Conf-740934--1

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

SUCCESSES AND FAILURES OF THE NUCLEAR SHELL MODEL

J. B. McGrory

Oak Ridge National Laboratory*, Oak Ridge, Tennessee

I have assumed that my job today is to convey some general impression on the usefulness and significance of the conventional nuclear shell model in studies of nuclear phenomena in terms of microscopic properties. Let me first define what I mean by a conventional shell model calculation. Nuclei are assumed to consist of neutrons and protons interacting through two-body interactions. An exact solution for such a system of more than two particles is intractable. To reduce the problem to manageable dimensions, it is assumed that a certain number of particles form an inert core, which generates a field in which the remaining active particles move. The active particles are assumed to be in only a few orbits outside this core, so that the problem is reduced to diagonalizing some Hamiltonian in the space generated by a few particles in a relatively few single-particle orbits. One interpretation of the results of calculations in such a model follows from the following train of thought. We assume that there exists a true Hamiltonian, H , for a nuclear system of neutrons and protons of which the dominant parts are the kinetic energy and the sum of all mutual two-body interactions. The eigenfunctions, ψ_E, of this Hamiltonian are real nuclear wave functions, and the eigenvalues, E , are real energy levels. With these wave functions, it is possible to calculate physical observables from matrix elements of some set of real observable operators, O . Thus,

$$H\,|\psi_E\rangle = E\,|\psi_E\rangle \tag{1}$$

and matrix elements $\langle\psi_E|O\,|\psi_E'\rangle$ correspond to observed transition rates. H is defined in an infinite Hilbert space. Is it possible to find effective operators $\mathcal{H}$ and $\mathcal{O}$ defined in a finite Hilbert space, so that the eigenfunctions, $|\chi_E\rangle$ of the effective $\mathcal{H}$ in this finite space have the property

*Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

MASTER

$$\mathcal{H}\ |\chi_E\rangle = E\ |\chi_E\rangle$$

and

$$\langle\chi_E|\mathcal{O}|\chi_{E'}\rangle = \langle\psi_E|O\ |\psi_{E'}\rangle.$$

If we can find a set of simple operators $\mathcal{H}$ and $\mathcal{O}$, simple in that they can be specified in terms of a relatively small set of parameters, which can account for a variety of observed phenomena over a relatively large number of nuclei, then we obviously have a phenomenological model which can be extremely useful in studies of systematic behavior of nuclei. In this sense the nuclear shell model has achieved a large measure of success. Much of this success is for one-body operators. For such operators, many detailed correlations in the nuclear wave function are not too important. The fact that nuclear observables involve wave function overlaps is crucial to the success of the shell model in the phenomenological sense.

This general approach, together with perturbation theory, is fundamental to the whole field of the calculation of "realistic" effective interactions. If one could find a systematic way to calculate the effective shell model parameters (one- and two-body matrix elements of $\mathcal{H}$ and $\mathcal{O}$) from known nucleon-nucleon and nucleon-electromagnetic field interactions, then one could start to talk of a real microscopic theory. The "success" of the shell model in this sense is still very unclear. Some of the uncertainties arise from purely mathematical considerations (mainly concerning perturbation theory convergence), and we'll hear in detail on this matter later in this meeting.

My main emphasis today is on the success or failure of the shell model, in a phenomenological sense, in describing systematic patterns of nuclear phenomena. I will first discuss conventional shell model calculations in the s-d shell and the f-p shell, wherein ^{16}O and ^{40}Ca are treated as closed cores. I choose these areas obviously because I am most familiar with them, and because most of the conventional shell model calculations in recent years have been in this region. In addition, most of the "realistic" interaction calculations are in this mass region, and so one can make some comments on the usefulness of the shell model in testing these interactions. I will first discuss some of the most popular types of theory-experiment comparisons, i.e. energy levels, electromagnetic moments, and

electromagnetic transition rates. Then I'll spend some time discussing shell model calculations of two-nucleon transfer cross sections. Such reactions are relatively sensitive to two-particle correlations in the wave functions and so may present a more challenging test of particular shell models. Finally, I'll spend some time discussing the application of the conventional shell model to states which involve core-excitation around ^{16}O and ^{40}Ca.

