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NUCLEAR STRUCTURE STUDIES WITH LARGE SHELL MODEL CALCULATIONS*

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The general topic I will talk about today is that of nuclear structure studies in the framework of the conventional nuclear shell model. The model is familiar to everyone, and I'll describe it only briefly. A finite set of single particle orbits is assumed to be "active" in a given calculation. Multiparticle basis states are formed by distributing nucleons in this set of orbits. Within the space of this set of orbits an effective Hamiltonian is diagonalized. The resulting eigenvalues are associated with the energies of low-lying states in a given nucleus, and the eigenvectors are associated with nuclear wave functions. As such, they are used to calculate various observables such as spectroscopic factors for nucleon transfer reactions or electromagnetic decay rates.

In recent years computer programs have been developed[1] which have enabled us to apply the shell model to the study of many of these observables in many nuclei, and in many mass regions, with considerable success. It is not my intention here to demonstrate these successes with a catalog of theory-experiment comparisons. The theoretical foundation of the shell model is still insecure. The model represents the first term in a perturbation series of which the convergence is not yet established. The model involves a severe truncation of the physical Hilbert space. In principle, this truncation should necessitate the use of complicated effective many-particle operators for the calculation of various observables. In practice, almost all shell model successes have been achieved with very simple one- and two-body effective operators. The calculation of all observables involves the calculation of some effective operator between two wave functions. The calculations therefore predict only relative properties between wave functions, and so the results say little about the details of individual wave functions.

In spite of these reservations, the model has been of extreme value in correlating many observed nuclear phenomena, and it seems only logical to take advantage of nature's apparent kindnesses to the fullest extent possible, and it is in this direction that this talk will aim. I will first discuss a recent advance in large shell model technology made by Whitehead.[2] He has developed a technique whereby it is now possible to study almost all the so-called s-d shell nuclei, with A = 18-38, in the full $(sd)^n$ space. Thus, a study of the systematics of nuclear properties in the middle of the shell, where some interesting qualitative changes in nuclear structure occur, is now practical, and I will comment on initial results in this direction. Then I will say something on the usefulness of shell model wave functions in the analysis of cluster transfer data which is being accumulated as heavy-ion experiments proliferate. Finally, I'll talk briefly about ways to further extend the domain of applicability of the shell model.

Let me first talk about shell model technology. In a shell model calculation, the physics involved is the selection of the model space to be used, and the effective Hamiltonian and observable operators to be used. After these two questions are answered, the Oak Ridge-Rochester code[1] proceeds to set up the set of basis states in the assumed model space, and then to calculate the complete N × N matrix of the effective Hamiltonian which has been selected. Then this matrix is transformed into the tri-diagonal form, wherein all elements are zero except the diagonal matrix elements and the elements immediately to the left and right of the diagonal elements. Once the matrix is in this tridiagonal form, there are standard algorithms by which the lowest n eigenvalues and eigenvectors for the

matrix can be determined. The Oak Ridge-Rochester program works in terms of quantities with good angular momentum so as to take full advantage of the efficiency of Racah algebra.

Whitehead has taken a significantly different approach. He has abandoned the mathematical elegance of group theory in favor of paying proper attention to what the Hamiltonian operator can tell you about its eigenvectors beyond their invariance properties, and also by recognizing and using those types of operations best suited to a computer. He has based his program on the so-called Lanczos Algorithm. The general idea is outlined here. Assume you have the operator H, and some initial trial vector ψ_1. Then carry out the following operations:

$$H\,\psi_1 = \alpha_{11}\psi_1 + \alpha_{12}\psi_2$$

$$H\,\psi_2 = \alpha_{21}\psi_1 + \alpha_{22}\psi_2 + \alpha_{23}\psi_3$$

$$H\,\psi_3 = \alpha_{32}\psi_2 + \alpha_{33}\psi_3 + \alpha_{34}\psi_4.$$

Here ψ_2 is the normalized part of H ψ_1 which is orthogonal to ψ_1. Similarly, ψ_3 is that part of H ψ_2 which is orthogonal to ψ_1 and ψ_2. If H is Hermitian, $\alpha_{12} = \alpha_{21}$. Since by definition $\alpha_{1\ell} = 0$ $\ell \geq 2$, $\alpha_{\ell 1}$ is 0 also. This procedure can be continued until a full N-dimensional basis is constructed. In this way, H is used directly to generate the basis space. The matrix is constructed directly in tri-diagonal form, so there is no need for the extra step of transforming the matrix to tri-diagonal form. What makes the method so useful is that the calculation can be carried out iteratively. At each stage, as a new basis vector, ψ_ℓ, is generated, one can obtain the eigenvalues and eigenvectors of the generated $\ell \times \ell$ tri-diagonal matrix by standard methods. It can be shown that with each successive step, the eigenvalues converge monotonically to the exact eigenvalues. The pleasant surprise is that in calculations carried out so far, there has been good convergence to the lowest eigenvalues after relatively few iterations, on the order of 40 or 50, and the number of iterations seems to be relatively independent of the dimension of the calculations. Thus, H is essentially used to generate a useful truncated space. The convergence rate does depend somewhat on the spacings of the low-lying eigenvalues. The calculations are done in a basis in which single particle orbits are labeled by j and j_z=m, and multiparticle states are specified only by what single particle orbits are occupied. This obviously leads to much larger dimensionalities than exist if states of good J and T are used. Whitehead and Sebe and Nachamkin[3] both tried to use the Lanczos method in shell model calculations in the late 1960's. They each tried to maintain some sort of group symmetries in their wave functions, and the time required to maintain these symmetries made the methods relatively unsatisfactory. It is somewhat ironic that real progress was made only when almost all symmetries were discarded. Detailed timings are not available for the program, but for ^{28}Si, in a full $(sd)^{12}$ space, one matrix encountered[4] had a dimension of 29,000 (in the m-scheme). The calculations took 3 minutes per iteration on a model IBM-360/195. Thus, the calculation took on the order of two hours of machine time. In ^{28}Si, in a basis with good J and T, the largest $(sd)^{12}$ space has a dimension of 3800. At present, there is no computer with a core large enough to enable the Oak Ridge-Rochester code to handle such a matrix. If there were an IBM-360/195 with such a large core, the Oak Ridge-Rochester program would take about six hours to obtain the lowest few eigenvalues and eigenvectors. As the matrices become even larger, the advantage of the Whitehead approach becomes much more appreciable, since it has only a linear dependence on dimension compared to the N^2 and N^3 dependences in the Oak Ridge-Rochester code.

