CONF-960832--1

RADIAL AND ORBITAL $q\bar{q}$ EXCITATIONS: "HIGHER QUARKONIA"

T. BARNES
Oak Ridge National Laboratory*
Oak Ridge, TN 37831

DEC 10 1996
OSTI

to be published in

Proceedings of Fourth Biennial Conference on Low Energy Antiproton Physics

Dinkelsbühl, Germany
August 27-31, 1996

MASTER

*Managed by Lockheed Martin Energy Research Corp. under contract DE-AC05-96OR22464 with the U.S.D.O.E.

DISCLAIMER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Radial and Orbital $q\bar{q}$ Excitations: "Higher Quarkonia"

T.Barnes[a]

[a]Theoretical and Computational Physics Group, Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6373, USA
and
Department of Physics, University of Tennessee, Knoxville, TN 37996, USA

Identification of the spectrum of non-$q\bar{q}$ exotica such as glueballs, hybrids and molecules will require a detailed understanding of radially and orbitally excited quarkonia, since these states are a background to non-$q\bar{q}$ states. The data likely to become available experimentally on these higher-mass resonances are their total widths and strong branching fractions to dominant modes. We advocate the use of these branching fractions to distinguish quarkonia from exotica; in this contribution we summarize our recent detailed ^{3}P$_0$ decay model calculations of the 374 two-body open-flavor decay modes of the 32 lightest $n\bar{n}$ states above 1S and 1P, which should be useful for this program.

1. INTRODUCTION

The existence of non-$q\bar{q}$ mesons is already well established. In multiquark systems the well-known $f_0(980)$ and $a_0(980)$ mesons clearly do not have masses or couplings consistent with $q\bar{q}$ states, and the alternative assignment as $K\bar{K}$ molecules [1] is widely believed to be at least qualitatively correct. More recently there has been great excitement over the possible discovery of meson resonances in the gluonic sector, specifically the LEAR glueball candidate $f_0(1500)$ [2], a possible exotic hybrid reported at BNL [3], and a possible nonexotic $\pi(1800)$ hybrid investigated by VES [4,5].

An ideal candidate for a non-$q\bar{q}$ meson would have exotic quantum numbers, such as I=2 or $J^{PC} = 1^{-+}$, that are completely forbidden to $q\bar{q}$. Unfortunately this is seldom the case, so we must use some property of a resonance to distinguish between $q\bar{q}$ and non-$q\bar{q}$ assignments. This underscores the importance of developing a clear and complete picture of the light meson spectrum: We cannot unambiguously claim discovery of non-$q\bar{q}$ exotica with conventional quantum numbers until we have understood the $q\bar{q}$ background.

Historically, quark model assignments for light hadrons were confirmed through measurements of the experimentally accessible electromagnetic, weak and strong couplings. As examples; the magnetic moments of the octet baryons made it clear that the qqq wavefunctions were approximately correct; the two-photon couplings of many $C = (+)$ $q\bar{q}$ mesons from the π^o to the χ_2 are consistent with quark model expectations for $q\bar{q}$ states; the relative branching fractions of $f_2(1275)$ and $f_2'(1525)$ to $\pi\pi$ and $K\bar{K}$ clearly show that these are approximately pure $n\bar{n}$ and $s\bar{s}$ tensor mesons respectively.

In the higher mass region of $\approx$ 1.5-2.5 GeV of current interest in the search for glueballs and hybrids, hadron experiments will probably tell us the mass, total width, and relative branching fractions of a resonance to a few final states, but we will have no information about other couplings. Thus it will be important to develop a detailed understanding of the strong decay mechanism and to have reasonably accurate numerical predictions for the branching fractions of $q\bar{q}$ states, to test a $q\bar{q}$ hypothesis for a given experimental resonance.

