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CONF-730312--2

SOME ASPECTS OF THE DWBA AS APPLIED TO HEAVY-ION REACTIONS*

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Let me start by saying that I have very little experience at computing heavy ion (HI) cross sections, by the DWBA or by other methods (except for the very minor claim to have been involved with some of the earliest applications of the DWBA to HI inelastic scattering^{1,2} and to the "exact" calculation of finite-range effects in a HI transfer reaction¹). This gives me the advantage of having a detached attitude toward the subject!

I do not wish to discuss in detail the relative merits of the DWBA versus a semi-classical approach. Clearly, the DWBA (as we usually understand it) is only applicable if the transition is predominantly a one-step (first order) process (heavy arrow in Fig. 1). If processes involving more than one step (inelastic or transfer, broken arrows in Fig. 1) are important, more is required. For light ions, so-called coupled-channel DWBA (or CCBA) calculations have been used,³ but the large number of partial waves often involved for HI and the probable importance of finite-range effects make the use of CCBA for HI rather difficult with currently available computing facilities (but see, however, the contributions to this symposium from Ascutto and Glendenning, and Low and Tamura). (A further complication with heavy ions is the possibility of excitations of both members of the pair.) Then we must resort to something else, like the semi-classical coupled channels described by Winther, et al.⁴

Even when the DWBA is appropriate, there is no question of the value of the semi-classical approach for gaining an understanding of the "physics" of what is going on. Nonetheless, for detailed comparisons with data, my

* Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

feeling is that, as long as the available computers will allow it, the usual partial wave approach of the DWBA is worth doing so as to give more control over the uncertainties in the calculation. I feel that as long as one can do "exactly" a calculation for a given model, one should do so; any deficiencies of the results can then be more readily identified with the basic physical assumptions of the model used (in the case of DWBA, the assumption of one-step and the use of optical potentials).

The application of the DWBA to HI reactions is a rapidly developing field -- some very interesting developments are being presented to this symposium -- so rather than attempt a "survey" I will restrict myself to setting the scene for others by outlining the formalism, defining some terminology, and drawing attention to a few features which seem to be of current interest. Consequently I have not attempted to compile an exhaustive list of references. I also refer you to detailed discussions given in the proceedings of other recent meetings.⁵⁻⁸

First, notation; not very basic but sometimes a source of confusion. Because the theory is an extension from that used for light ion reactions (especially deuteron stripping), I have continued to use the same notation.^{9,10} I must admit that it is not so mnemonic for heavy ions, where target and projectile are comparable in size; in this respect, Goldfarb's choice (G),¹¹ based upon two heavy "cores" c_1 and c_2 , has advantages. Another commonly used choice is due to Tobocman (T),¹² (but note a different version is used in Ref. 13). These three are:

$$\begin{array}{lll}
 \text{S: } a(= b+x) + A & \rightarrow B(= A+x) + b, & \text{e.g. } A(a,b)B \\
 \text{G: } a_1(= c_1+t) + c_2 & \rightarrow a_2(= c_2+t) + c_1, & \text{e.g. } c_2(a_1, c_1)a_2 \\
 \text{T: } I(= A+N) + B & \rightarrow F(= B+N) + A, & \text{e.g. } B(I,A)F.
 \end{array}$$

The transferred particle is x, t, or N respectively. To avoid confusion for myself, I shall continue to use my (S) notation.

Next, what is involved in the DWBA? The transition amplitude for the $A(a,b)B$ transfer reaction has the well-known form (of course, slight variations of this form have been introduced from time to time),

$$T = \int \frac{dr_{bB}}{r_{bB}} \int \frac{dr_{aA}}{r_{aA}} \chi_{bB}^{(-)*}(k_{bB}, r_{bB}) \langle Bb | V | Aa \rangle \chi_{aA}^{(+)}(k_{aA}, r_{aA}). \quad (1)$$