With the Glasgow programs, it is now possible to carry out complete $(sd)^n$ calculations for almost all nuclei between 18 and 40 and most of the reported work done with these programs have been related to s-d shell nuclei. By far the largest amount of effort with large shell model calculations has been carried out

in this region. There are two reasons for this intensity. The first is that since there are relatively few single particle orbits, all with relatively small j; i.e., $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$, the size of the matrices are "relatively" small. Secondly, the nuclei in this region display many systematic features that are typical of "collective" nuclei. There are "rotational" nuclei at the beginning of the shell, and "vibrational" nuclei at the end of the shell. The mass region A = 24-32 is an especially interesting one since it is there that a transition from "rotational" to "vibrational" occurs. With recent Glasgow calculations by Whitehead and Watt,[4] there now exist calculations of all even-even nuclei in the s-d shell, so it is possible to answer some questions as to whether or not the systematic behavior of all these nuclei can be reproduced with one shell model calculation. Insofar as excitation spectra are concerned, the answer is at best a qualified yes. In Figs. 1 and 2 I have plotted the low-lying spectra of ^{24}Mg, ^{28}Si, ^{32}S, and ^{36}Ar. (The A=24, 28, and 32 spectra were calculated by Whitehead and Watt. The A=36 result was reported by Wildenthal, et al.[5].) These are all calculations in which an inert ^{16}O core is assumed, and no truncation is made on the $(sd)^{A-16}$ space. For all these nuclei, ^{17}O single particle energies are used. For all the nuclei except the ^{36}Ar, the two-body part of the effective interaction is taken from the "realistic" interaction calculation[6] of Kuo. The ^{36}Ar calculation is made with a Kuo interaction, but with a larger oscillator parameter (i.e., $\hbar\omega$ = 12.5 MeV, while $\hbar\omega$ = 14.0 MeV for all the other cases). There are fairly well developed ground state rotational bands in ^{24}Mg and ^{28}Si. These are quite well described by the calculations. In ^{24}Mg there is an excited band, which would be designated as a K=2 band in the collective model, and the band head is degenerate with the ground state band 4^+. Such a band exists in the calculated spectrum, but it is too low with respect to the ground state band. There is a suggestion that excited states in ^{28}Si are too low also, but the excited band structure is not so obvious there. This feature that the excited band is too low is a consistent feature in the lighter s-d shell nuclei. It was known to occur also in ^{21}Ne and ^{22}Na,[7] and a contribution from the Glasgow group to this meeting on full s-d shell calculations of ^{24}Na and ^{26}Al hints at similar problems for these nuclei. I'll return to this point in a moment. The observed spectrum of ^{32}S and ^{36}Ar are suggestive of vibrational nuclei. For these two nuclei, the calculated results are not in too good agreement with experiment, especially for the "two-phonon" 0^+ state. It is quite possible that these many-particle calculations of energy levels are too sensitive a probe of the effective interaction which is used. As the number of particles increases, the effects of small errors in the two-body matrix elements becomes more and more important. A 100 keV or 200 keV difference in the interaction strength can become one or two MeV in ^{28}Si. In all the s-d shell calculations reported so far, it has been assumed that the effective interaction is at most a two-body operator. If there is a small three-body or higher particle-rank operator, it may not become important until the middle of the shell. It is also true that there are good reasons why the effective interaction should not be constant throughout a large mass region. There are known to be important renormalization effects due to core excitation, and in principle some of these effects should vary as particles are added in the active space. For example, the structure of the assumed 16-particle core could be quite different in ^{16}O and in ^{36}Ar, and if so, the single particle energies would be different for the two nuclei. It is known[5] that the calculated spectrum of ^{36}Ar becomes much more "vibrational" if the single particle energies are altered from the ^{17}O values. As I pointed out, the most systematic defect in these spectra calculations is that the first excited band in the rotational s-d shell nuclei is consistently too low with respect to experiment. There have been effective Hamiltonians found which give more satisfactory agreement for these bands. An interaction has been found[8] by best fit methods which leads to a better description of the spectrum of ^{22}Na and ^{24}Mg. In a contribution to this meeting Manakos, et al. show that with the Kuo-Brown interaction,[6] the position of these excited bands is sensitive to a part of the interaction that they refer to as the anti-symmetric L·S component, and that they can get significant improvement for the positions of excited band states by varying the strength of this component. Similarly, it has been possible[9] to obtain better agreement with the observed vibrational spectrum

of ^{32}S with a parameterized interaction. Thus, for the spectra, it has not been possible to describe all the s-d shell nuclei with one interaction so far, but most of the qualitative features of the s-d shell nuclei have been described with some interaction. As I pointed out above, it is possible that the spectra calculations are too severe a test. Soyeur and Zuker[10] have studied the properties of levels in ^{28}Si in various s-d model spaces with several different interactions. They find that two "realistic" interactions, the Kuo interaction and the Kahana-Lee-Scott interaction give significantly different spectra, but that the calculated E2 observables are quite similar for the two interactions. The structure of the wave function may be relatively stable to small differences in the interaction in "giant" shell model calculations, even when the calculated eigenvalues are quite sensitive. A result[9] consistent with this is found in ^{36}Ar, where, although the calculated excitation energies are in significant disagreement with observed energies with the Kuo interaction, the calculated relative B(E2)'s values are in quite good agreement with experiment. So far only calculated spectra for nuclei in the middle of the s-d shell in the full $(sd)^n$ space have been reported. Other observables and the structure of many other nuclei in the middle of the s-d shell have still to be studied in a complete s-d shell space, and it will be interesting to see the results of these calculations, which are now possible with the Glasgow code. In fact, the structure of nuclei around so-called minor-shell closures (i.e., ^{12}C, ^{28}Si, and ^{56}Ni) has consistently defied simple shell model descriptions. More satisfactory descriptions of these nuclei may now be possible with "giant" shell model calculations.