2. DECAY MODELS

2.1. The ^{3}P$_0$ Decay Model

Decays of light hadrons are usually described using the phenomenological ^{3}P$_0$ model [6], which assumes that open-flavor strong decays take place through $q\bar{q}$ pair production from the vacuum, with vacuum ($^3\mathrm{P}_0$) quantum numbers. (The "flux-tube" decay model is a simple generalization of the $^3\mathrm{P}_0$ model in which pair production is taken to be more likely near the axis of the initial $q\bar{q}$ pair.) As we have shown in recent work, the $^3\mathrm{P}_0$ model can be represented in terms of generalized Feynman diagrams with explicit wavefunctions attached to the external quark lines [7]. One of the two $^3\mathrm{P}_0$ Feynman diagrams for meson decay is shown in Fig.1.

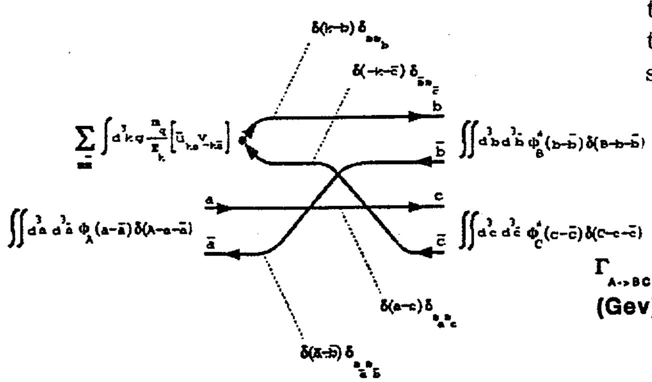

Figure 1. A Feynman diagram for meson decay in the $^3\mathrm{P}_0$ model.

Although the relation of this model to QCD is obscure, some theoretical work [7,8] suggests that it may simply be an approximate description of $q\bar{q}$ pair production from spectator lines through the confining interaction. The more obvious OGE mechanism leads to a linear superposition of $^3\mathrm{P}_0$ (color Coulomb) and $^3\mathrm{S}_1$ (transverse gluon) pair production amplitudes, and appears in most cases to be numerically weaker than pair production from scalar confinement [7]. Although the theoretical justification of $^3\mathrm{P}_0$-type decay models remains an open question, this decay mechanism nonetheless passes several nontrivial amplitude tests and is the numerically most accurate approach to calculating partial widths currently available.

2.2. Illustration of the $^3\mathrm{P}_0$ Decay Model

As a simple test application, in Fig.2 we show the decay rates predicted by the $^3\mathrm{P}_0$ decay model for eight strong decay modes of light, (mostly) well-established $q\bar{q}$ states below 1.5 GeV. The wavefunctions used for these states are simple harmonic oscillator forms, with an inverse length scale β; these are often used in decay calculations because they lead to closed-form decay rates. The length scale commonly used in the literature for light meson decays is $\beta = 0.4$ GeV. The overall scale of the rates is fixed by a pair production amplitude γ that is an adjustable parameter in the $^3\mathrm{P}_0$ model, here we set it to $\gamma = 0.5$ for illustration. (In the "higher quarkonia" above 1P we find that this overestimates widths of well established states, and will instead use $\gamma = 0.4$.)

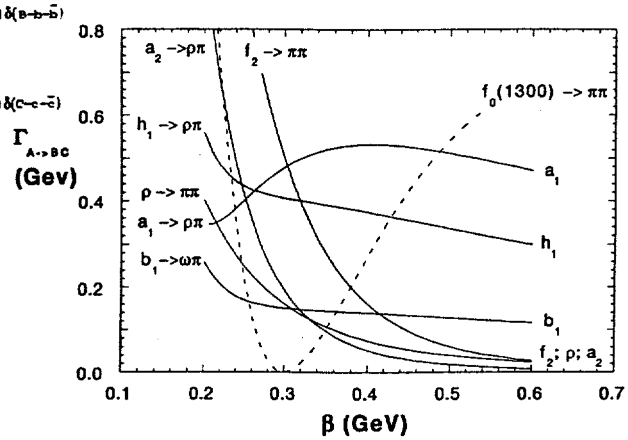

Figure 2. Decay rates of representative 1S and 1P $n\bar{n}$ mesons in the $^3\mathrm{P}_0$ model, with $\gamma = 0.5$.

