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EFFECTIVE THREE-BODY FORCES IN TRUNCATED SHELL-MODEL CALCULATIONS

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MASTER

INTRODUCTION

One of the main problems in nuclear physics is the determination of effective Hamiltonians to be used in shell-model calculations in a truncated vector space. However, a number of difficulties have arisen in trying to compute such effective Hamiltonians (1). In order to study this problem in more detail we have undertaken a series of calculations in which we can make a direct connection between a fixed Hamiltonian H, which "describes" the eigenenergies and eigenvectors of nuclei in a given mass range in some "large" shell-model vector space S, and an effective Hamiltonian $\mathcal{H}$ to be used to "describe" some of the eigenvalues and eigenvectors of the same nuclei in a smaller (truncated) vector space $\mathcal{S}$. In our calculations the results of the H-in-S model play the role of "experimental observables" to be reproduced by the $\mathcal{H}$-in-$\mathcal{S}$ model.

We investigated three different methods for constructing $\mathcal{H}$; these are

(i) lowest-order perturbation theory (i.e. perturbation theory to second order)

(ii) a projection technique, by which we construct an operator $\mathcal{H}$ in the truncated space $\mathcal{S}$ in terms of known eigenvalues and eigenvectors in the larger space S, and

(iii) a least-square search to fit to selected large-space eigenenergies.

We applied these methods to nuclei with A = 17-20, employing the following model. Using the experimental single-particle (s.p.) energies for A = 17 and the renormalized Kuo interaction (2), we calculated the eigenenergies and eigenvectors of nuclei with A = 18-20 in the full sd shell, i.e. $(d_{5/2}, s_{1/2}, d_{3/2})$. We then truncated the space to $(d_{5/2}, s_{1/2})$ and utilized the above three techniques to construct effective Hamiltonians in this smaller space.

One of the main reasons why these calculations were originally undertaken (3) was to investigate problems concerning the divergence of the perturbation expansion for the effective Hamiltonian $\mathcal{H}$ determined by means of microscopic perturbation theory (1). However, the three techniques used for constructing $\mathcal{H}$ also allow us to study the need for and the size of effective 3-body forces (and higher-body forces, if we wish) in nuclei containing three (or more) nucleons outside a closed-shell core. In this talk we will be concerned only with these effective 3-body forces and will discuss only our results which have to do with nuclei with A = 19 and 20. The results of all

our calculations including the 2-body matrix elements of $\mathcal{H}$ and the properties of the eigenvectors of the different $\mathcal{H}$ operators in the truncated space $\mathcal{S}$ will be presented in more detail in a forthcoming publication (3).

TECHNIQUES USED FOR CONSTRUCTING $\mathcal{H}$-IN-$\mathcal{S}$

The H-in-S model or large model consists of the results obtained by diagonalizing H in a "large" shell-model vector space S. We take Φ_I, Φ_J, etc. to be members of a set of orthonormal basis vectors spanning S and E_I to represent an H-in-S eigenvalue with eigenvector Ψ_I.

We want to construct an effective Hamiltonian $\mathcal{H}$ to be used within the smaller vector space $\mathcal{S}$, which we call the $\mathcal{H}$-in-$\mathcal{S}$ model or the small model. We take ϕ_i, ϕ_j etc. to be members of a set of orthonormal basis vectors spanning $\mathcal{S}$ and ε_i to present an $\mathcal{H}$-in-$\mathcal{S}$ eigenvalue with eigenvector ψ_i.

We will not discuss the lowest-order perturbation-theory (1,4) and least-square-search methods (5) in detail, since they are already well-known. In our lowest-order perturbation theory calculation we determine $\mathcal{H}$ to only second order in H, called $\mathcal{H}^{2ND}$, using more-or-less ordinary second-order perturbation theory (3). The main difference is that for convenience we symmetrize the energy denominator to give a Hermitian $\mathcal{H}$.

Our least-square-search method is similar to standard calculations of this type (5). We assume some simple parametrized form of $\mathcal{H}$ and determine its parameters by requiring a least-square fit to some chosen subset of the calculated large-model energies.

The projection method, like the least-square-search method, requires that we already know some of the large-model eigenvalues E_I with their eigenvectors Ψ_I. We then want to construct an $\mathcal{H}$-in-$\mathcal{S}$ model whose energy eigenvalues exactly match a selected subset of H-in-S eigenvalues E_I. Let $\{E_i\}$ be a subset of the large-model eigenvalues with corresponding small-model eigenvectors ψ_i. We assume that the ψ_i span $\mathcal{S}$. If the ψ_i are also orthonormal, then $\mathcal{H}$ is uniquely determined by

$$\mathcal{H} = \sum_i |\psi_i\rangle E_i \langle\psi_i| \,, \qquad (1)$$

which clearly satisfies $\mathcal{H}|\psi_i\rangle = E_i|\psi_i\rangle$ and exactly reproduces the E_i.

