

CALCULATIONS OF THE IMAGINARY PART OF THE OPTICAL POTENTIAL*†

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1. Introduction

There have been many attempts to calculate the imaginary part of the optical potential, especially for nucleons, but until recently these have been based upon Fermi gas models¹⁾ (i.e., calculate for nuclear matter and then use a local density approximation for finite nuclei).

More recently there have been calculations specifically for finite nuclei. Slanina²⁾ and Vinh-Mau et al.^{3,4)} both use a microscopic (particle-hole) description of the target excited states. A detailed discussion of the optical potential, including a calculation of the imaginary part, has been given by Cugnon⁵⁾. O'Dwyer et al.⁶⁾ report results for neutrons of 14 MeV and below incident on ⁵⁸Ni using a particle-vibration model and including only those intermediate ("compound") states in which the projectile was bound. Similar calculations are being made⁷⁾ for neutrons of low energy on ²⁰⁸Pb, including continuum intermediate states. I shall not discuss this work except to put it into context; rather I shall outline some semi-phenomenological calculations that I have been associated with^{8,9)} which attempt to get some feeling for the contributions to the imaginary potential for protons, deuterons and alphas from open inelastic and pick-up or stripping channels.

2. General Theory

There are numerous formalisms within which to express the optical potential; let me adopt the one of Feshbach¹⁰⁾ for the purpose of discussion. In this, the generalized optical potential for elastic scattering at energy E can be written

$$U_{\text{opt}}(E) = (\Phi_0 | V | \Phi_0) + (\Phi_0 | V \frac{1}{E - H_{\text{QA}} + i\epsilon} QV | \Phi_0) \quad (1)$$

$$= V_{00} + \Delta U \quad , \text{ say,}$$

where the limit $\epsilon \rightarrow 0$ is understood, Φ_0 is the ground state of the target and projectile, Q projects off the ground state, H is the total Hamiltonian and V is the interaction between the projectile and the target. (Antisymmetrization effects are not exhibited explicitly). V_{00} accounts for most of the real part of the potential, while ΔU contains the imaginary part since, formally,

$$\lim_{\epsilon \rightarrow 0} (E - H_{\text{QA}} + i\epsilon)^{-1} \rightarrow P(E - H_{\text{QA}})^{-1} - i\pi\delta(E - H_{\text{QA}}), \quad (2)$$

where P denotes the principal value. This also shows that only energy-conserving transitions (i.e. open non-elastic channels) contribute to the imaginary part. In general ΔU exhibits rapid fluctuations as the energy E is varied (because of "bound-states-embedded-in-the-continuum") which have to be removed by energy averaging before the potential (1) can be identified with phenomenological optical potentials. This introduces an imaginary part, corresponding to compound elastic scattering, even if no non-elastic channels are open. A standard prescription for averaging this compound nucleus part is to replace E by $E + iI$. The effects of various averaging intervals I were studied in ref.⁶⁾, for example. However, our calculations do not include the compound contributions.

ΔU is also non-local (as well as energy-dependent); most authors either do not calculate the scattering or else construct some local approximation to ΔU . In our

work, the scattering is calculated¹¹⁾ without approximation from the non-local ΔU . This makes the comparison with experiment less ambiguous. 3

The expression (1) is exact and hence useless in that form. For a feasible computation we need to make some approximations. We introduce a complete set of intermediate states α_m and then ignore all parts of the propagator which are not diagonal in that set. We do this by replacing the interaction V_α in the channels $\alpha = a + A$, which this set represents, by its diagonal matrix elements $(V_\alpha)_{nn}$, i.e. by the first order (and real) optical potential in those channels. So the corresponding Hamiltonian becomes

$$H \approx H_A + H_2 + T_\alpha + (V_\alpha)_{nn} \delta_{nn}. \quad (3)$$

(We also neglect the effects of Q when the intermediate states represent a rearrangement channel.)

Other authors^{2,3)} have neglected V altogether in the propagator so that the ΔU they calculate is simply second-order in V . We feel that our distorted-wave propagator is better than their plane-wave propagator; the differences are not negligible.

Of course, in a practical calculation we severely truncate the set of intermediate states to a few we believe to be important. This, plus the neglect of interchannel coupling (the off-diagonal parts of the propagator), makes us feel that a complex optical potential in the propagator is probably even a better approximation. The calculations we have just described are equivalent to solving coupled differential equations for the excitation of a set of states such that the only coupling each state experiences is to the ground state of the target. Solving the coupled equations themselves¹²⁾ for a more general situation would automatically include the non-diagonal parts of the propagator and also introduce in a natural way complex potentials both for the coupling and the propagator. (Of course, for a large number of coupled states this represents an unduly lengthy computation.) However, there are large uncertainties as to the imaginary potentials to be used, so rather than introduce any adjustable parameters we have so far restricted ourselves to real potentials in (1) and (3).