Evidently the partial widths predicted by the $^3\mathrm{P}_0$ model with $\beta = 0.4$ GeV and $\gamma = 0.5$ are approximately correct; the h_1 and a_1 are predicted to be the broadest, followed by the f_2 at about half the h_1 and a_1 widths, then the ρ and a_2. The least accurate is the ρ, with a relative width about 1/2 as large as experiment. Clearly this model gives a reasonable qualitative picture of widths, and the complicated variation with the length scale β shows that the observed pattern of widths is strongly wavefunction-dependent rather than just being due to SU(3) or SU(6) group-theory relations.

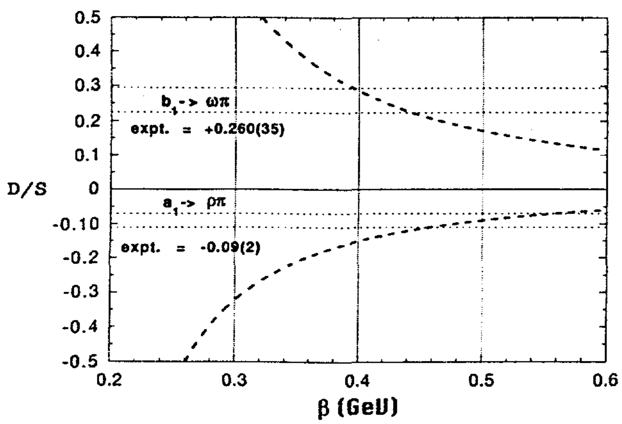

Figure 3. D/S amplitude ratio in the decays $b_1 \to \omega\pi$ and $a_1 \to \rho\pi$, showing theory (solid) and experiment (dashed).

A more striking test of the $^3P_0$ model is provided by the decays $b_1 \to \omega\pi$ and $a_1 \to \rho\pi$; these populate both S- and D-wave final states, and the D/S amplitude ratio can be measured experimentally. This ratio proves to be very sensitive to the quantum numbers of the $q\bar{q}$ pair produced in the decay. Fig.3 shows the D/S ratios for these two decays, and there is clearly reasonable agreement for $\beta \approx 0.4$ GeV. This test (for the b_1) was historically one of the more important in confirming the accuracy of the $^3P_0$ model. In contrast, OGE pair production, with mixed $^3P_0$ and $^3S_1$ quantum numbers, gives D/S ratios of the wrong sign [7].

3. DECAYS OF HIGHER QUARKONIA

3.1. Derivation of decay formulas

Until recently decay calculations in the $^3P_0$ model had only been carried out for the lowest few $q\bar{q}$ levels. These studies include the work of Busetto and Oliver [9], who studied decay modes of 2S radial vectors, Blundell and Godfrey [10], who considered $\approx$ 30 decay modes of excited quarkonia, notably those of the the $\xi(2230)$, and LeYaouanc *et al.* [11], who considered branching fractions of radially excited $c\bar{c}$ mesons into open-flavor states.

As we ascend in mass we find a rapid increase in the number of $q\bar{q}$ levels, which have a correspondingly large number of two-body modes due to the increased phase space. If we extend the application of the $^3P_0$ model to all $n\bar{n}$ states expected up to 2.1 GeV (somewhat above the expected mass of the lightest hybrid multiplet), we find a total of 44 states, which span 1S, 2S, 3S, 1P, 2P, 1D and 1F levels. Since the 1S and 1P states are mostly noncontroversial, we have concentrated on the decays of the remaining 32 states; assuming the decays are all open-flavor and quasi-two-body, we are led to the calculation of 374 decay modes.