Following a suggestion by Brandow (4,6), we take for the vectors ψ_i the projections of large-model eigenvectors on the small-model space, calling the resulting many-body Hamiltonian $\mathcal{H}^{PR}$. To construct $\mathcal{H}^{PR}$ we assume that $\mathcal{S}$ is a subspace of S and that we can calculate all the H-in-S eigenvalue and eigenvector pairs $\{E_I, \Psi_I\}$. We then pick a subset $\{E_i, \Psi_i\}$, whose dimensions in the (A,J,T)-subspaces of S exactly match the dimensions of the corresponding (A,J,T)-subspaces in $\mathcal{S}$. We choose this subset so that the generalized projections $P\Psi_i$ are all linearly independent and define the set of normalized small-model vectors,

$$\Psi_i^{PR} = P\Psi_i / \langle P\Psi_i | P\Psi_i \rangle^{1/2} . \qquad (2)$$

Although the vectors ψ_i^{PR} span $\mathcal{S}$, they are not generally orthogonal to each other. Therefore, it is convenient to construct a set of vectors $\tilde{\psi}_j^{PR}$, which are orthogonal to the set ψ_i^{PR}, i.e.

$$\langle \tilde{\Psi}_j^{PR} | \Psi_i^{PR} \rangle = \delta_{ij} . \qquad (3)$$

The "bi-orthogonal" set $\tilde{\psi}_j^{PR}$ is uniquely defined and can be easily constructed.

In analogy with eq. (1) we can construct the operator

$$\mathcal{H}^{PR} = \sum_i |\Psi_i^{PR}\rangle E_i \langle \tilde{\Psi}_i^{PR}| , \qquad (4)$$

which automatically satisfies $\mathcal{H}^{PR}|\psi_i^{PR}\rangle = E_i |\psi_i^{PR}\rangle$. The matrix elements of $\mathcal{H}^{PR}$ in the orthonormal representation ϕ are then given by

$$\langle \phi_r | \mathcal{H}^{PR} | \phi_s \rangle = \sum_i \langle \phi_r | \Psi_i^{PR} \rangle E_i \langle \tilde{\Psi}_i^{PR} | \phi_s \rangle , \qquad (5)$$

where the values E_i are known, selected H-in-S eigenvalues and the numbers $\langle \phi_r | \psi_i^{PR} \rangle$ and $\langle \tilde{\psi}_i^{PR} | \phi_s \rangle$ are the known expansion coefficients of ψ_i^{PR} and $\tilde{\psi}_i^{PR}$, respectively, in the basis representation ϕ.

The operator $\mathcal{H}^{PR}$ is not generally Hermitian. To simplify our numerical applications, we have used a Hermitian approximation to $\mathcal{H}^{PR}$, namely

$$\mathcal{H}^{AVPR} = \frac{1}{2}\left(\mathcal{H}^{PR} + \mathcal{H}^{PR\dagger}\right) . \qquad (6)$$

APPLICATION TO THE TRUNCATION $(d_{5/2}, s_{1/2}, d_{3/2}) \rightarrow (d_{5/2}, s_{1/2})$

We have applied the three methods discussed above to calculate effective nuclear Hamiltonians for A = 18-20. The truncation we treat is $(d_{5/2}, s_{1/2}, d_{3/2}) \rightarrow (d_{5/2}, s_{1/2})$, that is, we start with S spanned by all states of configurations $(d_{5/2}, s_{1/2}, d_{3/2})^{A-16}$, and let $\mathcal{S}$ be spanned by all states of configurations $(d_{5/2}, s_{1/2})^{A-16}$. Consequently there are no hole states in our calculations. We use the effective Hamiltonian suggested by Kuo (2) for H and the negatives of the measured binding energies with respect to ^{16}O of the lowest $5/2^+$, $1/2^+$ and $3/2^+$ states in ^{17}O for the s.p. energies. The full Hamiltonian H is a (1+2)-body operator, and its eigenvalues in S represent energies with respect to the ^{16}O ground state. We give our results in terms of a correction operator $\delta\mathcal{H}$, which represents the difference between $\mathcal{H}$ and H.