3. The Models

The physics of the problem enters primarily through our choice of the intermediate states. For example, we expect the most important absorptive processes for incident protons to be direct inelastic excitation of strong collective states of the target, direct rearrangement collisions (especially (p,d) pick-up and (p,np) or (p,2p) knock-out) and "compound nucleus formation". Our choice of representation for the intermediate states is determined by our assessment of the relative importance of these processes. Further, while any complete set will describe all of them, the actual set used will be so limited that we do not hesitate to use subsets from more than one representation and ignore their lack of orthogonality.

It remains to specify the interaction V and its matrix elements. For pick-up we took the standard zero-range form used in DWBA descriptions of these transfer reactions. For inelastic excitations, rather than use a microscopic description²⁻⁵⁾, with its concomitant uncertainties, we adopt the collective (or deformed potential) model. Thus the off-diagonal elements of V are generated by deforming the (real) first order potential V_{00} . This model has the advantage that it has successfully described the observed inelastic scattering and consequently the transition strengths (deformation parameters) are available from experiment. It has the disadvantage that, because it is a surface coupling, the resulting contributions to ΔU are guaranteed to peak near the nuclear surface and we do not gain any insight into the radial distribution. However, the problem of understanding the empirical success of the collective model in microscopic terms is itself not fully solved¹³⁾.

In the case of deuteron stripping⁶⁾, the interaction which appears includes the proton-target or neutron-target potential for the (d,p) or (d,n) process, respectively. If we allow these potentials to be deformed, as in the collective model, they can excite the target while the deuteron strips. This "core-excitation" process

appear to be important⁴⁾.

The details of the calculations are given elsewhere^{8,9)}; here we only discuss some of the results.

4. Proton Scattering

4.1. Inelastic excitations

Most of the calculations were made for 30 MeV protons on ^{40}Ca and ^{208}Pb . First we consider the effects of inelastic excitations. The excited states selected were those observed strongly in inelastic measurements plus, in some instances, others postulated at higher excitations with strengths chosen to exhaust the energy-weighted sum rules.

First, the effect of a single excited state was studied. Fig. 1 shows the absorption cross section for 30 MeV protons as the excitation energy of this state is varied. It shows interesting "resonance" phenomena which are associated with shape resonances for the proton in the intermediate state potential well (a 2f resonance for protons of 8.8 MeV and a broad 1g resonance for about 22 MeV protons). Such resonant behaviours are obscured when many excited states are included; they would be further damped if a complex propagating potential was used.

It was also observed that it made essentially no difference whether the strength was concentrated into one state or split amongst, say, 10 states (provided one of the latter did not fall on the sharp resonance in Fig. 1).

The contributions to the absorption cross section σ_A from single, observed excited states were found to be ~ 100 mb; the contributions from states with spin $\lambda \geq 5$ begin to fall off because of momentum mismatching.

Next we include 10 excited states of ^{40}Ca , with multipolarities $\lambda=1$ through 5, chosen to exhaust the corresponding sum rules. (The corresponding ΔU for the partial wave $L=4$ is shown as contour plots in Fig. 2. The general features are similar for other L .) This resulted in an absorption of $\sigma_A = 750$ mb, compared to that measured of 915 ± 38 mb. If a phenomenological "volume" local absorption of Woods-Saxon form and depth $W=2$ MeV is added (to represent crudely the compound formation), σ_A is raised to 8.6 mb. The differential cross sections for this latter case (labelled $U_0 + \Delta U + W$) are shown in Fig. 3. Also shown is $d\sigma/d\omega$ for $U_0 + W$ alone; the absorption for this case is 275 mb, whereas adding W to $U_0 + \Delta U$ only increased σ_A by 66 mb. This is a good example of the non-linear relation between σ_A and the imaginary potential.

The curve in Fig. 3 labelled U_3 is for an empirical potential which fits the observed scattering at this energy; thus it represents the experimental data. Qualitatively the calculated cross sections are in good agreement with the data. However, the solid curve is for U_2 , a conventional local potential which represents an attempt to find a fit (in the usual sense) to the scattering from the constructed $U_0 + \Delta U + W$. It was not possible to find a good fit to these theoretical scattering cross sections with a local potential, and Fig. 4 helps one to understand why. This shows the magnitudes of the partial wave scattering amplitudes. Clearly the angular momentum structure of the absorption for the theoretical potential is quite different from that obtained using the conventional Woods-Saxon potentials. The absorption for the theoretical potential is more strongly localized in the surface partial waves, especially $L=4$ and 5, and then decreases rapidly for $L > 5$. This latter effect reflects another difficulty; the best fits one can obtain to this scattering have local absorptive potentials which peak close to the radius of the real potential, instead of further out as do the empirical potentials. As we remarked earlier, the position of our theoretical imaginary potential is more or less determined by the collective model interaction used for V ; however, we believe using a microscopic model would merely exacerbate the problem since these models usually result in transition potentials which are somewhat inside the collective ones. It is possible that using a complex potential in the propagator would help to push ΔU out to larger radii. We also know from DWBA studies that stripping and pick-up processes tend to occur outside the nuclear surface -- but see below.

Another feature is that $\sigma_A \sim 800$ mb was only obtained by postulating high

excited states ($E_x \leq 20$ MeV) which exhaust the remainders of the sum rules. Such concentrations of strength seem improbable; more reasonable guesses only result in values for σ_A which are about one-half the observed one. Consequently we believe there must be other important absorptive processes acting.