Evaluation of such a large number of decay amplitudes is clearly a difficult theoretical task. It is further complicated by the presence of several amplitudes in many of the modes, complicated radial and orbital overlap integrals, and intricate angular momentum recombination problems. As an illustration of the increase in difficulty of these decay calculations, the relatively simple P-wave decay rate $\Gamma(\rho \to \pi\pi)$ is

$$\Gamma(\rho \to \pi\pi) = \frac{2^{10}}{3^6} x^3 \gamma^2 \pi^{1/2} \frac{E_\pi^2}{M_\rho} e^{-x^2/6} \qquad (1)$$

(where $x = |\vec{p}_f|/\beta$ and $E_\pi = M_\rho/2$), whereas the theoretically dominant πa_1 decay mode of a $^3D_1$ $\rho(1700)$ $q\bar{q}$ state has two amplitudes, and a total rate given by

$$\Gamma(\rho_D \to \pi a_1) = \frac{2^{12}5}{3^8} x \left(1 - \frac{5}{9}x^2 + \frac{673}{5400}x^4 - \frac{1}{450}x^6 + \frac{1}{12150}x^8\right) \cdot \gamma^2 \pi^{1/2} \frac{E_\pi E_{a_1}}{M_{\rho_D}} e^{-x^2/6} \ . \qquad (2)$$

The presence of nodes in the latter formula makes it clear that simple arguments about allowed modes which do not take the detailed momentum depedence of the decay amplitudes into account may lead to invalid conclusions regarding the important decay modes of these higher states.

To guard against error in deriving the many similarly complicated decay formulas we developed algebra programs that automatically evaluate these decay amplitudes and rates for general quarkonium quantum numbers in closed form, and also evaluate the total rates numerically. The detailed decay amplitudes found in this research project are presented in Ref.[12].

3.2. Specific resonances: $a_1(1700)$

As a first application of these $^3\mathrm{P}_0$ model results we consider the large 1^{++} signal recently reported near 1.7 GeV by BNL E818 [3] in πf_1 and by VES in $\rho\pi$ [5]. *A priori* the large πf_1 signal suggests a hybrid, and the flux-tube model does indeed predict that the lightest hybrid multiplet contains a 1^{++} level. Theoretical hybrid mass estimates are also consistent with this possibility [13,14]. Conversely this 1.7 GeV state may just be a $2^3\mathrm{P}_1$ quarkonium resonance, a first radial excitation of the a_1. This $q\bar{q}$ state is expected by Godfrey and Isgur at 1.82 GeV [15].

We can test the plausibility of hybrid and 2P quarkonium assignments for this resonance by comparing the decay amplitudes and branching fractions expected in each case. The branching fractions are given in Table 1. (The hybrid partial widths use the results of Close and Page [16] with masses adjusted to this case, and the quarkonium numbers are taken from Barnes *et al.* [12].) Evidently the branching fractions to $\rho\pi$ and $f_1\pi$ alone are not very characteristic and do not allow a choice between assignments. The branching fractions to $b_1\pi$ and $\rho\omega$, which have not been measured, would be much more useful in classifying this resonance.

At the amplitude level we find very characteristic differences between hybrid and quarkonium assignments. The $\rho\pi$ mode of an $a_1(1700)$ is allowed both S- and D-wave amplitudes, and for a 2P $q\bar{q}$ $a_1(1700)$ the S-wave amplitude has a zero quite close to the physical point given the standard wavefunction length scale $\beta = 0.4$ GeV (see Fig.4).