(Here, and in what follows, I will retain explicitly two subscripts so as to make perfectly clear which two particles the vectors refer to. This somewhat clumsy notation is often simplified in the literature.) The χ 's are distorted waves, generated from optical potentials, U_{aA} in the entrance channel and U_{bB} in the exit channel. The usual assumption is that the U are to be chosen so as to reproduce the observed elastic scattering in the corresponding channels. These model potentials U are almost invariably simple functions only of the relative distance between the centers of the colliding pair and contain no reference to internal degrees of freedom. It might be felt that this is very unreasonable for two heavy ions, and certainly the true wave function is much more complex when the ions begin to overlap. Fortunately the direct reactions tend to take place while the ions are still well separated because of strong absorption and/or Coulomb repulsion. In this region the optical potentials, by definition, tend to give a good account of the wave function of relative motion. A possible exception is that direct inelastic excitation (e.g. Coulomb excitation) may occur as the ions approach one another. This would appear as part of the absorption in the optical model, but these parts of the wave function may still contribute significantly to the transfer process.³ If the transfer itself may still be treated in first-order, the expression (1) may be generalized to include the inelastic scattering in the distorted waves, giving the so-called CCBA.³

Another consequence of the reaction taking place while the ions are still largely separated is the possibility of generating the tails of the optical potentials needed by simple folding procedures. For example, we take an effective nucleon-nucleon interaction $v(\underline{r}, \underline{r}')$ and construct the potentials

$$U_{aA}(\underline{r}_{aA}) = \int d\underline{r} \int d\underline{r}' \rho_A(\underline{r}) \rho_a(\underline{r}') v(\underline{r}, \underline{r}'),$$

etc., where $\rho_i(\underline{r})$ is the density distribution of nucleus i . This seems to be a reasonable procedure for the important tail of the real part of the potential; the imaginary part still has to be treated phenomenologically, determined by fitting to elastic scattering. (It is also an attractive idea to use such potentials for inelastic scattering near the Coulomb barrier, where we see interference between the Coulomb interaction and the tail of the nuclear potential.) The shapes of such potentials in the outer region are

not necessarily the same as the commonly used Woods-Saxon.

The nuclear matrix element (or "transfer function") represents integration over internal coordinates which are independent of the channel coordinates r_{aA} and r_{bB} . Hence it remains a function of r_{aA} and r_{bB} ,

$$\langle Bb|V|Aa\rangle = G(r_{aA}, r_{bB}). \quad (2a)$$

For a simple transfer between inert constituents, this transfer function is

$$\langle Bb|V|Aa\rangle \rightarrow \phi_B^*(r_{xA}) V \phi_a(r_{xb}) \quad (2b)$$

where the ϕ 's represent the relative motions of the nucleon (or cluster) x bound to the "cores" A or b . In a more realistic situation there will be spectroscopic factors for the separations $a \rightarrow b + x$ and $B \rightarrow A + x$, and, in general, a sum of terms like (2b), but the overall structure does not change.

In the post-interaction form, V becomes

$$V = V_{bB} - U_{bB} = V_{bx} + V_{bA} - U_{bB}, \quad (3)$$

$$\approx V_{bx}$$

where the last step represents the usual assumption that the dominant term is the potential V_{bx} which binds $b + x$ to form a . The corresponding assumption for the prior-interaction form yields $V \approx V_{xA}$, the potential binding $A + x$ to form B . (Before making this approximation, the post and prior forms of the amplitude may be shown to be equal. If the potentials U_{aA} and U_{bB} are similar, as they would be if x were small compared to the heavy cores b and A , this post-prior equality should be preserved approximately. Hence a comparison of post and prior calculations may provide one test of the validity of any additional approximations made.¹¹) There is the possibility that the neglected interaction terms may be important in some cases, such as for the transfer of clusters between light nuclei.¹⁴

There is a measure of uncertainty over the choice of the interactions V and the optical potentials U . Remembering that the amplitude (1) is an approximation (a model, if you will), it is not clear that the optimum choice

of interactions is the simple one usually made. For example, in general the V_{ij} could be complex. However, it is meaningless to start making arbitrary, uncontrolled, changes in these quantities without some very good reason.¹⁵ Hence I would strongly recommend sticking to the usual assumptions until such reasons come along.