Much of the success of the shell model in the s-d shell is due to the existence of a relatively strong symmetry underlying the structure of many of the states in these nuclei. The use of representations of the SU(3)-group for light s-d shell nuclei was suggested in 1958 by Elliott.[11] It has since been used with considerable success[11,12] as a basis for truncation in s-d shell calculations. It has proven to be a useful exercise to transform the jj-coupled shell model wave functions for the s-d shell nuclei with $18 \leq A \leq 22$ and $34 \leq A \leq 38$ into representations which are labelled, partially, by the SU(3) symmetry. I have transformed many of our eigenvectors from the jj-coupling basis we use into a representation specified by the labels [f], (λ,μ), L,S,J, where [f] is the space symmetry, (λ,μ) is the SU(3)-group label, and L, S, and J are the usual orbital angular momentum, spin, and total angular momentum. This has been accomplished by diagonalizing the operator

$$P_{ij} + Q_2 + aL\cdot L + bS\cdot S \tag{1}$$

in the jj-coupled basis wherein J is already a good quantum number. Here P_{ij} is the Majorana space exchange operator, and Q_2 is the two-body Casimir invariant operator for the SU(3)-group. Given the eigenvectors and the states $|[f], (\lambda,\mu), L,S,J\rangle$ in the same j-j coupled basis, it is trivial to expand the eigenvectors in terms of the SU(3) states by calculating obvious overlaps. I would first like to use this transformation to analyze some recent calculations of logft values for β-decay in s-d shell nuclei reported by Lanford and Wildenthal.[14] I think it provides a rather nice illustration to summarize the status of shell model calculations in this region. In Fig. 3 I have plotted all the measured logft values for β-decays involving s-d shell nuclei with $A \leq 22$ and $A \geq 35$. The abscissa indicates the mass of the nucleus in which the transition is observed. I have omitted all decays where the Fermi operator can contribute, since these transitions provide no significant test of the calculation. I have further omitted all decays for nuclei with $23 \leq A \leq 34$, because the only calculations made for these decays are made in a truncated space, and it is known[13] these calculations can be very sensitive to truncation effects. The logft's for the transitions shown in Fig. 3 vary from 3 to 6, implying a variation in the transition matrix elements over four orders of magnitude. The features of the data here are very well reproduced. Many of these transitions involve small fractions of the total possible

Gamow-Teller strength, and so involve strong cancellation effects. It is possible to understand much of the qualitative structure in terms of the space symmetry and the L values of the states involved in the transition. The Gamow-Teller decay rate is partially governed by matrix elements of the operator

$$\sum_{i=1}^{A} \sigma_i \tau_i^{\pm}. \tag{2}$$

This operator has no spatial dependence. From this it follows that neither L nor the space symmetry can change between the initial and final states of a given decay. The logft values in Fig. 3 can be roughly divided into those less than 4 (strong transitions) and those greater than 4 (weak transitions). In every case of a weak transition, the space symmetry of the dominant components of the wave function changes between the initial state and the final state, or the predominant L values change. For the eight "strong" transitions, in six cases, the initial and final states are dominated by the same [f],L symmetry. Thus, most of the dominant qualitative features are "understood" in terms of a dominant symmetry. There are two exceptions. The "strong" transitions for A = 19 and A = 20 are transitions from ^{19}O and ^{20}O to states in ^{19}F and ^{20}F. The large components in these states change the [f] or L symmetry between initial and final states, but apparently the small components add in a coherent fashion to give a relatively strong transition. I think this slide is a succinct summary of many s-d shell model results; i.e., there is relatively good quantitative agreement between the shell model calculation and experiment for low-lying states, and much of the qualitative structure can be "understood" in terms of a dominant symmetry. Here for the β-decay process it is space symmetry and orbital angular momentum that is important. For excitation spectra and E2 observables, the SU(3)-symmetry plays an important role. But, as with the oxygen isotope beta decays here, there are a significant number of cases where the dominant symmetry does not explain the data, and where the more complete model calculations are necessary. The fact that symmetries play such a dominant role in these calculations is in itself strong evidence of the validity of the shell model in this region. The existence of these symmetries depends on the fact that there are discrete particles distributed in a relatively few single particle orbits with distinct ℓ and s.

The transformation of the shell model wave functions into the SU(3) representation may have a very practical value in the analysis of cluster transfer data. This subject is discussed at some length in these proceedings by Arima. Let us use as an operational definition of an α-cluster a system of two neutrons and two protons in a spatially fully symmetric state, [f] = [4], with the motion of the four particles with respect to the center of mass of the four particles in a relative ℓ=0 state, and with a total angular momentum L with respect to the shell model origin. If the four particles of this cluster are in the first unoccupied oscillator shell outside a doubly closed shell, and the same oscillator parameter is assumed for particles in both the closed shell and the cluster, then the wave function for the four particles is identical to the so-called leading representation of the SU(3) group for four particles in the occupied shell. Thus, in this picture, the α-cluster outside ^{16}O is, in the SU(3) representation defined above, in the state $|[f] = [4]\ (\lambda,\mu) = (8,0),\ L = 0,\ S = 0,\ T = 0,\ J = 0\rangle$. If this is a reasonable picture of the α-cluster, then the spectroscopic factor for the transfer of α-particle onto ^{16}O to form an s-d shell state in ^{20}Ne is proportional to the intensity of this leading representation in the populated state. In the s-d shell model calculation of ^{20}Ne, the ground state rotational band is dominated by the $(\lambda,\mu) = (8,0)$ state. In Table I, I have shown the percentage of $(\lambda,\mu) = (8,0)$ in each member of the ground state rotational band of ^{20}Ne. These numbers have been used by Ichimura, et al.[15] to calculate cross sections for transitions to these states with reasonable success. It is reasonable to use a similar model for three-particle cluster transfer. Then, in the s-d shell, the spectroscopic factor for t or ^{3}He transfers from ^{16}O to ^{19}F or ^{19}Ne is proportional to the intensity

Table I. Intensity of Leading SU(3) Representation in States in "Ground State Rotational Bands" in ^{19}F, ^{20}Ne, ^{37}K, ^{36}Ar. The model for ^{19}F and ^{20}Ne is the $K+^{17}O$ model described in Ref. 7. The model for ^{37}K and ^{36}Ar is the 12.5 p + ^{17}O model in Ref. 9