Application of the Lowest-Order Perturbation Method

Because S and $\mathcal{S}$ are so simply related in our calculations, there are no 0-body or 1-body corrections in converting H to $\mathcal{H}$. Also, since H is only a (1+2)-body operator, $\mathcal{H}^{2ND}$ has particle rank no higher than 3. The 2-body corrections to H (i.e. $\delta\mathcal{H}^{2ND}_{2\text{-body}}$) correspond to

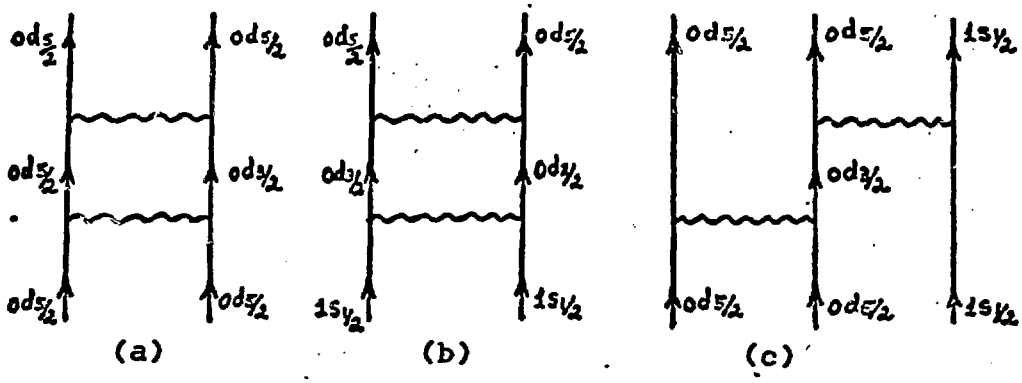

Fig. 1: Typical Feynman-Goldstone diagrams which contribute to the perturbation expansion for the effective Hamiltonian $\mathcal{H}$ for the $(0d_{5/2}, 1s_{1/2}, 0d_{3/2}) \rightarrow (0d_{5/2}, 1s_{1/2})$ truncation problem.

Diagrams (a) and (b) imply contributions to the 2-body-operator part of $\mathcal{H}$. Diagram (c) implies a contribution to the 3-body-operator part of $\mathcal{H}$.

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Table 1

Contributions to some matrix elements $\langle\phi_r(3\text{-body})|\mathcal{H}|\phi_s(3\text{-body})\rangle$ of various effective Hamiltonians $\mathcal{H}$. In the columns headed "Basis states", the symbols s and d imply $s_{1/2}$ and $d_{5/2}$, while the symbol $(d^2)_J$ implies $(d_{5/2})^2_J$. Off-diagonal matrix elements are marked with parentheses. All matrix elements are between fully antisymmetrized basis states.

	Basis States		Kuo Hamiltonian	Deviation of various $d_{5/2}s_{1/2}$ effective Hamiltonians from the Kuo Hamiltonian (in MeV)				
J, T	ϕ_r	ϕ_s	H(1+2)-body (in MeV)	$\delta\mathcal{H}^{2ND}$ 2-body	$\delta\mathcal{H}^{AVPR}$ 2-body	$\delta\mathcal{H}^{2ND}$ 3-body	$\delta\mathcal{H}^{PR'}$ 3-body	$\delta\mathcal{H}^{AVPR}$ 3-body
1/2,1/2	s^3	s^3	- 17.54	- 1.46	- 1.18	0.02	0.70	0.70
		$(d^2)_0 s$	(1.19)	(0.37)	(0.23)	(0.81)	(0.20)	(0.35)
		$(d^2)_1 s$	(- 0.73)	(- 1.51)	(- 1.60)	(- 0.29)	(0.91)	(0.44)
		d^3	(0)	(0)	(0)	(- 0.40)	(- 0.15)	(0.13)
	$(d^2)_0 s$	$(d^2)_0 s$	- 17.86	- 2.93	- 2.91	- 0.33	0.19	0.19
		$(d^2)_1 s$	(1.93)	(- 0.40)	(-0.89)	(1.01)	(1.36)	(1.64)
		d^3	(2.27)	(0.72)	(0.45)	(0.35)	(0.36)	(- 0.14)
	$(d^2)_1 s$	$(d^2)_1 s$	- 14.58	- 3.31	- 3.97	- 0.60	0.36	0.36
		d^3	(- 1.85)	(- 0.63)	(- 0.44)	(- 0.37)	(-0.32)	(0.23)
	d^3	d^3	- 15.29	- 1.42	- 1.17	- 0.18	0.35	0.35
3/2,3/2	$(d^2)_2 s$	$(d^2)_2 s$	- 14.22	- 0.95	- 0.75	0.19	0.24	0.24
		d^3	(- 1.44)	(- 0.55)	(-0.44)	(0.12)	(0.29)	(0.27)
	d^3	d^3	- 14.71	- 0.81	- 0.80	0.17	0.24	0.24

ladder diagrams, such as those shown in figs. 1(a) and 1(b). The corrections $\delta\mathcal{H}^{2ND}_{3\text{-body}}$ correspond to diagrams such as the one illustrated in fig. 1(c). Since H is a (1+2)-body operator, $\delta\mathcal{H}^{2ND}_{3\text{-body}} \equiv \mathcal{H}^{2ND}_{3\text{-body}}$. Table 1 lists some 3-body matrix elements of $\mathcal{H}^{2ND}_{3\text{-body}}$ between the basis states ϕ.