Even when fitted to the elastic scattering, the optical potentials are not unambiguous -- we encounter the well-known ambiguities for strong-absorption scattering. These potentials will differ in yielding different wave functions χ in the nuclear interior. Insofar as these interior regions do not contribute to the transfer reaction, the various potentials may be expected to yield the same transfer cross sections.

Another assumption, not made explicit above, is that one may ignore exchange effects due to antisymmetrization between the colliding nuclei (except for those implicit in the phenomenological optical potentials). Again these effects may be important for light nuclei.¹⁴ They may also result in the explicit appearance of transfer contributions to the elastic scattering of two nuclei of similar mass,¹⁶ such as $^{12}\text{C} + ^{13}\text{C}$ or $^{16}\text{O} + ^{12}\text{C}$.

When the transferred particle x is not a single nucleon but rather a cluster, it is usually assumed for computational convenience that V_{bx} (or V_{xA}) can still be represented by a potential dependent only on the position of the center of mass of the cluster. Some exceptions to this have been reported.¹⁷

The coordinates involved in the amplitude (1) are shown in Fig. 2. The amplitude is an integration over two independent coordinate vectors; this is part of the difficulty in evaluating it exactly. The other part results from the need to express all the wave functions in terms of the same two vectors. Exact evaluation tends to use the pair r_{aA} and r_{bB} because these enter the distorted waves which one would prefer not to tamper with (but see the contribution to this symposium by McMahan and Tobocman). Then the matrix element (2), with $V \rightarrow V_{bx}(r_{xb})$, must be re-expressed through the relations

$$r_{xA} = \alpha[r_{aA} - \gamma r_{bB}], \quad r_{xb} = \alpha[\delta r_{aA} - r_{bB}] \quad (4)$$

where

$$\alpha = \frac{aB}{x(A+a)}, \quad \gamma = \frac{b}{a}, \quad \delta = \frac{A}{B}$$

and the letters denote the masses of the corresponding particles. The

technology for doing this is a little complicated and involves some multipole expansions because, in general, the ϕ 's are not scalars. It is well understood,⁹ but we will not go into it here.

When approximations are to be introduced, other choices of variables may be more convenient. For example, in terms of \tilde{r}_{bA} and \tilde{r}_{xb} ,

$$\begin{aligned}\tilde{r}_{aA} &= \tilde{r}_{bA} + \frac{x}{a} \tilde{r}_{xb}, \\ \tilde{r}_{bB} &= \tilde{r}_{bA} - \frac{x}{B} \tilde{r}_{xA} = \frac{A}{B} \tilde{r}_{bA} - \frac{x}{B} \tilde{r}_{xb}.\end{aligned}\quad (5)$$

The last term in each case is due to the center of mass of the composite system being shifted (by "recoil") from the center of mass of the heavy core; this shift is small when x is one (or a few) nucleons and a, B are heavy nuclei. If we drop these terms from the arguments of the distorted waves,

$$\tilde{r}_{aA} \approx \tilde{r}_{bA}, \quad \tilde{r}_{bB} \approx \frac{A}{B} \tilde{r}_{bA}, \quad (6)$$

we get the so-called no-recoil approximation, and the amplitude (1) can be written

$$T_{\text{no recoil}} = \int \chi_{bB}^{(-)*} \left(k_{bB}, \frac{A}{B} \tilde{r}_{bA} \right) F(\tilde{r}_{bA}) \chi_{aA}^{(+)} \left(k_{aA}, \tilde{r}_{bA} \right) d\tilde{r}_{bA} \quad (7)$$

which has the same form as the usual so-called zero-range DWBA amplitude (but it does not involve a zero-range approximation). With (2b) and (3), the no-recoil transfer function F is, without further approximation,

$$F(\tilde{r}) = \int \phi_B^*(\tilde{r} + \tilde{r}') V_{bx}(\tilde{r}') \phi_a(\tilde{r}') d\tilde{r}'. \quad (8)$$

A technique for evaluating (8) in general has been given by Sawaguri and Tobocman;¹² an approximate treatment for use at "low" energies (below the Coulomb barrier) is also available.¹⁸