	^{20}Ne			^{19}F			^{36}Ar			^{37}K		
[f],(λ,μ)	[4],(8,0)			[3],(6,0)			[4],(8,0)			[3],(6,0)		
	J	E(MeV)	%	2×J	E(MeV)	%	J	E(MeV)	%	2×J	E(MeV)	%
	0	0	78	1	0	86	0	0	47	1	1.43	59
	2	1.45	82	3	1.26	72	2	1.87	55	3	0	36
	4	3.86	76	5	0.36	75	4	3.74	50	5	2.03	43
	6	8.18	80	7	5.12	68	6	8.80	65	7	1.97	30
	8	12.35	72	9	2.78	81	8	13.63	91	9	7.12	48
				11	6.98	0				11	6.01	90
				13		70						

for the state [f] = [3] (λ,μ) = (6,0) in the final state. Such numbers are shown for the ground state rotational band in ^{19}F. We see that all but the $11/2^+$ state are dominated by this (6,0) state. Two reactions involving this transfer have been reported. The Penn group[16] have studied the $^{16}O(^6Li,t)^{19}F$ reaction, and their results are consistent with the shell model prediction; i.e., all but the $11/2^+$ state in the ground state band are populated. The 1972 Annual Report from Munich[17] reports briefly on a study of the (α,p) reaction. In that reaction all states, including the $11/2^+$ state, are populated. The (α,p) experiment was done with 20 MeV α-particles, so the question of how "direct" the reaction is must be considered. In any case, these numbers should be useful in analyzing this type of data. A similar analysis should be useful for the analogous hole transitions on ^{40}Ca. Thus, the S-factors for the three particles and four-particle clusters to ^{37}K, ^{37}Ar, or ^{36}Ar should again be proportional to the intensity of the (λ,μ) = (6,0) and (8,0) in the states in these nuclei. These numbers also are presented in Table I. The percentages shown there indicate that although the SU(3) representations are not as "good" here as they are in the lighter s-d shell nuclei, there is still significant cluster "strength" in these low-lying levels. For the higher-spin states in both the A=36 and A=37 nuclei, the SU(3) purity becomes quite large. The positions of these states are not known, to my knowledge. These numbers suggest that the three- and four-particle cluster transfer reactions on ^{16}O and ^{40}Ca to s-d shell nuclei would provide good tests for studies of the reaction used to effect such transfers.

One interesting facet of the heavy-ion transfer work reported so far is the selective population of high-spin states. There is evidence that the conventional shell model is particularly effective in predicting the occurrence and positions of these levels. Some of this evidence is summarized in a contribution to this conference by Gogelein, et al., and in a recent note by Goode and Zamick.[18] A particularly striking example of this is found in the study of the reaction $^{10}B(^{16}O,\alpha)^{22}Na$ carried out by the Oak Ridge tandem group.[19] The spectrum of the positive parity levels calculated[19] in a complete $(sd)^6$ space is compared with the observed levels in Fig. 4. All states in the ground state K=3 band up to the 10^+ state have been tentatively identified. There is a distinct kinking feature in the spectrum which presumably arises because only odd-spin levels are present in the next higher $K=0^+$ band, and so band mixing is possible only for these odd-spin states, which are the ones which are depressed in the ground state band.

The successes of the shell model are sufficient to justify some effort to

extend the capabilities of the model as far as possible. But even with significant advancement in technology such as I discussed earlier, there are still limitations on what shell model calculations are practical. For example, the dimensions of the $(fp)^n$ problem still make the exact shell model diagonalization impractical for $n \geq 7$. It is still necessary, and certainly desirable, to find truncation schemes which reduce the magnitude of the shell model calculation significantly and/or which provide some insight into the structure of the states of interest. It would, of course, be highly desirable to find groups which are as useful in other mass regions as the SU(3) group is in the s-d shell. Hecht[21] and his collaborators have had some success with so-called pseudo-$\ell \cdot s$ and pseudo-SU(3) calculations in the N=82 region. I would like to take the last minutes of my talk to talk on the possibility of using extensions of the weak coupling ideas which have been used with considerable success by Arima and his co-workers,[20] as well as many others.

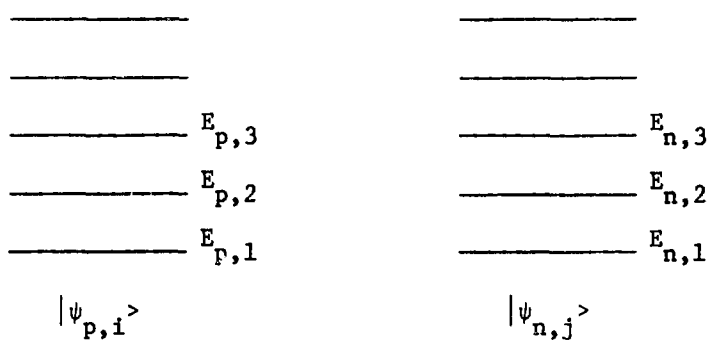
The general idea is outlined as follows. Suppose the shell model calculation is too big to handle in the desired model space. Then divide the model space into two smaller subspaces in which exact calculations are possible. One can then calculate two sets of "exact" eigenvalues and eigenvectors. Next form product wave functions consisting of one eigenvector from each of the smaller subspaces. The product wave functions span part of the complete large space. The question is, is it possible to select a relatively small number of subspace vectors such that the product space generated from this set of subspace vectors has a significantly large overlap with some set of low-lying eigenstates in the complete space.

Wong and Zuker[22] have studied such an approach to shell model calculations in the s-d shell. They studied the nuclei ^{20}Ne, ^{22}Na, and ^{24}Mg, where exact shell model calculations are possible. They treated the ground state band of ^{20}Ne as the coupling of low-lying eigenstates of ^{18}F. They first carried out the conventional shell model calculation of ^{18}F as two particles outside an inert ^{16}O core. In this space there are 28 two-particle states. In their truncated calculation of ^{20}Ne, they included the lowest 11 ^{18}F eigenstates. They then formed all possible states of products of two of these 11 states, and diagonalized the effective Hamiltonian in this product space. The results are summarized in Fig. 5. In this figure are indicated the dimension of the calculation in the complete $(sd)^4$ space and in the truncated space, and the overlap of the approximate eigenvector with the exact eigenvector. It is seen that the qualitative features of the excitation spectra of the ground state rotational band are reproduced in the approximate calculation. The dimensions of the approximate calculations are from 1/2 to 1/3 of the full $(sd)^4$ dimensions, and the overlaps of the approximate wave function with the exact wave function varies from 0.995 to 0.932. These results are only mildly encouraging, since the reduction in dimensions effected here is not sufficient to suggest that really large calculations can be treated in this way. However the results for ^{24}Mg are more encouraging. If ^{24}Mg is calculated in a product space generated from the lowest 0^+, 2^+, 4^+, and 6^+ states in ^{20}Ne, the calculated ground state, $J = 0^+$, eigenvector, which is 4-dimensional, has a 0.961 overlap with the ground state eigenvector obtained in a full $(sd)^8$ space with dimension 325. The main difficulty with this particular type of calculation is that the two subspaces are not inequivalent; i.e., they contain orbits in common. The required treatment of anti-symmetrization effects adds a great deal to the complexity to the calculation, and limits its usefulness for carrying out calculations in spaces where the complete calculation is impossible.