In our truncation, the denominators for $\mathcal{H}^{2ND}$ have magnitudes ranging from 3.34 to 10.16 MeV. Because of these small energy denominators, we did not expect $\mathcal{H}^{2ND}$ to constitute a good effective Hamiltonian for our problem.

Application of the Projection Method

In constructing $\mathcal{H}^{PR}$ we had to project $(d_{5/2}, s_{1/2}, d_{3/2})$ eigenvectors Ψ_I onto the $(d_{5/2}, s_{1/2})$ space. To do this, we simply threw away all components involving one or more $d_{3/2}$ particles and then computed $\mathcal{H}^{PR}$ by using eq. (5). The first step in our calculations was to choose an appropriate subset of the $(d_{5/2}, s_{1/2}, d_{3/2})$ eigenstates Ψ_I to use in eq. (5). Let us call the chosen eigenstates in this subset Ψ_i. These Ψ_i should be the best candidates for "non-intruders", where we expect non-intruder states to have "large" projections in $\mathcal{L}$. Unfortunately, it is not always obvious how to choose the best set of candidates for "non-intruders". With one exception, our chosen states correspond to the lowest eigenvalues E_I for each A, J, T.

For our calculations, it is easy to see that $\mathcal{H}^{PR}_{1\text{-body}} = H_{1\text{-body}}$ and $\mathcal{H}^{PR}_{0\text{-body}} = H_{0\text{-body}} \equiv 0$. Using eq. (5), we computed $\mathcal{H}^{PR}_{(1+2)\text{-body}}$ from which we determined $\mathcal{H}^{PR}_{2\text{-body}}$ (which is non-Hermitian), by subtracting the two-body matrix elements of $\mathcal{H}^{PR}_{1\text{-body}}$. Similarly, we obtained $\mathcal{H}^{PR}_{(1+2+3)\text{-body}}$ using eq. (5) and our projected wave functions for A = 19. To determine $\mathcal{H}^{PR}_{3\text{-body}}$, we would need to calculate the matrix elements of the non-Hermitian 2-body operator $\mathcal{H}^{PR}_{2\text{-body}}$ between three-body states ϕ. But, this is impossible with our present computer codes. Hence, we decided to compute an approximation to $\mathcal{H}^{PR}_{3\text{-body}}$. First, we symmetrized $\mathcal{H}^{PR}_{2\text{-body}}$, so as to get $\mathcal{H}^{AVPR}_{2\text{-body}}$ (see eq. (6)), then we used our shell-model codes to calculate 3-body matrix elements of $\mathcal{H}^{AVPR}_{2\text{-body}}$, and finally by subtraction from $\mathcal{H}^{PR}_{(1+2+3)\text{-body}}$, we computed

$$\mathcal{H}^{PR'}_{3\text{-body}} \equiv \mathcal{H}^{PR}_{(1+2+3)\text{-body}} - \mathcal{H}^{AVPR}_{(1+2)\text{-body}}. \qquad (7)$$

Table 1 shows some of the 3-body (A=19) matrix elements of $\mathcal{H}^{PR'}_{3\text{-body}}$ and $\mathcal{H}^{AVPR}_{3\text{-body}}$ between the basis states ϕ. One should see ref. 3 for details on how the choice of the "non-intruder" states Ψ_i influence the magnitude and the non-Hermiticity of the matrix elements obtained for $\mathcal{H}^{PR}_{(2+3)\text{-body}}$.

Application of the Least-Square-Search Method

Clearly the least-square-search Hamiltonian depends upon the search criteria employed. Our results are for two different searches, called SRCH.A and SRCH.B, which differed in the selection of energy data that were least-square fitted. However, in both searches $\mathcal{H}$ was restricted to be a Hermitian (1+2)-body operator, the two s.p. energies were fixed at the same ^{17}O energies used in H and the

variable parameters were the 16 2-body matrix elements of $\mathcal{H}_{2\text{-body}}$.

The criteria of SRCH.A were patterned after those of a typical least-square-search to fit experimental data for A = 18-20 (5). Hence, the H-in-S levels selected for least-square fitting corresponded to levels of A = 18-20 which "seemed important" to fit among experimental data.