Once $F(\tilde{r})$ is available, standard ("zero range") DWBA codes may be used to evaluate (7). This is a great advantage. However it was pointed out long

ago¹⁹ that the no-recoil approximation (6) may easily cause large errors in the phases of the distorted waves. One way to see this is to write the distorted waves as

$$\begin{aligned} \chi_{\vec{k}, \vec{r}+\delta\vec{r}} &= e^{i\delta\vec{r}\cdot\vec{\nabla}} \chi_{\vec{k}, \vec{r}} \\ &\approx e^{i\delta\vec{r}\cdot\vec{K}(\vec{r})} \chi_{\vec{k}, \vec{r}} \end{aligned} \quad (9)$$

where $\vec{K}(\vec{r})$ is the local momentum at the point \vec{r} . Then (1) again reduces to the form (7) except that the effective transfer function is now

$$F(\vec{r}) \rightarrow F_{\text{eff}}(\vec{r}) = \int e^{iQ(\vec{r})\cdot\vec{r}'} \phi_B^*(\vec{r}+\vec{r}') V_{\text{bx}}(\vec{r}') \phi_a(\vec{r}') d\vec{r}' \quad (10)$$

where

$$Q(\vec{r}) = \frac{\vec{x}}{B} K_{\text{bB}}(\vec{r}) + \frac{\vec{x}}{a} K_{\text{aA}}(\vec{r}) \quad (11)$$

introduces the extra phase due to the recoil effects. The classical picture of the transfers occurring predominantly for grazing collisions in which initial and final orbits are well matched tells us that the important K_{aA} and K_{bB} are parallel and similar in magnitude, also that the important \vec{r} are of order of the sum of the two nuclear radii. Then the main contributions to the overlap integral (10) will be for $\vec{r}' \approx R_a$, the radius of the nucleus a. The K will be somewhat less than the asymptotic momenta k because of the Coulomb barrier, but still of the same order. Consequently quite large phase shifts may be involved, of the order of the momentum per nucleon times R_a . Further, the phase shift will increase as the energy increases since the K will increase.

The expectation that these recoil effects may be important is further supported by complete calculations of the amplitude (1) which do not use the approximation (6) and some of which are reported to this symposium by DeVries. Whether the full calculations²⁰ can be avoided successfully, perhaps by the use of approximate corrections,²¹ remains to be seen. A number of authors have tried very simple prescriptions^{11,22} which, even if not very accurate, did give some indication that the recoil effects were important. A typical example¹¹ was to assume the transfer took place along the line of centers (especially for sub-Coulomb collisions), at the distance d of closest approach. Hence in Eqs. (5) instead of neglecting \vec{r}_{xb} we put it parallel to \vec{r}_{ba} with a

magnitude

$$\tilde{r}_{xb} = -(R_a/d)\tilde{r}_{bA},$$

where R_a is the radius of nucleus $a = b + x$. Then instead of the no-recoil approximation (6) we have

$$\tilde{r}_{aA} \approx \tilde{r}_{bA} \left(1 - \frac{x}{a} \frac{R_a}{d} \right), \quad \tilde{r}_{bB} \approx \frac{A}{B} \tilde{r}_{bA} \left(1 + \frac{x}{A} \frac{R_a}{d} \right).$$

For sub-Coulomb collisions these can be translated¹¹ into effective wave numbers k'_{aA} and k'_{bB} , and for these collisions the approximation is moderately successful.¹¹ At higher energies and for light targets it is not so useful. Part of the reason is that any colinear approximation like this automatically neglects the unnatural parity terms to be discussed below.