Such calculations are more straightforward in cases where the space can be divided into inequivalent subspaces. One such case would be to work in a neutron-proton formalism and to split the complete space into a neutron space and a proton space. Then the effective Hamiltonian can be split up as follows:

$$H = H_{p-p} + H_{n-n} + H_{p-n} \tag{3}$$

in an obvious notation. Assume that the neutron-neutron and proton-proton parts of the interaction can then be treated more or less exactly. This leads to a set of neutron and proton eigenstates



where i and j includes all labels of the proton and neutron eigenstates and $E_{p,i}$ and $E_{n,j}$ label indicate the energies of the eigenstates. If we construct the product states of the form

$$|\psi^{i,j}_{n,p}>_J \equiv |\psi_{n,i} \times \psi_{i,j}>_J \tag{4}$$

where the × implies angular momentum coupling, then the matrix elements of the complete Hamiltonian between such states takes the form

$$\begin{aligned} <\psi^{i,j}_{n,p}|H|\psi^{i',j'}_{n,p}>_J &= (E_{n,i} + E_{p,j})\ \delta_{ii'}\ \delta_{jj'} \\ &+ <\psi^{i,j}_{n,p}|H_{np}|\psi^{i',j'}_{n,p}>_J. \end{aligned} \tag{5}$$

Then the last matrix element in (5) can be written

$$\begin{aligned} <\psi^{i,j}_{n,p}|H_{np}|\psi^{i',j'}_{n,p}>_J &= \sum_{\substack{aa' \\ bb' \\ \Delta}} G(aa'bb'\Delta)\ <\psi^{i}_{n}||(A^a \times B^{b'})^{\Delta}||\psi^{i'}_{n}) \\ &\times <\psi^{j}_{p}||(A^b \times B^{b'})^{\Delta}||\psi^{j'}_{p}>. \end{aligned} \tag{6}$$

In (b), $G(aa'bb'\Delta)$ is a function of the two-body matrix elements of H_{np}, a and a' are summed all neutron states, b and b' are summed over all proton states, and A and B are proportional to the usual single particle creation and destruction operators respectively. From this expression we can see that there are two ways in which a weak coupling scheme can be useful. If the neutron-proton interaction is weak relative to the proton-proton and neutron-neutron interaction, then only states $|\psi^{i,j}_{n,p}>$ and $|\psi^{i'j'}_{n,p}>$ which are close to each other in energy will mix, and one can use energy denominator arguments alone to truncate. There are reasons[20] to expect the neutron-proton interaction is weak when there is a neutron excess, so that the active proton orbits are different from active neutron orbits. A second possibility is that one can divide the complete set of proton (and/or

neutron) eigenstates into several subsets so that all the states within each subset are "collective" with respect to the hole-particle operator, $A \times B$. By this I mean that the set of proton states can be divided into sets α and β, and a state in α will have non-vanishing matrix elements of the operator $A \times B$ only with another member of set α. If there is a subset of proton states and of neutron states which are "collective" with respect to the hole-particle operator in this sense, then the eigenstates obtained by diagonalizing the effective Hamiltonian in the product space formed from states within the "collective" neutron space and states within the "collective" proton space will have an exact overlap with the space spanned by some set of eigenstates of the Hamiltonian in the complete space. There is no obvious reason that such subsets of proton or neutron eigenstates would exist. What might be possible is to construct a truncated basis along the lines suggested by this expression. One could assume that the lowest few states in the proton and neutron spaces would be important in the low-lying neutron-proton wave functions, include them in the truncated space, plus all other states generated by acting on these low-lying states with the $A \times B$ operator. This space would then include the first-order effects of coupling higher states to the low-lying eigenstates. However, the number of $A \times B$'s can be quite large, so this might not be of practical use. A more useful approach is suggested by assuming that H_{np} is a surface delta interaction (SDI). Such an interaction has been quite successful in shell model calculations. For such an interaction, (6) becomes,[23] again schematically,

$$\langle\psi^{i,j}_{n,p}|H^{SDI}_{np}|\psi^{i',j'}_{n,p}\rangle_J = -G\sum_k (-)^{J_i+J_{j'}-J}\sqrt{\frac{(2J_i+1)(2J_j+1)}{(2J+1)}} \tag{7}$$

$$W(J_iJ_jJ'_iJ'_j;kJ)\ \langle J_i\|Q^K\|J_{i'}\rangle\langle J_j\|Q^K\|J_{j'}\rangle.$$

Here, G is the strength of the neutron-proton interaction, Q^K is a normalized spherical harmonic, and only the angular coordinates of the constituents of the matrix elements are considered. In this case a perfect truncation scheme would exist if there were a subset of proton (and/or neutron) states which were "collective" with respect to the surface multipole operator in the same sense that we spoke of collectivity above. There are cases where such collectivity does exist to a fair extent. If one treats four identical particles in the (f-p) shell in a full shell model calculation with a surface delta interaction for the residual interaction and degenerate single particle energies, it turns out that on the order of 80-90% of all the relevant surface multipole strength is contained within 18 low-lying states.[23] Thus a truncation to these 18 states should be a very "good" truncation scheme. I have made a similar calculation for four particles on the f-p shell with the "realistic" Kuo-Brown f-p shell interaction and "experimental" single particle energies, and in this case the "collectivity" is significantly diluted. I have calculated all the surface multipole operators between the lowest six eigenstates and all other eigenstates in the space. I have analyzed the numbers as follows. For each low eigenstate and each multipole, I calculate the matrix elements for each Q^K with all other states of a given J. Define the total strength as the sum of absolute values of all the matrix elements to states with that J value. I then see how many matrix elements each contribute more than 1% of this total strength, and what fraction of the total strength is contributed by all such matrix elements. A typical result is that 10-12 final states each contribute more than 1% of the total strength, and the total contribution of these 10-12 states is about 80% of the total strength. I have not yet proceeded further to see if the surface multipole matrix elements from these 10-12 states connect mainly among themselves. Even assuming they do, this still says that roughly 100 states should be included in the truncation scheme, involving the $(fp)^4$ space of neutrons or protons, and this is not a severe enough truncation to be practical. What I do suspect may be a useful approach is to select a set of low-lying proton and neutron states, and add to them all linearly independent states which can be

found by acting on these low-lying states with the relevant surface multipoles. In the f-p shell, if we include as many as 10 low-lying states, we will generate at most 30 more states with the surface multipole operator. This basis will then include the strong first-order coupling of higher states to the low-lying eigenstates. This good first-order approximation, plus the expected relatively weak two-body interactions between particles in different orbits in nuclei with neutron excesses, offer a hopeful route to the application of large shell model techniques to the many interesting mass regions now inaccessible to standard shell model calculations.