In SRCH.B we used another approach; we least-square fitted those H-in-S eigenvalues E_i which we thought should be easiest to fit from perturbation theory arguments, assuming a Hermitian operator $\mathcal{H}$. Consequently, we included a level E_I in our least-square-fit data set if and only if:

(a) its intensity in the $(d_{5/2}, s_{1/2})$ space $\mathcal{S}$, i.e. $\langle P\Psi_I | P\Psi_I \rangle$, was $\geqslant 0.6$,
(b) its normalized $\mathcal{S}$-projected wave-function had a squared overlap of < 0.02 with the normalized $\mathcal{S}$-projected wave function of every lower-lying H-in-S level, and
(c) all lower-lying states Ψ of the same A, J, T satisfied (a) and (b).

For more details about how SRCH.A and SRCH.B were carried out one should see ref. 3.

RESULTS FOR A = 19 AND 20

As mentioned earlier, table 1 lists some typical 3-body matrix elements contributing to the A = 19 Hamiltonian matrices, e.g. those for J = T = 1/2 and J = T = 3/2.

After obtaining the matrix elements of H-in-S by our three techniques, we then diagonalized $\mathcal{H}$ in the space $\mathcal{S}$ to obtain the

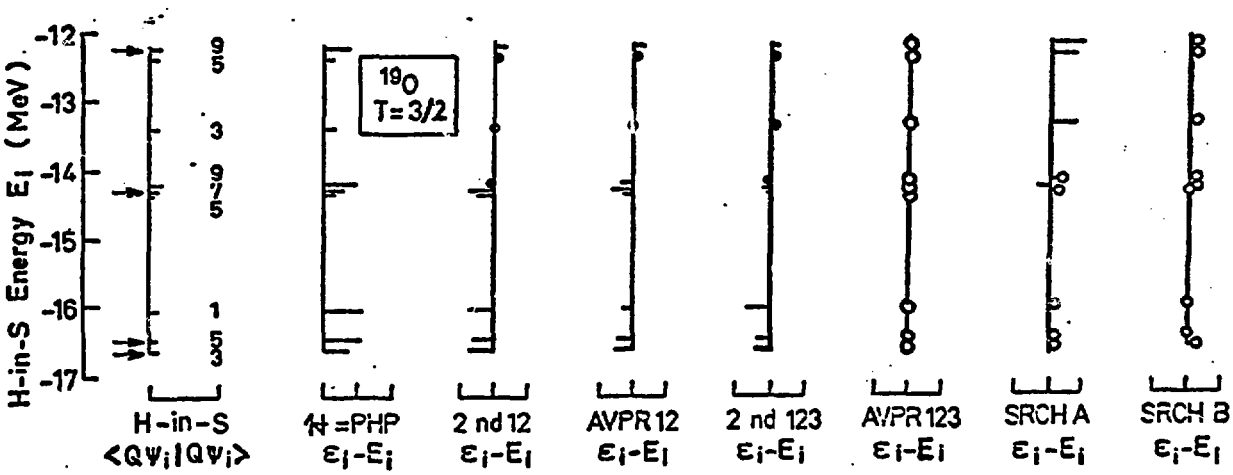

Fig. 2: Comparison of ^{19}O T = 3/2 eigenstates as obtained in the large (H-in-S) and small ($\mathcal{H}$-in-$\mathcal{S}$) models. The notation is explained in the text. In column 1, one horizontal unit represents 100% intensity. In columns 2 onward, one horizontal unit represents 1 MeV.