Similar calculations were made for ^{208}Pb with even more marked results. Fig. 5 shows the $|\eta_L|$ for a case containing 14 excited states which exhaust the sum rules for $\lambda \leq 8$. $U_0 + \Delta U$ is the theoretical potential, U_1 is an empirical potential which represents the observed scattering and U_2 is one attempt to find a local potential fit to the theoretical scattering. Such a fit could only be obtained for scattering angles up to about 60° ; at large angles the calculated scattering is more than an order of magnitude too large. Nonetheless, the calculated $\sigma_A = 1458$ mb compared to the measured 1865 ± 98 mb. Fig. 5 shows the absorption is much too strongly concentrated in the surface partial waves.

4.2. Pick-up terms

There have been some previous studies of the effects of (p,d) pick-up on proton elastic scattering by solving very limited sets of coupled equations¹⁴). We have made a few calculations in our formalism, constructing the ΔU for 30 MeV protons on ^{40}Ca using as intermediate states positive energy deuterons plus the single-hole neutron states in ^{39}Ca . Pick-up from the $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1p$ and $1s$ shells results in $\sigma_A = 682$ mb; leaving out the $1p$ and $1s$ only reduces this by about 10% to 613 mb. Consequently pick-up can produce almost as much absorption as the 10 inelastic channels. However the elastic angular distributions are very different from the observed ones. There is strong absorption for $L = 4$ and 6 but almost none for $L = 5$, while $L = 0$ through 3 have $|\eta_L|$ twice as large as the empirical optical potential gives. This situation is not improved much if we include both the pick-up and the 10 inelastic channels (see Fig. 6). Then σ_A is increased to 917 mb. The low partial waves, $L \leq 3$, are now strongly absorbed but the $L = 5$ remains weakly absorbed ($|\eta_5| = 0.84$). Comparing this to Fig. 4, where $L = 5$ is the most strongly absorbed and $L = 6$ is weakly absorbed, it appears that the pick-up process is somehow dominating the scattering. The pick-up contributions to ΔU have much more structure than the inelastic contributions shown in Fig. 2, there being several "hills and valleys".

We were unable to find a local optical potential fit to the theoretical scattering with pick-up alone, even for just the forward angles. A qualitative fit was obtained to the forward scattering with pick-up and inelastic absorption but with unphysical parameters and even then the η_L structure was not reproduced, especially for $L = 5$ and 6.

Hence, although we can obtain the observed absorption cross section, clearly we do not have a good description of the physical processes involved. The value of σ_A can be a poor indicator; provided we include a sufficient number of absorption channels, σ_A will saturate at a value close to the observed one simply from geometrical reasons.

5. Alpha Scattering

Some tentative calculations were made⁸) for the absorption due to inelastic scattering of 43 MeV alphas from ^{58}Ni . Five levels, 2^+ and 3^- , with excitation energies ≤ 7 MeV were chosen with deformation parameters taken from experiment. The first order potential V_{00} was a sum of 4 (real) nucleon optical potentials folded into the alpha particle. These inelastic states easily accounted for the measured absorption of $\sigma_A = 1430$ mb; indeed using the lowest 2^+ alone exceeded this value. However the calculated angular distributions of the elastic cross sections were in poor agreement with experiment, especially being much too large at large angles. The partial wave amplitudes indicated too much absorption in the surface region and too little in the interior; $|\eta_L| \geq 0.1$ for low L instead of being almost zero as expected.

In this case, calculations were made for 11.8 MeV deuterons on ^{58}Ni to see the effects of stripping in intermediate states. Single nucleon capture into six levels of the $(2p, 1f)$ and $(3s, 2d, 1g)$ shells with excitations up to 7.8 MeV were allowed. In addition, the possibility of exciting the target into one of 5 excited states, at the same time as the deuteron strips, was allowed. Simple inelastic effects without stripping were not included, neither was deuteron break-up in which both nucleons remained in continuum states. While the latter probably is important, computational limitations precluded it.

The observed absorption cross section is 1400 mb. Stripping into these 6 states, without core excitation, gave $\sigma_A = 453$ mb. Including core excitation almost tripled this value, to 1622 mb. (The biggest increase was due to the lowest 2^+ state; this alone increased σ_A to 1168 mb.) However, once again the angular distribution of the elastic cross sections were in poor agreement with experiment, again being much too large at large angles. Examining the $|\eta_L|$ amplitudes shows this can also be traced to too little absorption for small L and too much for large L .

7. Conclusions

The studies reported here are certainly incomplete and somewhat inconclusive. It is shown that it is easy to reproduce a measured absorption cross section, but this is not a good indicator. Differential cross sections and partial wave amplitudes are generally in poor agreement with experiment. The two main approximations made are the truncation of the set of intermediate states and the neglect of scattering between different intermediate channels by the propagator. Hopefully these effects can be represented to some degree by the use of complex coupling and propagating potentials, and this possibility will be explored. If not, we shall seem to be obliged to solve the coupled equations explicitly, a formidable undertaking.