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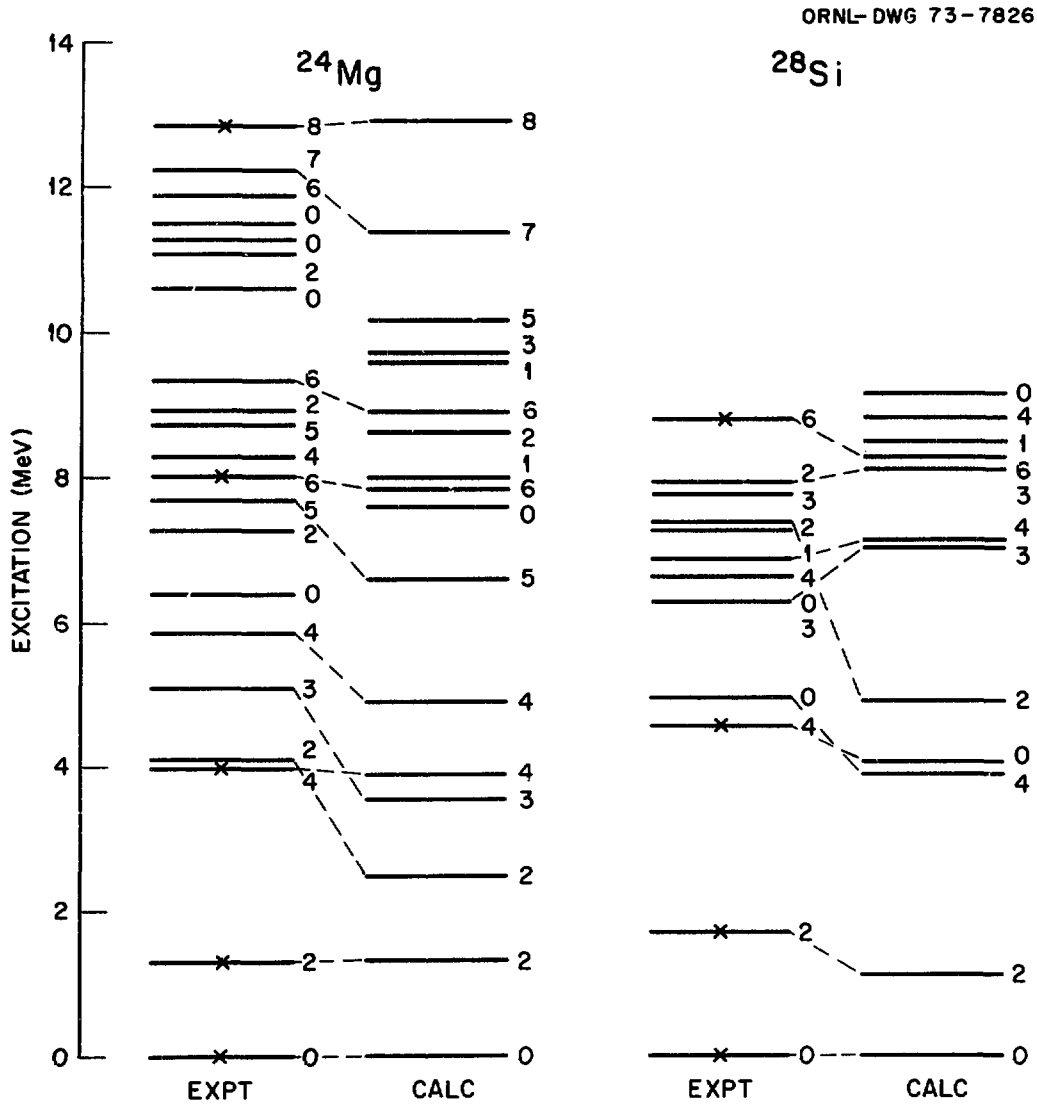

Fig. 1. Observed and Calculated[4] Spectra of Low-Lying Positive Parity States in ^{24}Mg and ^{28}Si. States in the ground state bands are indicated by crosses.