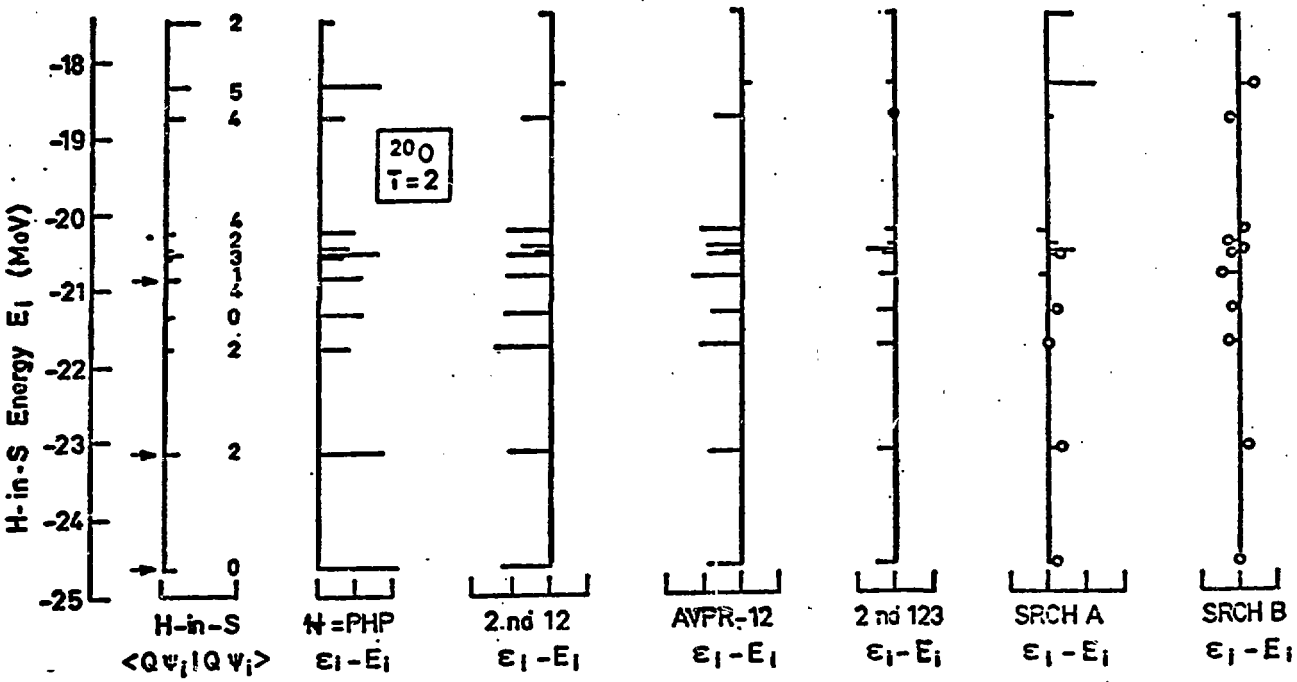

Fig. 3: Comparison of ^{20}O T = 2 eigenstates as obtained in the large (H-in-S) and small ($\mathcal{H}$-in-$\mathcal{S}$) models. The notation and horizontal scales are the same as in fig. 2.

corresponding eigenvalues and eigenvectors. In figures 2-5 we compare low-lying H-in-S eigenstates with low-lying $\mathcal{H}$-in-$\mathcal{S}$ eigenstates. Figures 2-3 display results for low-lying levels of the oxygen isotopes A=19 and 20 (maximum T nuclei). Figures 4-5 show results for low-lying levels of F(A=19) and Ne(A=20) (non-maximum T nuclei).

In each figure, column 1 shows the low-lying H-in-S eigenvalue spectrum. The length of the level-line represents

$$\langle Q\Psi_i|Q\Psi_i\rangle = \langle 1-P\Psi_i|1-P\Psi_i\rangle , \qquad (8)$$

which is the intensity of the H-in-S eigenvector <u>outside</u> $\mathcal{S}$. Consequently, we expect better $\mathcal{H}$-in-$\mathcal{S}$ fits for <u>short</u> lines in column 1.

Column 2 and all columns to the right of it give information about $\mathcal{H}$-in-$\mathcal{S}$ eigenstates. Hence, $\mathcal{H}$ = PHP means that the original, unrevised Hamiltonian H was used in $\mathcal{S}$. The remaining column-labels include obvious abbreviations, such as, 2ND.12 means $\mathcal{H}^{2ND}_{(1+2)}$-body' etc. Each $\mathcal{H}$-in-$\mathcal{S}$ state is plotted at the <u>same</u> energy ordinate as its proposed partner in the H-in-S spectrum. In these columns, the length and direction of each level-line indicates the <u>deviation</u> of the $\mathcal{H}$-in-$\mathcal{S}$ eigenvalues $\mathcal{E}_i$ from the H-in-S eigenvalue E_i. Thus, short lines indicate <u>relatively good</u> fits, either expected (column 1) or actually found (column 2 onward).

In column 1, the numerical labels represent J for even-A nuclei, 2J for odd-A nuclei. Symbols ➤ indicate those levels which