The angular momentum selection rules are determined by the nuclear matrix element or transfer function, Eq. (2) etc. Denote the spin of nucleus i by I_i ; then the changes or transfers of angular momentum induced by the gain or loss of the transferred particle x are defined as*

$$\begin{aligned} \tilde{J}_{BA} &= I_B - I_A, & \text{or} & \quad |I_B - I_A| \leq J_{BA} \leq I_B + I_A \\ \text{and} & & & & & & & & & (12) \\ \tilde{J}_{ab} &= I_a - I_b, & \text{or} & \quad |I_a - I_b| \leq J_{ab} \leq I_a + I_b. \end{aligned}$$

The difference L between the two spin changes J_{ij}

$$\tilde{L} = \tilde{J}_{BA} - \tilde{J}_{ab}, \quad \text{or} \quad |J_{BA} - J_{ab}| \leq L \leq J_{BA} + J_{ab} \quad (13)$$

is taken from the orbital angular momentum of relative motion of the colliding pair,

* Reluctantly, a change of notation is introduced here, but I felt uncomfortable with the notation I have previously used when a, b were light ions.^{9,10} The correspondence with this other notation is: $I_A = J_A$, $I_B = J_B$, $I_a = s_a$, $I_b = s_b$, $J_{BA} = J$ or j , $J_{ab} = S$ or s , $L = L$ or l .

$$\underline{L} = \underline{L}_{aA} - \underline{L}_{bB} \quad (14)$$

(In light ion reactions it is this L-transfer which characterizes the angular distributions.) Thus the transfer function G, Eq. (2), behaves under rotations of the coordinate system like an angular momentum L (or a sum of such terms if more than one L is allowed by the selection rules (12) and (13)). Further, its parity π is determined by the internal parities π_i of the four nuclei,

$$\pi = \pi_A \pi_B \pi_a \pi_b \quad (15)$$

The selection rules (12)-(15) are general. Further selectivity may be imposed by structure considerations. Suppose the transferred particle x is in a "single particle" orbit with $\underline{j}_a = \underline{l}_a + \underline{s}_x$ in nucleus a, and in an orbit with $\underline{j}_B = \underline{l}_B + \underline{s}_x$ in B, if x has a spin s_x . Then we are restricted to $J_{BA} = j_B$ and $J_{aA} = j_a$ while

$$\underline{L} = \underline{j}_a - \underline{j}_B \quad \text{or} \quad |j_a - j_B| \leq L \leq j_a + j_B \quad (16)$$

Further, the orbital character of L ensures that also

$$\underline{L} = \underline{l}_a - \underline{l}_B \quad \text{or} \quad |l_a - l_B| \leq L \leq l_a + l_B \quad (17)$$

The transfer function then has the form of Eq. (2b) with the ϕ 's being the corresponding single-particle wave functions. (Note that V is a scalar.)

The parity change is

$$\pi = (-)^{l_a + l_B} \quad (18)$$

However, nota bene, in general we need not have $\pi = (-)^L$. The transfer function (2) is a function of two variables and hence its parity is not tied to its rotational properties. This is no longer true when the no-recoil approximation is used; the transfer function (8) is a function of one variable, hence a term with angular momentum L must be proportional to $Y_L^M(\hat{r})$ and have parity

$\pi = (-)^L$. This may be called "natural parity". Recoil effects allow "unnatural parity" terms with L such that $\pi = (-)^{L+1}$. A simple example of this occurs when both ϕ_a and ϕ_b are p-states and hence

$$\phi_a(\underline{r}_{\underline{xb}}) \sim \underline{r}_{\underline{xb}}, \quad \phi_b(\underline{r}_{\underline{xA}}) \sim \underline{r}_{\underline{xA}}.$$

We can form products with $L = 0$ and $L = 2$ with natural parity; e.g., the scalar $L = 0$ is simply $\underline{r}_{\underline{xb}} \cdot \underline{r}_{\underline{xA}}$. We can also form the unnatural parity $L = 1$ axial vector $\underline{r}_{\underline{xb}} \times \underline{r}_{\underline{xA}}$. However, as soon as we make the two vectors parallel (or integrate over the angle between them as in Eq. (8)) this unnatural parity term vanishes. Consequently the no-recoil approximation loses these terms; DeVries has more to say about that at this symposium.