Another feature missing from our work is the inclusion of bound intermediate states, i.e. the "compound nucleus" process^{6,7}). It seems likely that this would correspond mostly to the "volume" absorptive potential in the optical model, whereas the processes we have considered are essentially surface reactions.

However, we should not be completely disappointed that our results cannot be reproduced by a conventional, local, optical potential. The latter are known to encounter difficulties sometimes in fitting data, especially for light nuclei (including ^{40}Ca) and it is possible that a residuum of some of the effects we have seen could remedy these deficiencies, e.g. specific effects of a particular state or a particular process which would vary from nucleus to nucleus and from energy to energy.

- 1). A recent example is B. Sinha and F. Duggan, Nucl. Phys., to be published.
- 2). D. A. Slanina, The Optical Potential in Proton-Nucleus Scattering, thesis, Michigan State University (1969).
- 3). M. Bruneau and N. Vinh-Mau, in Methods and Problems of Theoretical Physics (North-Holland, Amsterdam, 1970).
- 4). N. Vinh-Mau and A. Bouyssy, contribution to Munich conference (1973).
- 5). J. Cugnon, Theorie Dynamique et Calcul du Potentiel du Modele Optique, thesis, Universite de Liege (1972); See also, J. P. Jeukenne, A. Lejeune and C. Mahaux, Proc. Symp. "Present Status and Novel Developments in Nuclear Many-Body Problem," Rome (1972).
- 6). T. F. O'Dwyer, M. Kawai and G. E. Brown, Phys. Lett. 41B (1972) 259.
- 7). A. Lev and W. P. Beres, Bull. Am. Phys. Soc. 18 (1973) 717.
- 8). M. Reeves, Microscopic Analysis for Elastic Scattering of Deuterons and Alphas, thesis, University of Tennessee (1970); Oak Ridge Report CTC-32 (1970).
- 9). C. L. Fao, M. Reeves, and G. R. Satchler, Nucl. Phys. A207 (1973) 182.
- 10). H. Feshbach, Ann. of Phys. 19 (1967) 287.
- 11). M. Reeves and L. W. Owen, J. Comp. Phys. 4 (1969) 572.
- 12). R. S. Mackintosh, Nucl. Phys. A164 (1971) 398.
- 13). G. R. Satchler, Phys. Lett. 39B (1972) 495;
R. S. Mackintosh, Nucl. Phys. A175 (1971) 526;
A. Seegen, Nucl. Phys. A195 (1972) 415.
- 14). T. Ohmura et al., Prog. Theor. Phys. 44 (1970) 1242;
G. H. Rawitscher and S. M. Mukherjee, Am. Phys. 68 (1971) 57;
R. S. Mackintosh, Phys. Lett. 44B (1973) 437, and to be published.