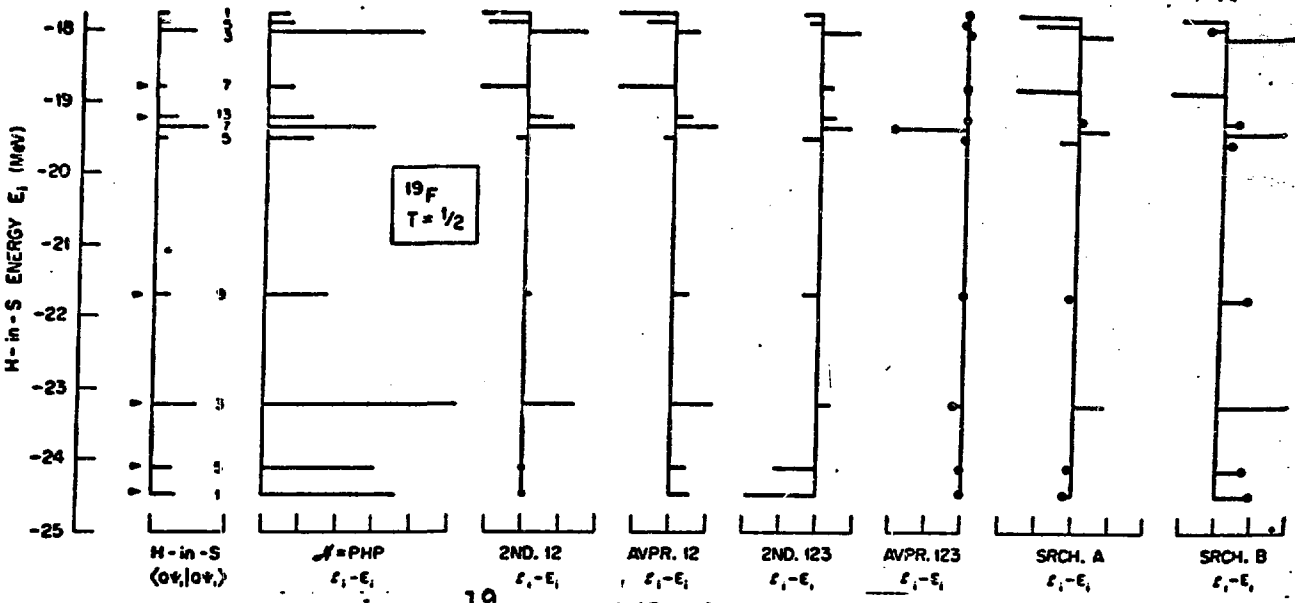

Fig. 4: Comparison of ^{19}F T = 1/2 eigenstates as obtained in the large (H-in-S) and small ($\mathcal{H}$-in-$\mathcal{S}$) models.

we consider to be ground-state-band members. In all the columns black dots on or near the vertical zero-line replace level-lines that would be too short to see. In the $\mathcal{H}$-in-$\mathcal{S}$ columns, small open circles mark those levels whose H-in-S energies were explicitly used in constructing $\mathcal{H}$ (as in the projection procedure or in a least-square search).

In examining columns 2 onward, remember that the level-lengths $\varepsilon_i - E_i$ represent deviations of $\mathcal{H}$-in-$\mathcal{S}$ eigenvalues from H-in-S eigenvalues. In these columns the unmarked units on the horizontal scales represent 1 MeV per unit.