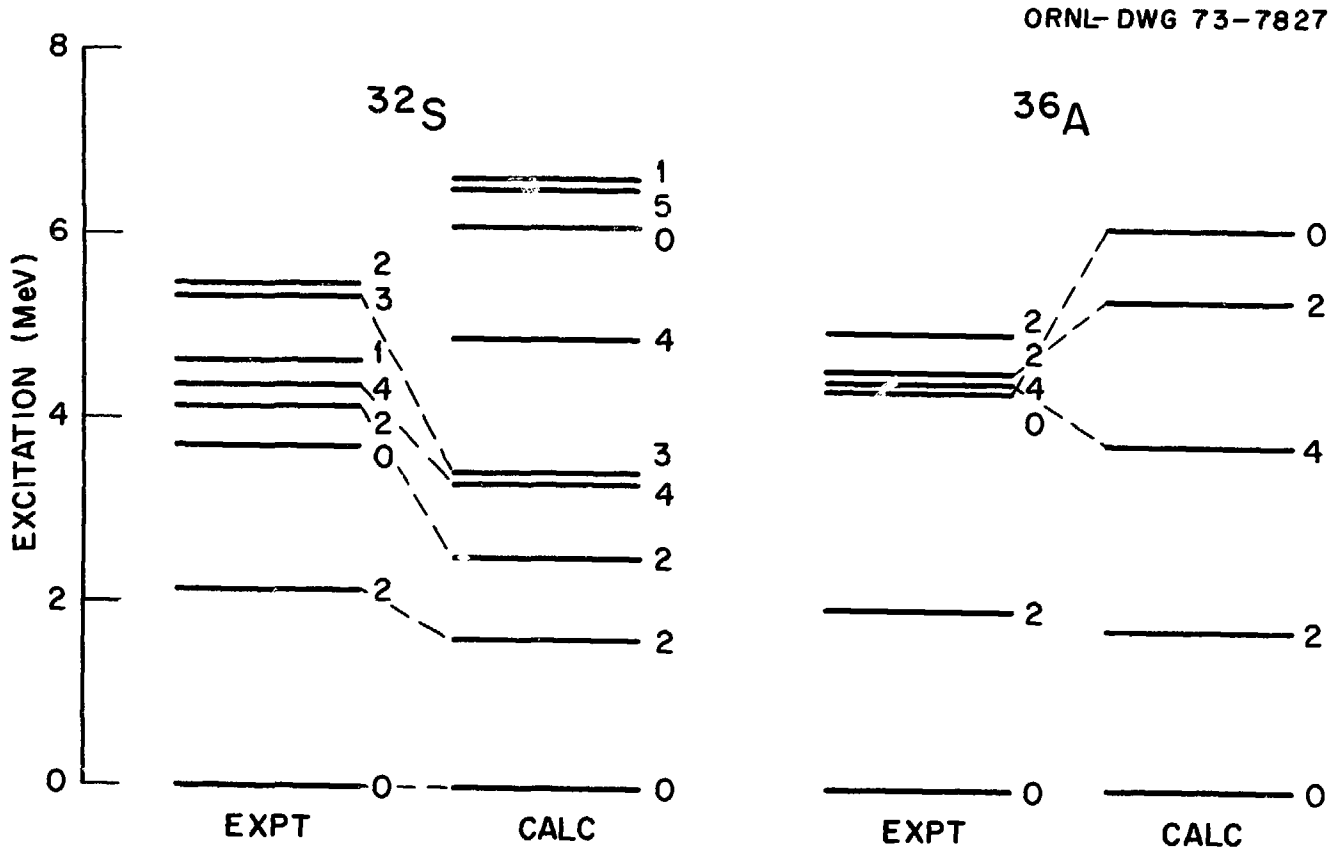

Fig. 2. Observed and Calculated[4,5] Spectra of ^{32}S and ^{36}Ar.

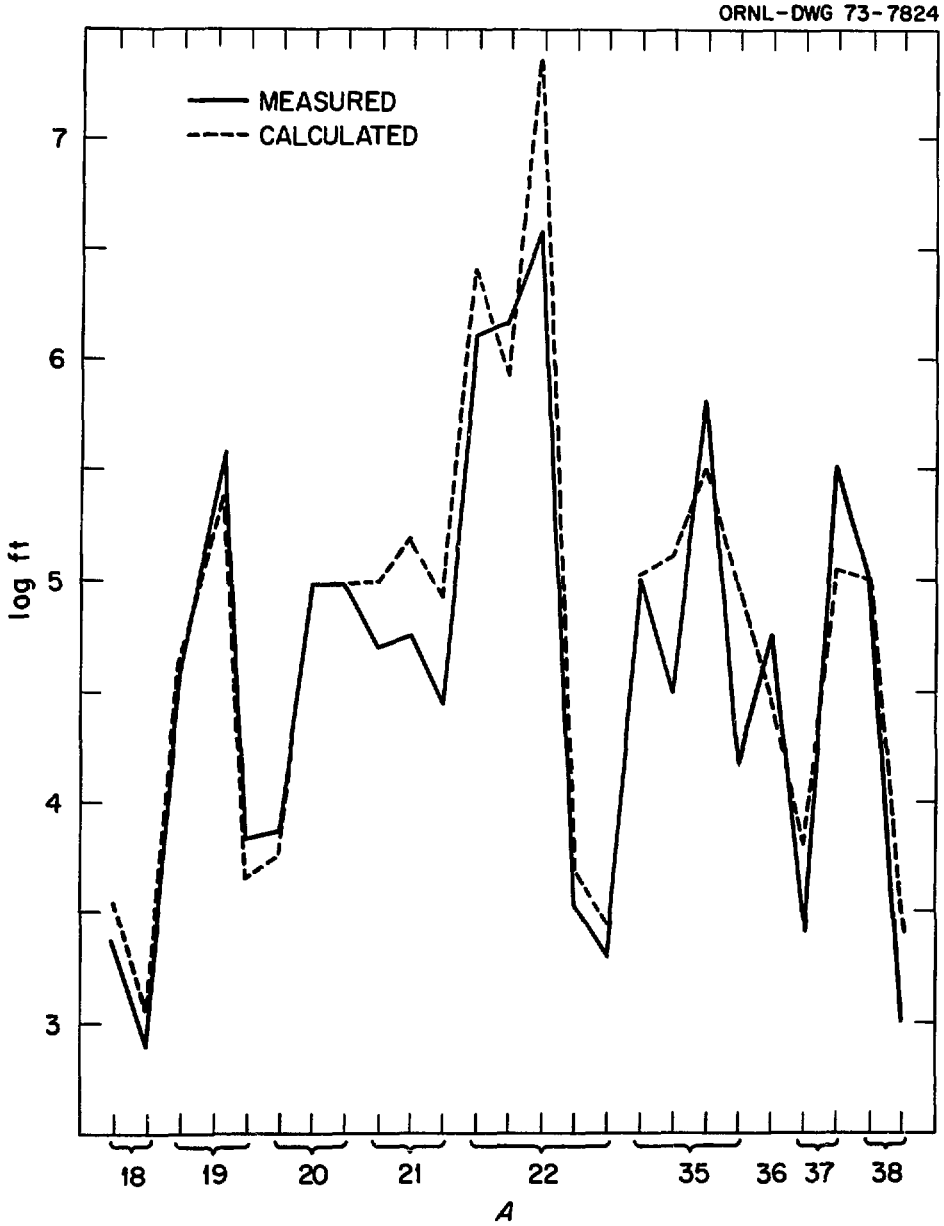

Fig. 3. Measured and Calculated[14] Logft Values for Gamow-Teller β-decay Transitions in s-d Shell Nuclei with A $\leq$ 22 and A $\geq$ 35. The label on the abscissa indicates the mass of the nucleus in which the decay is observed.