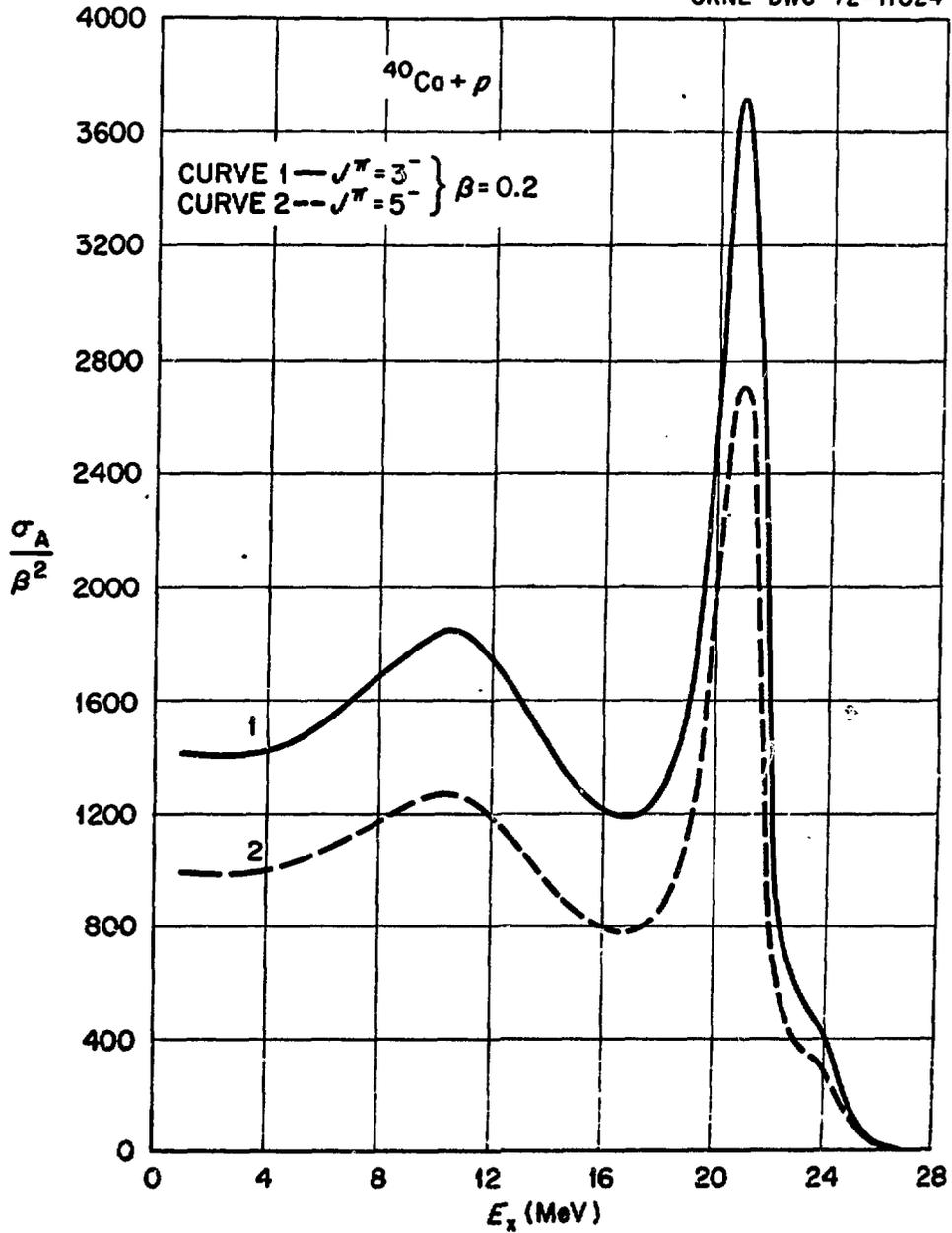


Figure 1

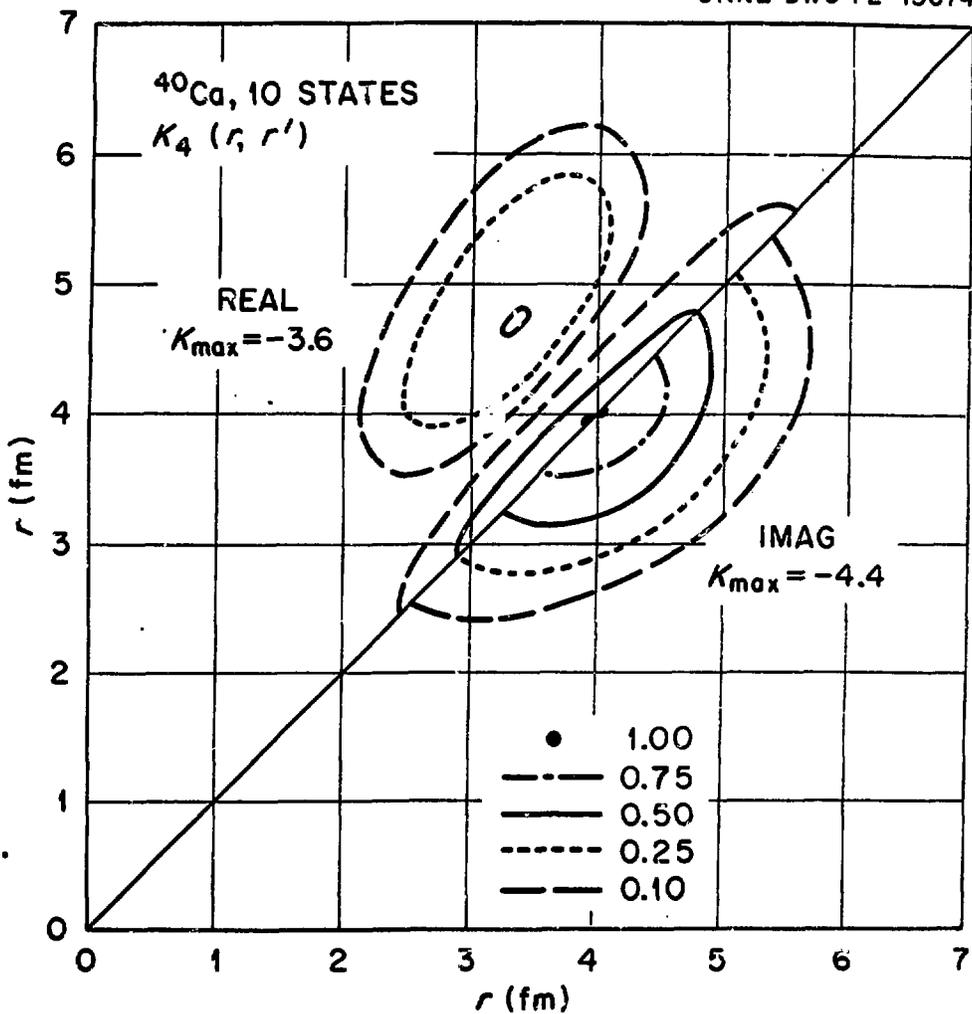