The ratio is predicted to be

$$\frac{S}{D}\bigg|_{a_{1r}\to\rho\pi} = -\frac{3^2\cdot 5}{2^{1/2}\cdot 7}\,\frac{\left(1-\frac{4}{9}x^2+\frac{4}{135}x^4\right)}{x^2\left(1-\frac{2}{21}x^2\right)}\,, \quad (3)$$

which has a zero at $x = (3/2)^{1/2}5^{1/4}(5^{1/2} - 2^{1/2})^{1/2} = 1.660$. In comparison, the physical point for $a_1(1700) \to \rho(770)\pi(138)$ is $|\vec{p}| =$ 667 MeV, hence $x = 1.67$ (for $\beta = 0.4$ GeV),

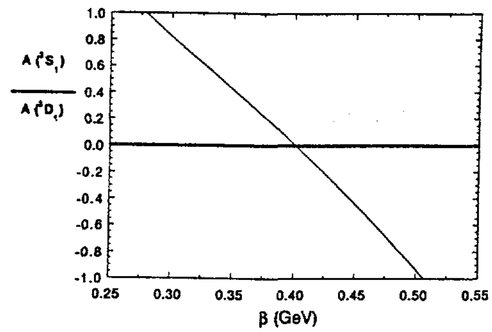

Figure 4. S/D amplitude ratio in the decay $2^3\mathrm{P}_1$ $a_1(1700) \to \rho\pi$.

essentially at the theoretical zero in the S-wave amplitude.

This remarkable prediction of a small $(\rho\pi)$ S-wave and dominant D-wave agrees with experiment: The VES data shows that the $a_1(1700) \to \rho\pi$ transition is dominated by the $(\rho\pi)_D$ final state (Fig.5).

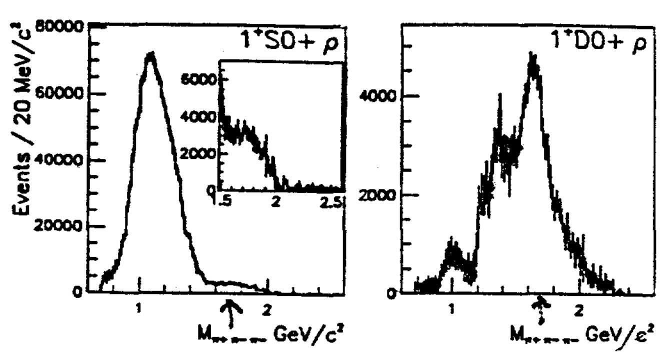

Figure 5. $a_1(1700)$ dominance of the $\rho\pi$ D-wave.

For a hybrid one instead predicts more conventional S-wave dominance, so the $a_1(1700)$ appears to be confirmed as a $2^3\mathrm{P}_1$ radial excitation of the

Table 1
Partial widths (in MeV) of 2P and hybrid $a_1(1700)$ states.

	$\rho\pi$	$\rho\omega$	$\rho(1465)\pi$	$b_1\pi$	$f_0(1300)\pi$	$f_1\pi$	$f_2\pi$	K*K	total
$a_{1(2P)}$	57.	15.	41.	41.	2.	18.	39.	33.	246.
$a_{1(H)}$	30	0	110	0	6	60	70	20	≈ 300

$a_1(1260)$.

3.3. $\pi(1800)$

Although there has been evidence of a $\pi(1800)$ resonance for some time, it has only recently attracted attention as a hybrid candidate. This was suggested by the VES observation of the $\pi(1800)$ in $\pi f_0(1300)$ shown in Fig.6, combined with the surprising absence of the simpler S+S mode $\rho\pi$. ($\pi f_0(1300)$ is an S+P final state; these are preferred theoretically for hybrids in flux-tube model decay calculations, whereas S+S modes like $\rho\pi$ are expected to be weak [16,13]).

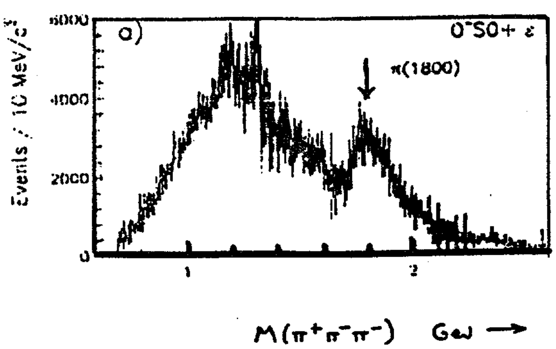

Figure 6. The $\pi(1800)$ in $(\pi\pi)_S\pi$.