Insofar as conventional shell model calculations are concerned, the s-d shell is one of the most extensively studied regions of the periodic table[1-7]. With the capability for large space calculations developed first by the Oak Ridge-Rochester group[8] and considerably extended recently by the Glasgow group[9], it is now possible to do shell model calculations of almost all nuclei with $8 \leq N, Z \leq 20$ in untruncated $(sd)^{A-16}$ model spaces. Thus, very extensive studies of systematic trends are possible. The first step of the shell model calculation is the diagonalization of the chosen effective Hamiltonian, so that at the first step, eigenvalues are obtained. Thus the natural first comparison is for energy levels. Although this is the natural first test, it is not obviously the best comparison. Soyeur and Zuker[10] demonstrated that in really large shell model calculations, such as treating ^{28}Si as 12 particles in the s-d shell outside an inert ^{16}O core, the excitation energies can be very sensitive to the chosen effective interaction. In the ^{28}Si case, changes of several MeV in the excitation energy of several states occurred for two interactions for which the two-body matrix elements looked very similar. This does not really reflect great sensitivity in the total eigenvalues, merely a sensitivity in eigenvalue differences. The eigenvalues for the two interactions for a given state differed by only a few percent. Their results also implied that the calculated E2-observables were relatively <u>insensitive</u> to the differences in the interactions. Thus, one must treat energy level comparisons in large shell model calculations with caution. Having issued such a caveat, we proceed with an uncautious comparison of energy levels.

In the s-d shell nuclei with $A \leq 28$, there is a large body of experimental information on ground state rotational bands, and in many cases on excited rotational bands. Some qualitative features of these bands have been found to be particularly sensitive to differences in the effective Hamiltonians which are used in the shell model calculations. The situation

is easily summarized. The shell model yields spectra[1,5] in which rotational bands clearly exist (i.e., they show J(J+1) spacings, and the calculated E2-observables are similar to rotational band E2-observables). The calculated structure within the rotational bands is relatively insensitive to what effective Hamiltonian is used[1,5]. The relative position of the bands as a whole is sensitive to the interaction[1,5], and interband transitions are sensitive. When one uses the Kuo[12] realistic interaction, the first excited rotational band is consistently calculated to occur at too low an excitation energy relative to the ground state band. Wildenthal[13] has found an empirical effective s-d shell interaction which does not suffer this defect to nearly so large an extent. He starts with the Kuo interaction, as expressed in terms of 63 two-body matrix elements. He treats certain subsets of these 63 matrix elements as free parameters which are adjusted to reproduce optimally a set of energy levels in nuclei with A = 18-22 in complete $(sd)^{A-16}$ model spaces. Thus, he essentially forces the interaction to reproduce correctly the position of the excited bands in these 18-22 nuclei. When this interaction, now referred to as the Preedom-Wildenthal (PW) interaction, is used in nuclei with A = 23-28, the resulting spectra are in significantly better agreement with experiment than is the case with the Kuo interaction. One example of this is shown in Fig. 1. Here, shell model calculations[5] of ^{25}Mg in the complete $(sd)^9$ space are compared with experiment. The complete spectrum calculated with the PW interaction is shown. The spectrum of states in the excited K=1/2 band as calculated with the Kuo interaction is shown. The calculated position of the $J=1/2^+$ state is arbitrarily set equal to the observed $J=1/2^+$ energy. The significant point here is that the ground state J=5/2 state is calculated, with the Kuo interaction, to be more than 2 MeV above the excited band head $J=1/2^+$ state. ^{26}Mg presents another example of the sensitivity of the calculations[11] to the effective interaction. In this case it turns out that the quadrupole moments of the two low-lying $J=2^+$ states are very sensitive to the interaction, as are interband transitions between states in the two low-lying bands in ^{26}Mg. Results calculated in mass 26 with the PW interaction are generally superior to those calculated with the Kuo interaction.