Figure 2

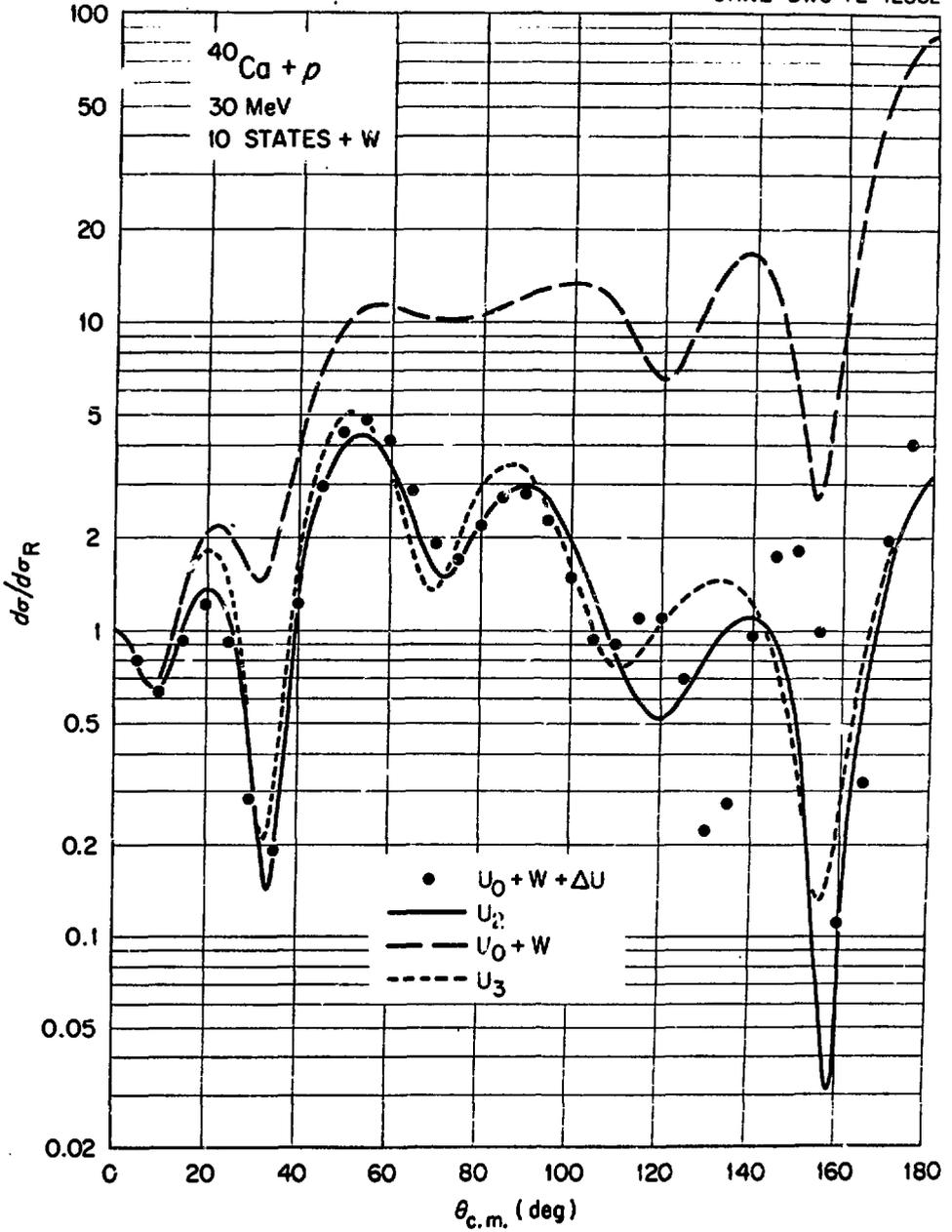


Figure 3