The approximate treatment of recoil embodied in Eqs. (9) and (10) does allow the unnatural terms to reappear, for example through the first, odd parity, term in the expansion of the phase factor,

$$e^{i\mathbf{Q} \cdot \underline{r}'} = 1 + i \mathbf{Q} \cdot \underline{r}' + \dots \quad (19)$$

Keeping terms to this order (with the additional approximation $\underline{k}_{\underline{bb}} \approx \underline{k}_{\underline{bb}}$, $\underline{k}_{\underline{aA}} \approx \underline{k}_{\underline{aA}}$) has been suggested by Nagarajan²¹ as an approximate treatment of the recoil effects. It may be appropriate at this point to clear up a misunderstanding of semantic origin. Nagarajan states²¹ that an exact calculation of recoil effects "will violate all selection rules (Eqs. (16)-(18)) and there is no limitation on the ℓ -transfer". His statement is true for his definition of ℓ -transfer; however, this is not the usual definition. It seems to me that the useful ℓ -transfer refers to the rotational properties of the internal nuclear matrix element or transfer function (2), and is the L defined in Eqs. (13) and (17). This is the quantity of interest for nuclear spectroscopy, as Eq. (17) shows in particular. Nagarajan's ℓ -transfer characterizes the rotational properties of the effective transfer function (10); besides the angular momentum in the nuclear wave functions this also receives contributions from the relative motion (distorted waves) through the phase factor $\exp(i \mathbf{Q} \cdot \underline{r}')$. Clearly this latter factor contains all partial waves from zero to infinity, hence Nagarajan's comment. This comes about because with the no-recoil form (6) and (7) the distorted waves describe the relative motion of A with respect to b, instead of A with respect to the center of mass of $a = b + x$ (and of b

with respect to A instead of $B = A + x$). The relative angular momenta "lost" in this way are then incorporated into the phase factor in the effective transfer function (10).

In some transfer reactions with light ions, it has been found to be important to treat the form factor (transfer function) correctly in the tail region. This is even more true for heavy ions because the transfers take place at relatively large separations, and especially true for the transfer of clusters. For single nucleon transfer, the shape of the tail of the transfer function is largely determined by the separation energies, however its magnitudes depends upon the short-range behavior of the bound-state wave functions and the interaction. It is at this point that the simple picture of an inert core plus an active nucleon, which lies behind the form (2b), may not be adequate. These problems are exacerbated when more than one nucleon is transferred and, again, have already been encountered for light ion reactions (e.g. for 2-nucleon transfer). Work on these problems is only just beginning.^{17,23} In particular, when dealing with the transfer of a cluster like an "alpha particle", it is tempting to think of the cluster as having an independent structure like that of a free alpha while the transfer takes place; this allows one to assign a definite form to the wave function for the center of mass of the cluster. But this is almost certainly wrong. To what degree it is wrong we do not yet know.

In this context, the new approach of Bonche and Giraud²⁴ is of interest. They use a generator-coordinate description of the distorted waves, within the framework of the DWBA. This is equivalent to a two-center shell model description and apparently it greatly simplifies the calculation of the transfer function. Present calculations assume the transferred cluster lies at the center of mass of the two heavy cores, which is probably not adequate for accurate results. A more complete description of this method has just been published.²⁴