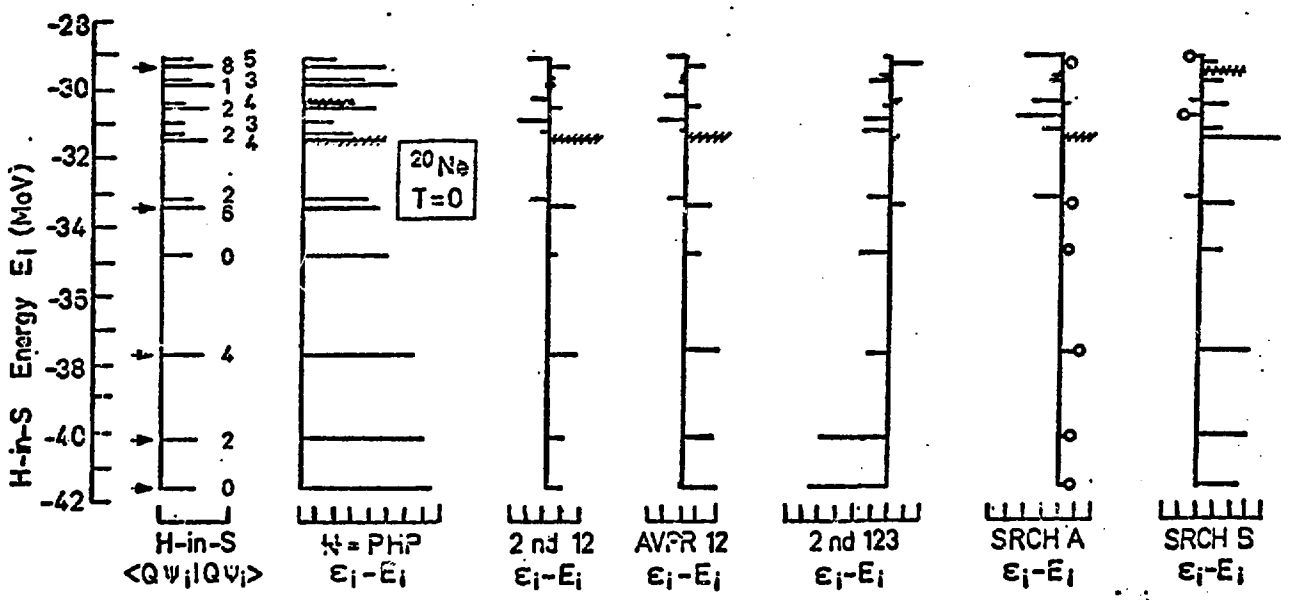

Fig. 5: Comparison of ^{20}Ne T = 0 eigenstates as obtained in the large (H-in-S) and small ($\mathcal{H}$-in-$\mathcal{S}$) models. The notation and horizontal scales are the same as in fig. 2.

DISCUSSION OF RESULTS

Since the projection method (by construction) exactly reproduces in the truncated space the selected large-space eigenenergies, the effective Hamiltonian determined by this method indicates how big an effective 3-body term is needed to do this. From table 1 (columns 8 and 9), we see that the effective 3-body terms computed by the projection method are fairly large in magnitude, generally being several tenths of an MeV, and are slightly non-Hermitian. However, the effective 3-body terms are generally smaller than the effective 2-body terms. Hence, the qualitative features of the large-space A=19 spectra are given by the (1+2)-body parts of $\mathcal{H}^{PR}$, as one can see in figs. 2 and 4. However, the effective 3-body terms are needed in order to reproduce the A=19 large-space eigenenergies.

From table 1 and figures 2-5 we see that columns 2ND.12 and AVPR.12 strongly resemble each other. This shows that, in spite of small denominators, the simple second-order perturbation method determines the 2-body part of the effective Hamiltonian reasonably well, i.e. it gives results qualitatively similar to those produced by the projection method.

On the other hand, the 3-body part of the $\mathcal{H}$ determined by simple second-order perturbation theory is noticably different from the 3-body part of $\mathcal{H}$ computed by the projection method (see table 1, columns 7-9). This difference can also be seen in figs. 2-5. For the maximum-T cases (figs. 2-3) the 2ND.123 results are an improvement over the 2ND.12 results, while for non-maximum-T cases (figs. 4-5) the 2ND.123 results are somewhat worse than those from 2ND.12.

Consequently, the determination of the 3-body part of $\mathcal{H}$ by simple second-order perturbation theory appears to be fairly unsuccessful, particularly for non-maximum T. Although we are not sure of the reason why second-order perturbation theory seems to work for determining the 2-body part of $\mathcal{H}$, yet fails to give the 3-body part, we think it may be connected with the fact that so many more terms appear in third order for the 3-body part of $\mathcal{H}$ than for the 2-body part. For this reason, it is possible that the effect of using such small energy denominators may be much more important for the 3-body calculation than the 2-body calculation, indicating that one must probably go to higher order in the interaction to determine the 3-body part of $\mathcal{H}$ in perturbation theory.

Although the SRCH.A and SRCH.B effective Hamiltonians are not much better than the other solutions for $\mathcal{H}$ for A=18 (these results are not shown here), they are generally better for A=19 and 20, particularly for the maximum-T cases (figs. 2-3). For the non-maximum-T cases (figs. 4-5), neither SRCH.A or SRCH.B is consistently better compared to 2ND.12 or AVPR.12.

The generally improved results for A=19 and 20 given by the SRCH.A and SRCH.B effective Hamiltonians would seem to indicate that the least-square search (1+2)-body effective Hamiltonians are "better" than the (1+2)-body effective Hamiltonians determined by simple second-order perturbation theory and by the projection method, since they contain the influence of the effective 3- and 4-body

terms in some average or smeared-out way.

In conclusion, when the number of valence nucleons is increased beyond 2, a 3-body term exists in the effective Hamiltonian $\mathcal{H}$ and is needed in order to obtain accurate agreement with the experimental spectra. This conclusion is consistent with the results of previous calculations to investigate effective 3-body forces in nuclei (7). However, this 3-body term in $\mathcal{H}$ is generally smaller than the 2-body part of $\mathcal{H}$, so that the (1+2)-body part of $\mathcal{H}$ gives the basic qualitative features of the 3-particle spectrum. Lowest-order perturbation theory is unsuccessful in accurately reproducing the 3-body part of $\mathcal{H}$. Finally the least-square-fitted (1+2)-body effective Hamiltonian appears to be more successful in reproducing the experimental spectra for 3 or 4 valence nucleons than the lowest-order perturbation-theory and projection-method effective Hamiltonians, since the former contains the influence of the effective 3- and 4-body forces in some average way.

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