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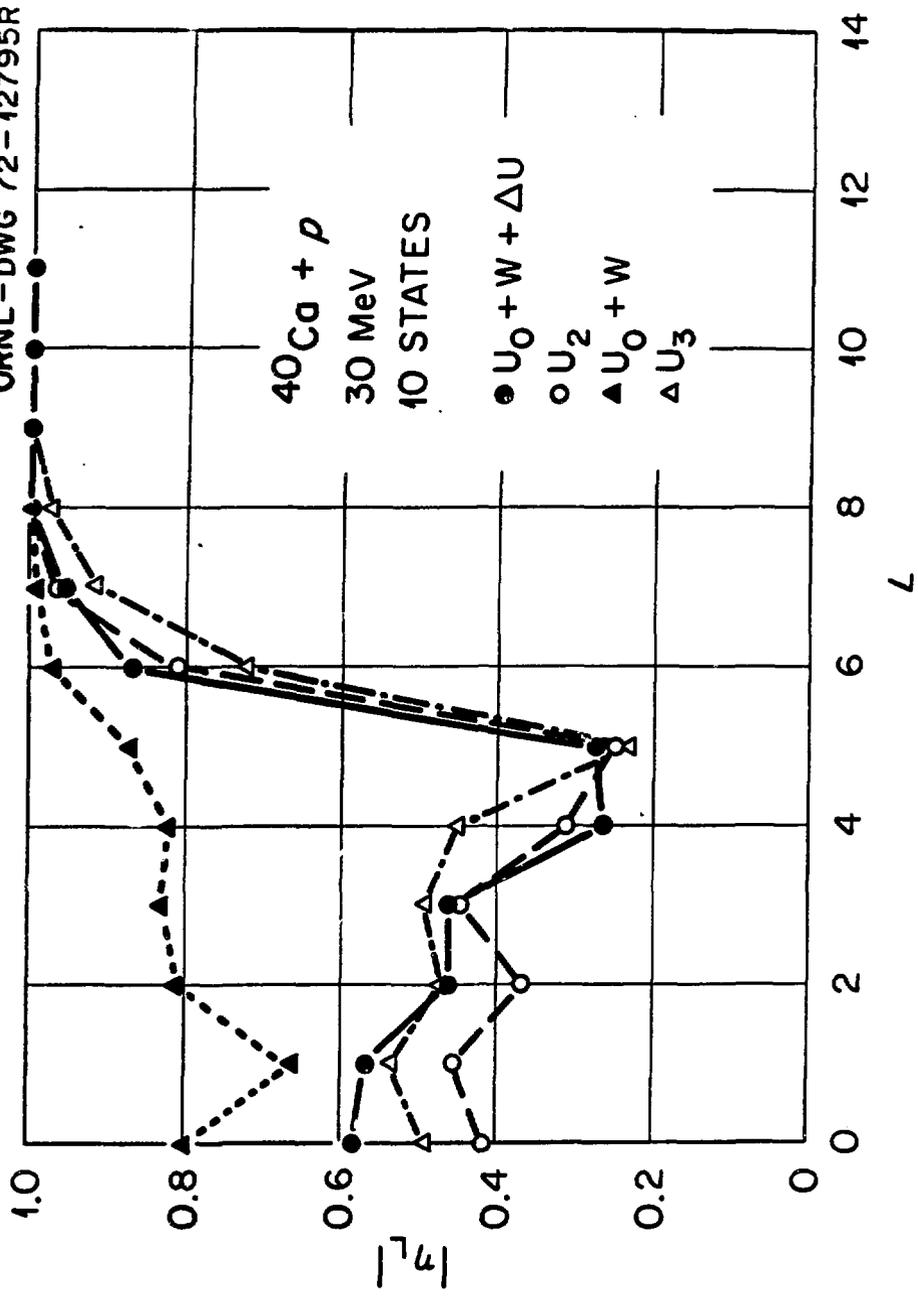