In Fig. 2, the two-body matrix elements of the Kuo interaction and the PW interaction are compared[5]. The matrix elements are expressed in LS coupling. We see that the central and tensor force matrix elements are

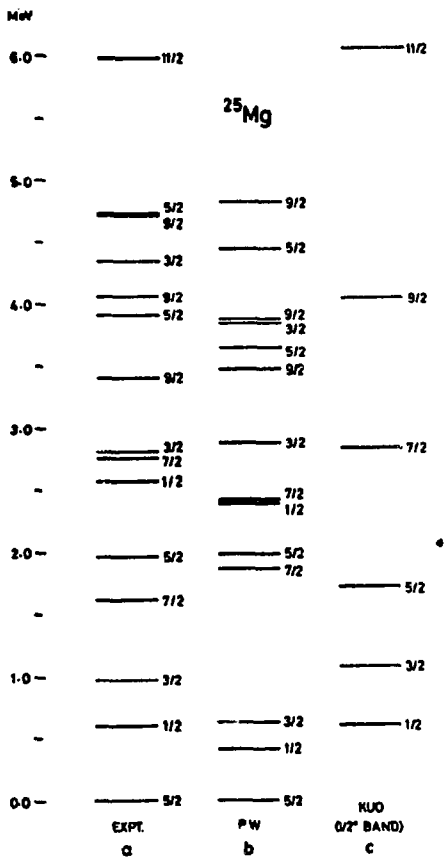

Fig. 1. Observed and Calculated Spectrum of T=1/2 States in ^{25}Mg.

virtually identical. There are several striking differences in the two-body L•S terms. Thus, the interactions in general are quite similar, but the few differences which do exist are apparently quite important with regard to band head energies. The main difference in the Kuo and the PW interactions is more transparent in the j-j coupling representation. The center-of-gravity (c-g) of the interaction of two particles in two different orbits is defined in

$$c\text{-}g = \frac{\sum_{J,T} (2J+1)(2T+1)\langle j_1 j_2 JT|H|j_1 j_2 JT\rangle}{\sum_{J,T} (2J+1)(2T+1)}$$

The PW interaction differs from the Kuo interaction in that the c-g of the $d_{5/2}$-$s_{1/2}$ and $d_{5/2}$-$d_{3/2}$ interactions is less attractive[13] by roughly 0.5 MeV. In calculations involving many particles, such a shift can be very significant. The interaction of one $s_{1/2}$ particle with n $d_{5/2}$ particles is roughly proportional to n times the center-of-gravity of the $d_{5/2}$-$s_{1/2}$ interaction. Thus, in ^{28}Si, a change of 0.5 MeV in this center-of-gravity leads to a difference of $\sim$ 5 MeV in the relative excitation of the $(d_{5/2})^{12}$ and the $(d_{5/2}^{11}, s_{1/2})$ configuration. Thus, a change in this center-of-gravity is n times more significant than a change in the single-particle energy of a comparable magnitude. Thus, we see here that with a simple change in two parameters, the two centers-of-gravity, the Kuo interaction in the s-d shell is apparently significantly improved. A similar "defect" has been noted in the f-p shell[14]. These types of calculations may help to focus in on possible defects in the realistic interactions, and suggest directions for the effective interaction