Since the competing quarkonium assignment is a 3S *second radial excitation* of the pion (expected by Godfrey and Isgur at 1.88 GeV [15]) one should view such arguments with caution. The nodes in a radial wavefunction can lead to nodes in decay amplitudes near the physical point which "notch out" naively expected modes. The partial widths to the six allowed open-flavor two-body modes of a 3S $\pi(1800)$ and a $\pi(1800)$ hybrid are given in Table II. We find that the $\rho\pi$ mode of a 3S $q\bar{q}$ $\pi(1800)$ does indeed have a node not far from the physical point (specifically at $\beta \approx 0.32$ GeV), so the weakness of the $\rho\pi$ mode is not a good argument against a quarkonium assignment. More characteristically, we find that the $f_0(1300)\pi$ mode from a $q\bar{q}$ state should be weak. In contrast it is predicted to be dominant for a hybrid. Thus the $\pi(1800)$ passes an important test for a hybrid, and it will be important to search for this state in other modes such as $\rho\omega$, which is predicted to be dominant for $q\bar{q}$ but weak for a hybrid.

Of course this test is only as good as the $^3P_0$ model, and one should not exclude the possibility that anomalous branching fractions such as the large coupling $\pi(1800) \to f_0(1300)\pi$ are simply indications that the decay model is inaccurate. There is theoretical evidence for example that the $^3P_0$ decay model may be inaccurate in the related channel $^3P_0 \to {}^1S_0 + {}^1S_0$ [7], due to an especially large OGE decay amplitude. This may invalidate the application of the simple $^3P_0$ model to the $\pi(1800)$, and require a more careful treatment incorporating OGE decay amplitudes.

3.4. $\rho(1465)$ and $\rho(1700)$

There is long-standing confusion over the nonstrange vector resonances near 1.5 GeV [17]. Quark potential models typically expect 2^3S_1 radial excitations of the ρ and ω around 1.4-1.5 GeV, and in addition there should be $^3D_1$ $n\bar{n}$ vectors near 1.7 GeV [15]; members of this 1D multiplet such as the $\pi_2(1670)$, $\omega_3(1667)$ and $\rho_3(1691)$ are well established. The complications in the 1^{--} sector arise from the presence of both 2^3S_1 and $^3D_1$ levels: These are expected to be relatively broad resonances, which will overlap experimentally, and hence will be difficult to identify unambiguously. The physical resonances may in addition be significantly mixed linear combina-

Table 2
Partial widths (in MeV) of 3S and hybrid $\pi(1800)$ states.

	$\rho\pi$	$\rho\omega$	$\rho(1465)\pi$	$f_0(1300)\pi$	$f_2\pi$	K*K	total
$\pi_{3S}(1800)$	30.	74.	56.	6.	29.	36.	231.
$\pi_H(1800)$	30	0	30	170	6	5	≈ 240

tions of these diagonal-L basis states. Clearly it will be useful to know what final states pure initial 2^3S_1 and $^3D_1$ basis states would couple to, and with what relative strengths.

This is summarized in Table 3, which shows the decay modes of hypothetical pure 2^3S_1 $\rho(1465)$ and $^3D_1$ $\rho(1700)$ states. Both states are expected to appear in $\pi\pi$, and in apparent confirmation have recently been reported in a Crystal Barrel study of $\bar{p}n \to (\pi^-\pi^o)\pi^o$ [18]. The most striking differences are in the 4π modes. A $^3D_1$ state is predicted to have very large couplings to $h_1\pi$ and $a_1\pi$; for a 2^3S_1 state these are very weak. The $\omega\pi$ mode is produced mainly from 2^3S_1, although a significant contribution from $^3D_1$ is expected. Finally, a hypothetical hybrid near this mass would decay strongly to $a_1\pi$, but $h_1\pi$ should be absent, due to the singlet selection rule that $(S_{q\bar{q}} = 0) \not\to (S_{q\bar{q}} = 0) + (S_{q\bar{q}} = 0)$ in the $^3P_0$ decay model. The reported dominant coupling of the $\rho(1465)$ to $a_1\pi$ combined with a weak coupling to $h_1\pi$ [17] supports the suggestion that there may be an important hybrid component in this state [19].