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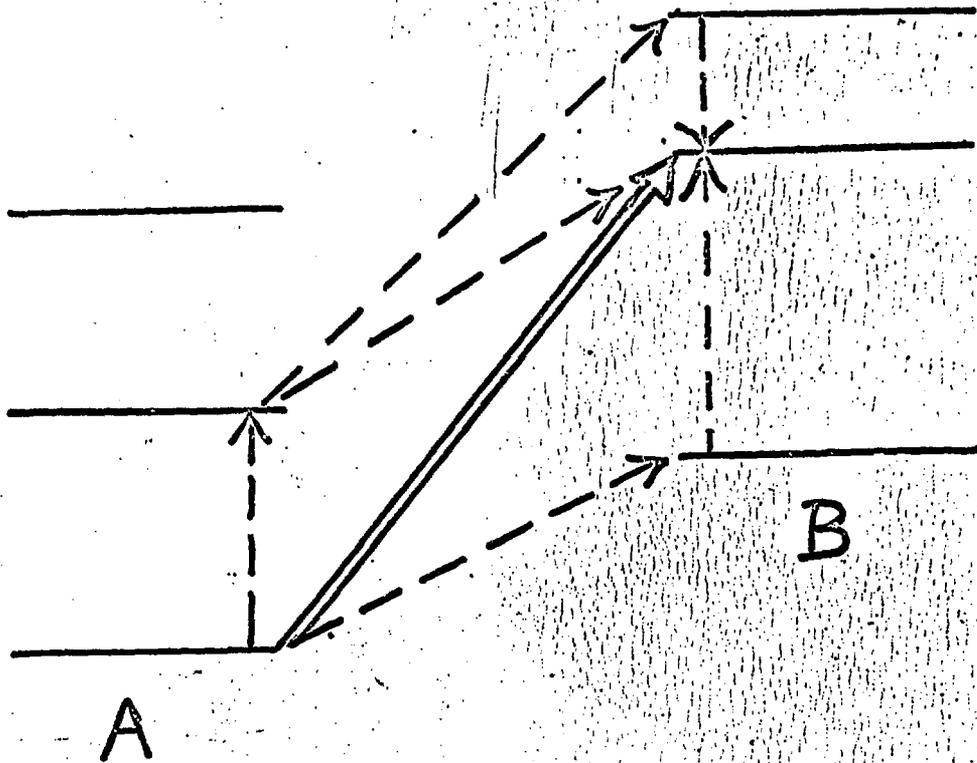


Figure 1. A direct (one-step) transfer (broad arrow) and some possible multi-step processes (broken lines) involving excitations in A and B.

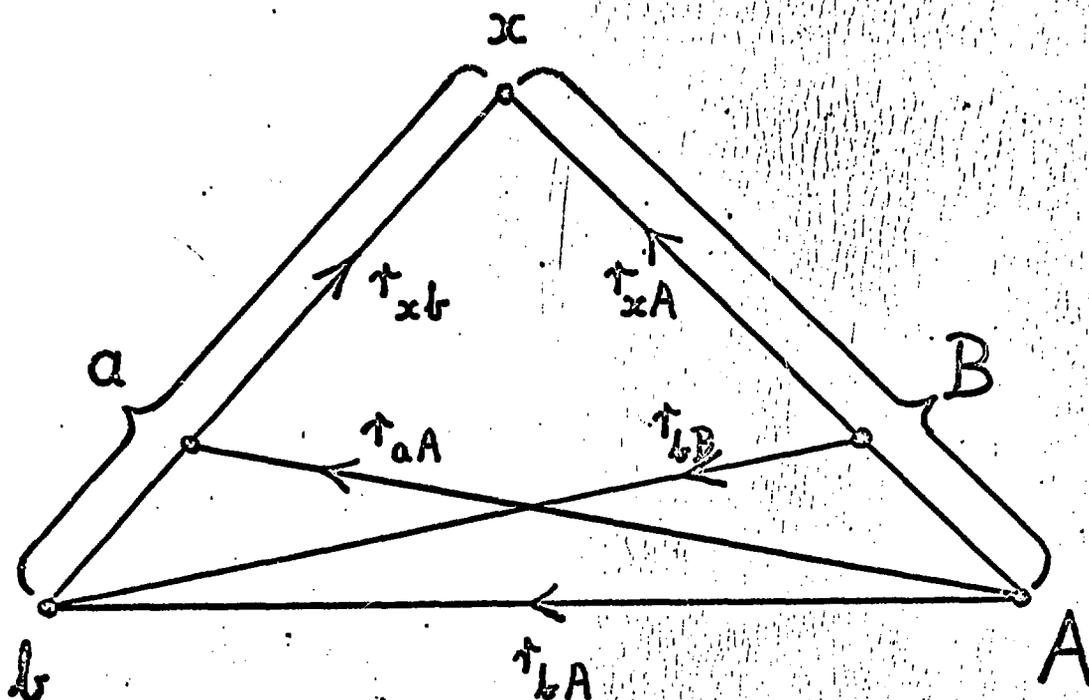


Figure 2. Coordinates for transfer reaction $A(a,b)B$ with $a = b + x$, $B = A + x$.