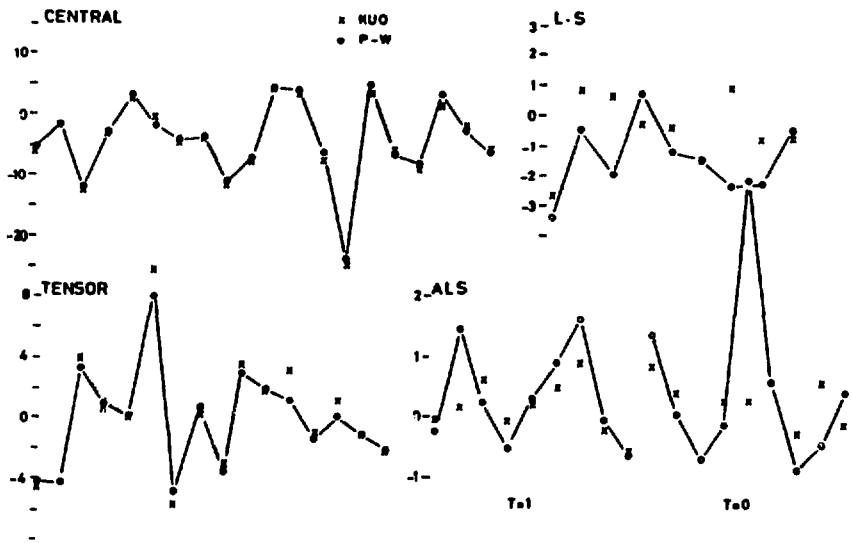

Fig. 2. Comparison of Matrix Elements of Preedom-Wildenthal and Kuo Interactions in LS Representation.

theorists to follow. Thus, in the s-d shell, the phenomenological shell model successfully describes the qualitative features of the excitation spectra of low-lying collective states. The only "realistic" interaction tested systematically for all nuclei in this region fails to reproduce the relative positions of the bands, but a simple prescription has been found to repair this defect.

In the f-p shell, pure $f_{7/2}{}^{n}$ models have been very successfully exploited by Talmi[16], by McCullen, Bayman, and Zamick[17], and by others to describe the spins and energies of many nuclei. Complete $(fp)^{A-40}$ calculations have been reported only for nuclei with $A \leq 44$[18]. Configuration mixing within the s-d shell is essential to the development of the rotational structure observed. Insofar as energy levels and spins in the light f-p shell nuclei are concerned, configuration mixing is not so important. Perhaps the best summary of the situation is shown in Fig. 3. This compares the observed spectrum of ^{44}Sc with the complete $(fp)^4$ calculation using essentially the Kuo-Brown effective interaction derived for this region. Of all the low-lying states observed which are believed to have good shell model counterparts, all but two are accounted for equally well by both the

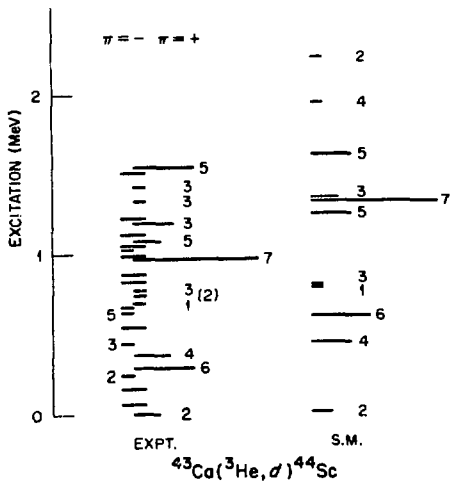

Fig. 3. Calculated and Observed Spectrum of Low-Lying States in ^{44}Sc. In the experimental spectrum states under π=− are negative parity states. π=+ labels positive parity states. The calculated spectrum includes only positive parity states.

$f_{7/2}{}^4$ and the $(fp)^4$ calculations. The only improvement insofar as energy levels are concerned as a result of configuration mixing, is to account for the second 3^+ and 5^+ states. What is more significant here is that more than half the states in the low energy spectrum are not accounted for by either model. Many of these intruder states are negative parity, and are obviously indescribable in an $(fp)^4$ model, but many are positive parity, and are not so ruled out. This is but one of several pieces of evidence I'll discuss of the relative complexity of the ^{40}Ca-core region compared to the ^{16}O-core region.