In summary, a comparative study of $h_1\pi$, $a_1\pi$ and $\omega\pi$ should allow the identification of the 2^3S_1 and $^3D_1$ initial $q\bar{q}$ basis states in this possibly strongly mixed system. The modes $\rho\pi$ and $b_1\pi$ can play a similar rôle in the excited-ω sector [12].

3.5. f_0 $n\bar{n}$ candidates: $f_0(1300)$ and $f_0(1710)$.

The scalar sector has been of great interest recently with the discovery of an unusual $f_0(1500)$ state, which has been promoted as a scalar glueball candidate [2]. The mass alone rules out an $n\bar{n}$ assignment, since the 1P and 2P multiplets lie at about 1.2-1.3 GeV and 1.7 GeV respectively. Since the $f_0(1500)$ has large couplings to nonstrange final states such as $\pi\pi$ (relative to KK), we can probably reject an $s\bar{s}$ assignment as well. This state also has problems as an ideal glueball, notably in its flavor-nonsinglet couplings to $\pi\pi$ versus KK. Models of this state proposed by Amsler and Close [2] typically require a large mixing between glueball and quarkonium basis states, so the entire scalar sector may show large flavor mixing, analogous to the light $\eta - \eta'$ system.

We may be able to clarify the situation by studying the decay couplings of naive flavor-pure basis states at the expected $n\bar{n}$ masses, to see whether or not such approximately unmixed states exist. The spectrum shows evidence for a broad $f_0(1300)$ in $\pi\pi$ phase shifts, which may be a relatively pure $n\bar{n}$ state, and the "$\theta(1710)$" scalar seen in ψ-radiative decays might *a priori* be considered a 2^3P_0 $n\bar{n}$ candidate. We will also consider a 2P tensor assignment for the $\theta(1710)$.

Our $^3P_0$ model calculations predict that $^3P_0$, 2^3P_0 and 2^3P_2 $n\bar{n}$ states should all be very broad, with total widths of ≈ 500 MeV. The $\pi\pi$ modes all have nodes, but these are rather far from the preferred value $\beta = 0.4$ GeV and so should not alter our conclusions. We find that a $^3P_0$ $n\bar{n}$ $f_0(1300)$ should indeed decay strongly to $\pi\pi$, but its KK and $\pi\pi$ branching fractions should be comparable; this may be altered in practice by flavor mixing. The 2^3P_0 $f_0(1700)$ state is predicted to decay dominantly to the broad 4π modes $\pi\pi(1300)$, πa_1 and $\rho\rho$, so it would be interesting to search for evidence of these in processes that produce the $\theta(1710)$. We find however that the KK and $\eta\eta$ branching fractions of a 2^3P_0 $f_0(1700)$ are very small, so the $\theta(1710)$ appears implausible as a 2^3P_0 $n\bar{n}$ state. The KK and $\eta\eta$ modes are also very small for an $n\bar{n}$ tensor 2^3P_2 $f_2(1700)$, which should decay dominantly to $\rho\rho$.

In summary, the $f_0(1300)$ appears consistent with a $^3P_0$ $n\bar{n}$ state, although its relative $\pi\pi$ and KK branching fractions may indicate $n\bar{n} \leftrightarrow s\bar{s}$ mixing. In contrast, the $f_0(1500)$ and $\theta(1710)$ do not appear to have the decay couplings expected for quarkonia.

ness of this approach should increase as our understanding of strong decay amplitudes improves.

As a final comment we noted that many of the "missing mesons" in the quark model may be observable if one reconstructs the appropriate virtual two-body decay modes, and the appropriate modes are indicated by our numerical results. We suggest the unknown 2^{--} states as two obvious possibilities for such a "treasure hunt".