Figure 4

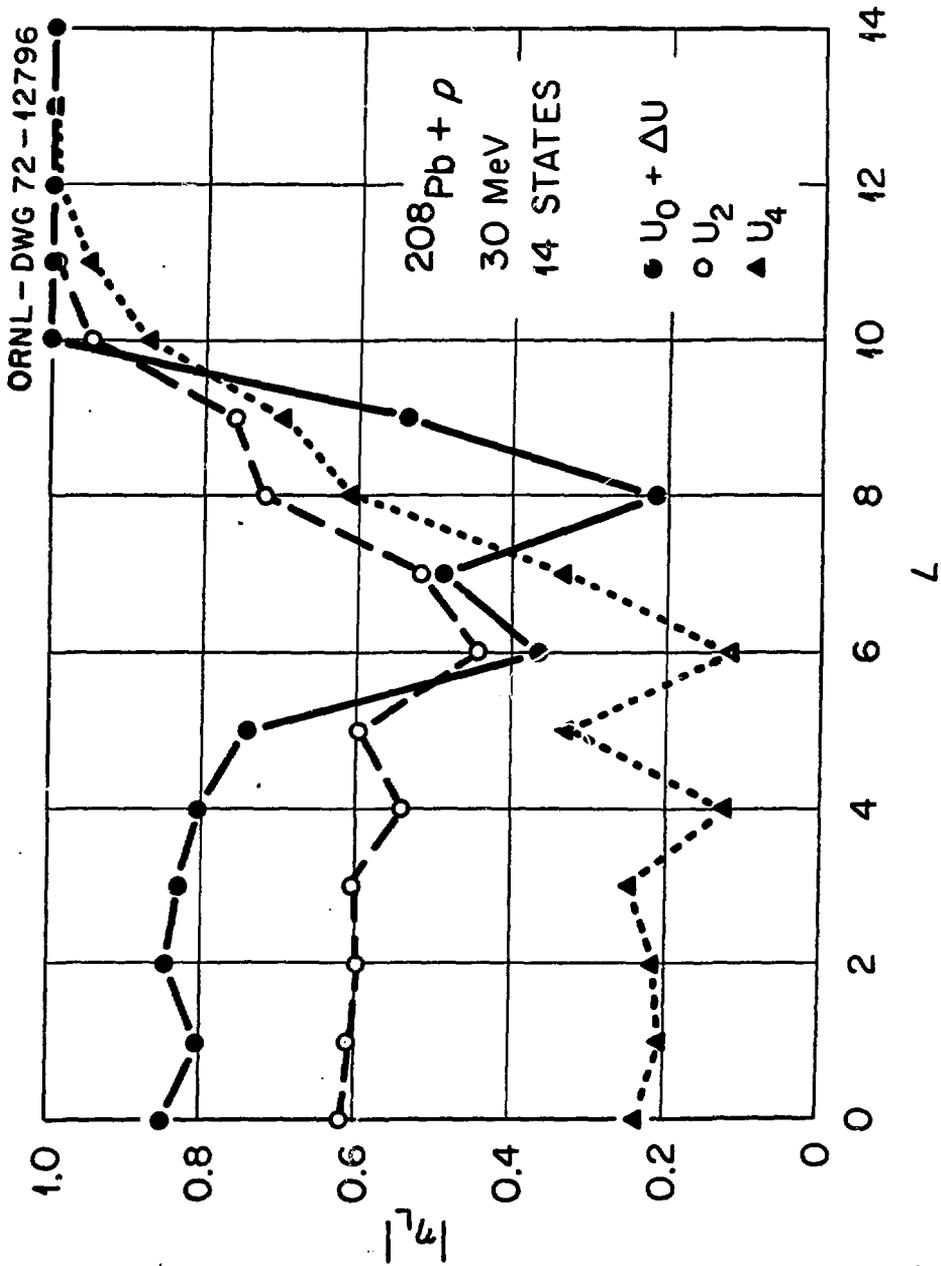


Figure 5

$^{40}\text{Ca} + p$ 30 MeV

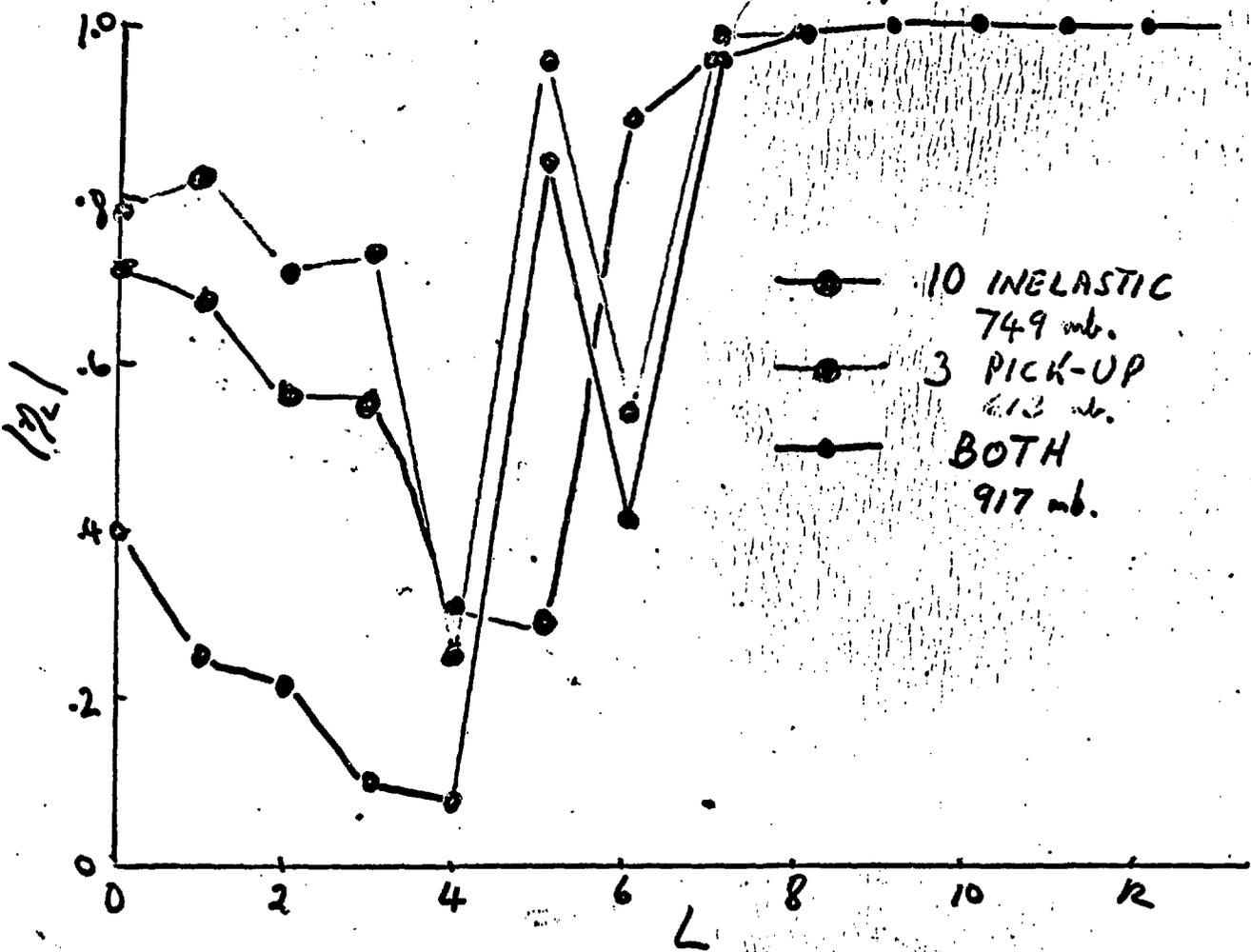


Figure 6