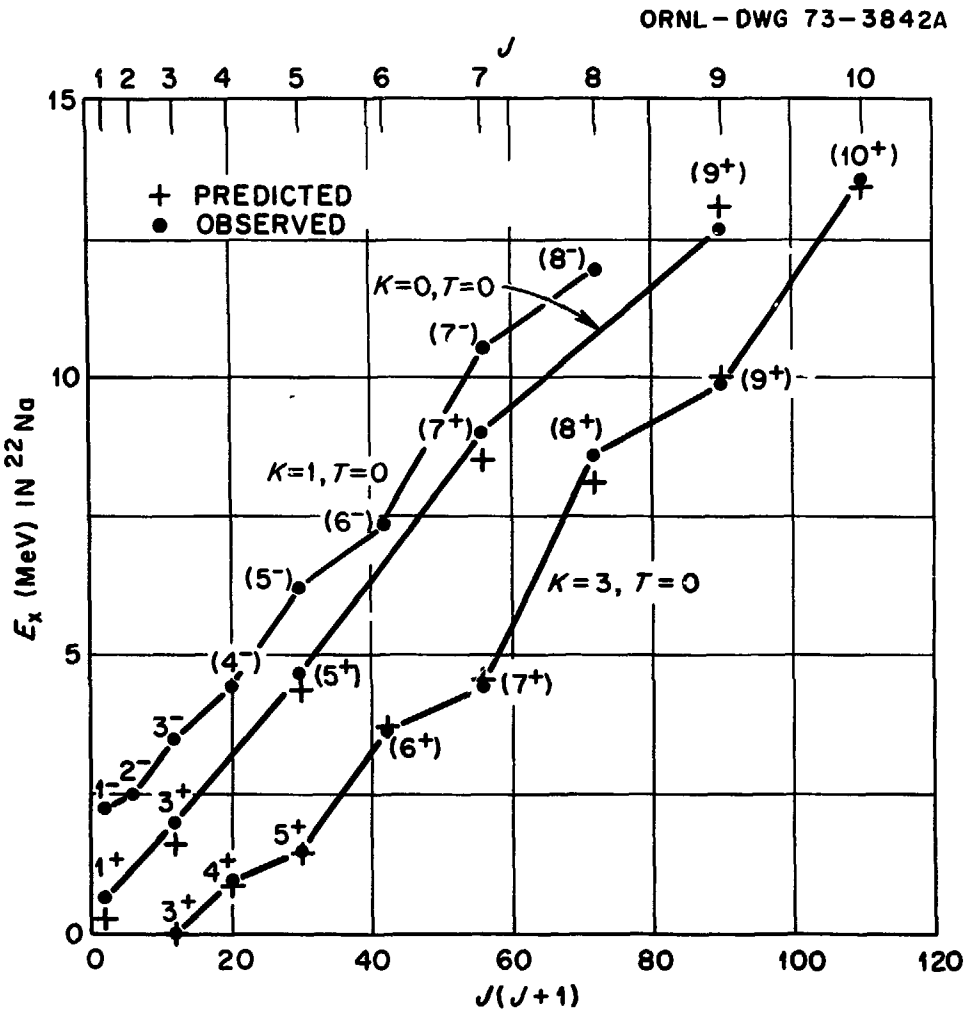

Fig. 4. Calculated and Observed[19] Positions of States in Lowest Three Known "Bands" in ^{22}Na.

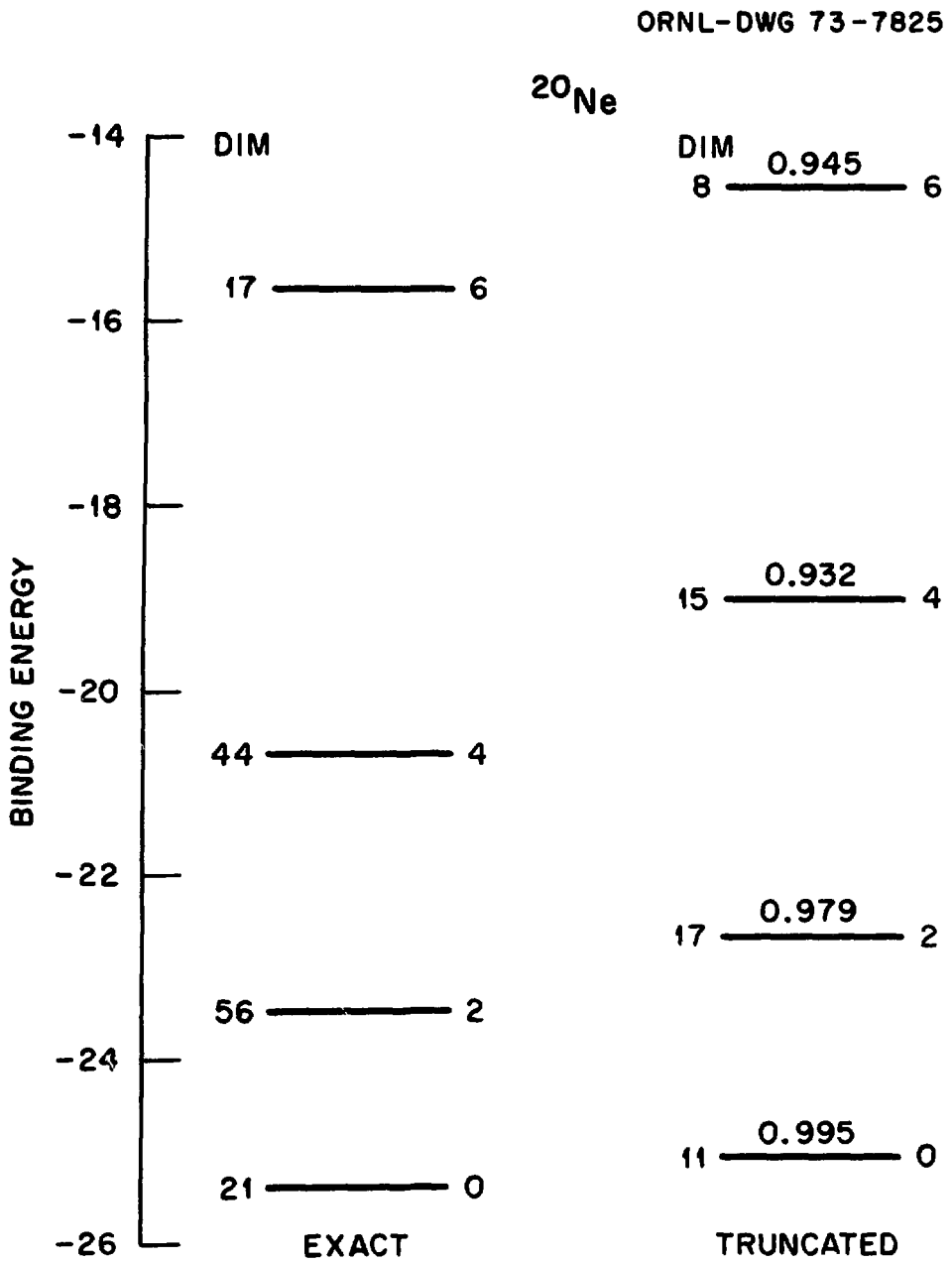

Fig. 5. Binding Energies of States in Ground State Band of ^{20}Ne as Calculated[22] in Complete $(sd)^4$ Space (EXACT) and in "Weak Coupling" Approximation (Truncated) Discussed in Text. The numbers in column headed DIM are the dimensions of the space for the indicated J-value in the two models. The numbers over the levels in the "truncated" spectrum are the overlaps of the approximate wave functions with the exact wave functions.