5. ACKNOWLEDGEMENTS

I would like to thank my colleagues C.Amsler, D.V.Bugg, S.U.Chung, F.E.Close, P.R.Page and E.S.Swanson for useful discussions relating to this material. I am also very grateful to H.Koch, K.Peters and associates for their kind invitation to present this material at LEAP96 in Dinkelsbühl. This research was sponsored in part by the U.S.Department of Energy under Contract No. DE-AC05-96OR22464 managed by Lockheed Martin Energy Research Corp.

REFERENCES

1. J.Weinstein and N.Isgur, Phys. Rev. D41, 2236 (1990).
2. C.Amsler *et al.*, Phys. Lett. B342, 433 (1995); V.V.Anisovich *et al.*, Phys. Lett. B323, 233 (1994); C.Amsler and F.E.Close, Phys. Lett. B353, 385 (1995); Phys. Rev. D53, 295 (1996).
3. J.H.Lee *et al.*, Phys. Lett. B323, 227 (1994).
4. A.M.Zaitsev *et al.*, p.1409 in Proc. XXVII International Conf. on High Energy Physics, Glasgow, (Institute of Physics, U.K., 1994, P.Bussey and I.Knowles eds.).
5. D.V.Amelin *et al.*, Phys. Lett. B356, 595 (1995).
6. L.Micu, Nucl. Phys. B10, 521 (1969); A.LeYaouanc, L.Oliver, O.Pène and J.-C.Raynal, Phys. Rev. D8, 2223 (1973). See also P.Geiger and E.S.Swanson, Phys. Rev. D50, 6855 (1994), for a recent study of $^3P_0$ versus $^3S_1$ amplitudes.
7. E.S.Ackleh, T.Barnes and E.S.Swanson, "On the Mechanism of Open-Flavor Strong Decays", hep-ph/9604355, ORNL-CTP-96-03 (April 1996), Phys. Rev. D54, 6811 (1996).
8. E.Eichten, Phys. Rev. D17, 3090 (1978).
9. G.Busetto and L.Oliver, Z.Phys. C20, 247 (1983).
10. H.G.Blundell and S.Godfrey, Phys. Rev. D53, 3700 (1996).
11. A.LeYaouanc *et al.*, Phys. Lett. B72, 57 (1977).
12. T.Barnes, F.E.Close, P.R.Page and E.S. Swanson, "Higher Quarkonia", hep-ph / 9609339, ORNL-CTP-96-09, RAL-96-039 (Sept.1996).
13. N.Isgur, R.Kokoski and J.Paton, Phys. Rev. Lett. 54, 869 (1985).
14. The original mass estimate for the lightest hybrid multiplet in the flux-tube model was 1.9(1) GeV [13]. A subsequent Hamiltonian Monte Carlo study of the flux-tube model by T.Barnes, F.E.Close and E.S.Swanson, Phys. Rev. D52, 5242 (1995), found 1.8-1.9 GeV, and a recent lattice QCD simulation by P.Lacock, C.Michael, P.Boyle and P.Rowland, hep/lat-9611011, reported 2.0(2) GeV for the 1^{-+} $s\bar{s}$-hybrid level. We would expect $n\bar{n}$-hybrids about 0.25 GeV lower.
15. S.Godfrey and N.Isgur, Phys. Rev. D32, 189 (1985).
16. F.E.Close and P.R.Page, Nucl. Phys. B443, 233 (1995).
17. For a recent review of the excited vectors as seen in e^+e^- see A.Clegg and A.Donnachie, Z. Phys. C62, 455 (1994).
18. A.Abele *et al.*, "High-mass ρ-meson states from $\bar{p}d$ annihilation at rest into $\pi^-\pi^o p_{\text{spectator}}$, Phys. Lett. (to appear).
19. A.Donnachie and Yu.S.Kalashnikova, Z. Phys. C59, 621 (1993).