d}{d\tau} T_{cc'}(r_\alpha|r_\beta,\tau) = \sum_{c''} \Bigg[ i\eta_c\eta_{c'} \left(\frac{2M_{c''}}{\hbar^2}\right) \frac{dr_>}{d\tau}$$

$$+ \left[\frac{d}{d\tau} \ln(\eta_c\eta_{c'})\right] T_{c''c'}(r_\alpha|r_\beta,\tau)\delta_{c''c}$$

$$- \left(\frac{2M_{c''}}{\hbar^2}\right) B_{c''} \left[ \frac{\eta_c}{\eta_{c'}} T_{c''c}(a_{c''}|r_\beta,\tau) \frac{dr_\beta}{d\tau} + \frac{\eta_c}{\eta_{c''}} T_{c'c''}(r_\alpha|a_{c''},\tau) \frac{dr_\alpha}{d\tau}\right]$$

$$+ \sum_{c'''} \left( \left(\frac{2M_{c'''}}{\hbar^2}\right) \frac{\eta_{c'}}{\eta_{c'''}} \int_{r_\alpha}^{a_{c'''}} dr[E_{c'''}\delta_{cc'''} - U_{c''c'''}(r,\tau)]\, T_{c'''c}(r|r_\beta,\tau) \frac{dr_\beta}{d\tau}\right.$$

$$\left. + \frac{dr_\alpha}{d\tau} \frac{\eta_c}{\eta_{c'''}} \int_{r_\beta}^{a_{c''}} dr[E_{c'''}\delta_{cc'''} - U_{c''c'''}(r,\tau)]\, T_{c'''c'}(r|r_\alpha,\tau) \right)$$

$$-i \sum_{c'''} \int_0^{a_{c''}} dr \frac{T_{cc''}(r_\alpha|r,\tau)}{\eta_{c''}(r_\alpha)} \left\{\frac{d}{d\tau}[E_{c'''}\delta_{cc'''} - U_{c''c'''}(r,\tau)]\right\} \frac{T_{c'''c}(r|r_\beta,\tau)}{\eta_{c'''}(r_\beta)}$$

$$-i \sum_{c'''} (\eta_{c''}\eta_{c'''})^{-1} T_{cc''}(r_\alpha|a_{c''},\tau) \left\{ \left[\left(\frac{2M_{c''}}{\hbar^2} B^2_{c'''} + E_{c'''}\right)\delta_{c''c'''} - U_{c''c'''}(a_c|\tau)\right] \frac{da_{c''}}{d\tau}\right.$$

$$\left. + \frac{dB_{c''}}{d\tau}\right\} T_{c'''c'}(a_{c''}|r_\beta,\tau) \Bigg] \quad , \qquad (1)$$

where $r_>$ is the largest of the two radial coordinates, $r_\alpha$ and $r_\beta$, and $B_c$ is the complex, momentum dependent Kapur-Peierls[13] boundary conditions, and

$$\eta_c = \left(\frac{\hbar^2}{M_c}\frac{P_c}{a_c}\right)^{1/2} . \tag{2}$$

The sums are carried out over the intermediate channels; $P_c$ is the penetration factor, $a_c$ is the channel radius, and $U_{c''c''}(r,\tau)$ is the channel potentials. By discretization of the radial integrals in Eq. (1), one obtains a set of Ricatti equations which, together with the proper initial conditions, defines an initial value problem for the transition matrix. Alternatively one can expand the T-matrix in shell-model eigenfunctions. In this case one makes $\tau = B_c$, and one marches the solution from $\tau_o$ equal to the shell-model logarithmic derivative at the channel radius to the correct boundary conditions, $B_c$.

As one of the many applications of the present formalism, we show a method to generate the Kapur-Peierls resonance parameters from a set of R-matrix parameters. The poles and residues of the R-matrix derive from an eigenvalue problem in which the boundary conditions, $b_c$, are real numbers. The corresponding magnitudes in the Kapur-Peierls theory arise from complex momentum dependent boundary conditions. Again we make $\tau = B_c$, $\tau_o = b_c$, and expand the T-matrix in terms of its residues, $g_{\lambda c}$,

and poles $\varepsilon_\lambda \equiv \mu_\lambda - iV_\lambda$. Introduction of this expansion into Eq. (1) yields:

$$\frac{d}{dB_c} g_{\lambda c} = -\frac{1}{2} \sum_{c''} \sum_{\lambda' \neq \lambda} (\varepsilon_{\lambda'} - \varepsilon_\lambda)^{-1} (\eta_c \eta_{c''})^{-1} [g_{\lambda c''} g_{\lambda' c} + g_{\lambda c} g_{\lambda' c''}] g_{\lambda' c} \quad , \tag{3}$$

and

$$\frac{d}{dB_c} \varepsilon_\lambda = \sum_{c''} (\eta_{c''} \eta_c)^{-1} g_{\lambda c} g_{\lambda c''} \quad , \tag{4}$$

with the initial conditions

$$g_{\lambda c} (B_c = b_c) = \left(\frac{P_c}{ka_c}\right)^{1/2} \Gamma_{\lambda c}^{1/2} \quad (k = \text{momentum, cm}^{-1}) \tag{5}$$

and

$$\varepsilon_\lambda (E_c = b_c) = E_\lambda \quad . \tag{6}$$

Numerical solutions can be obtained easily by the Runge-Kutta technique for any value of the angular momentum. To obtain useful analytical approximations, one can expand residues and poles in Taylor series around $B_c = b_c$. The derivatives $[d(n)/dB_c^n]_{b_c}$ are then computed from Eqs. (3) and (4). As a check this procedure has been applied for S-wave neutrons ($b_c = 0$), yielding the Adler-Adler[14] formulae for the poles and residues of the collision matrix.

The main advantages of the present nuclear reaction formalism are:

(a) It deals directly with "observable" quantities such as the T-matrix or its residues and poles. The solution of Eq. (1) by discretization preserves the unitarity of the collision matrix,

(b) One has a large degree of flexibility in the choice of the boundary conditions. Hence, one can use shell-model function or R-matrix states indistinctively,

(c) It does not involve time-consuming iterative calculations, and

(d) It is very suitable for the performance of a parameter search procedure.

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