I have discussed so far only excitation energies. There is available a considerable amount of data on ground state binding energies with respect to ^{16}O. Cole, et al.[7] have calculated the binding energies of a large number of s-d shell nuclei in complete $(sd)^{A-16}$ shell model spaces. They report on calculations with the Kuo and PW interactions. Their results are tabulated in Table 1 for those nuclei for which the binding energies have been determined experimentally. We see that the Kuo interaction leads to more and more overbinding with increasing A. The comparison of theory with experiment for the PW interaction is significantly better. In general, the binding energies calculated with the PW interaction are within 1 MeV of the observed binding energies. Slightly better agreement is obtained by Gross and Talmi[19] with a modified surface delta effective interaction which includes seven parameters. They obtain an r.m.s. deviation between theory and experiment of 0.47 MeV for 18 observed binding energies. An alternative method for predicting binding energies is the well known[20] Garvey-Kelson mass formula. Where there is experimental information available, the Garvey-Kelson formula as

Table 1. Calculated and Observed Mass Excesses (M-A) MeV in s-d Shell Nuclei with A $\leq$ 33. The heading Kuo refers to calculations with the Kuo effective interaction. The heading PW refers to calculations with the Preedom-Wildenthal interaction.

Nucleus	J	(M-A) MeV Experiment[a]	Calculated[a] Kuo	Calculated[a] PW
^{20}O	0	3.80	2.89	3.74
^{21}O	5/2	9.30	6.90	8.31
^{22}O	0	11.50	6.83	9.47
^{22}F	4	2.83	1.30	3.09
^{24}Ne	0	- 5.96	-10.60	- 6.25
^{25}Ne	1/2	- 2.07	- 8.53	- 2.24
^{26}Na	2	- 6.85	-15.27	- 7.83
^{27}Na	3/2	- 5.78	-17.78	- 6.97
^{28}Na	2	- 1.26	-16.99	- 2.22
^{29}Na	3/2	2.73	-17.47	2.46
^{30}Na	0	8.37	-15.04	11.03
^{29}Mg	3/2	-11.70	------	-12.37
^{30}Al	2	-15.89	-36.77	-18.20
^{31}Al	5/2	-[illegible].06		-16.83
^{33}Si	3/2	-20.56	-53.58	-21.44

[a]Observed and calculated numbers are taken from Ref. 6.

parameterized by Thibault and Klapisch[21)] yields slightly better agreement with the data than does the shell model calculation with the PW interaction. Where there is no experimental information available, Garvey-Kelson and the shell model are in agreement to within 1.5 MeV.

After energy considerations, the next traditional shell model-experiment comparison is made for single-nucleon-transfer reaction spectroscopic factors. Detailed comparisons for such numbers abound in the literature (see, for example, Refs. 1-3), and I'll not say more about such comparisons here. To my knowledge, the analyses of such data have not been of great use in testing various realistic interactions.

Now we move to the calculation of various electromagnetic observables, and first of all to E2-observables. The first question that arises is that

of the effective charge, which is commonly pointed to as a shell model failure. In the conventional model of s-d shell nuclei, the oxygen isotopes consist of only neutrons outside an inert ^{16}O core. The neutrons should not interact with the electric field, but experimentally, there are strong E2-transitions observed between low-lying states in the oxygen isotopes. This is a manifestation of the fact that the neutron can polarize the core and excite protons, which can then radiate. The usual shell model prescription is to assume that the neutron has an effective charge, ε_n. For similar reasons the proton has a total charge $1+\varepsilon_p$, where ε_p represents the effective charge on the proton due to core-polarization due to the proton core interaction. The usual approximation is to assume that particles in all single-particle orbits carry the same effective charge. In this approximation, the shell model effective operator for E2-observables has the following form

$$\mathcal{O}^{E2} = \sum_{i=1}^{A} \left[(\tfrac{1}{2}+t_z^1(i))\ (1+\varepsilon_p)\ r_i^{\,2}\ Y^2\ (\Omega_i) + (\tfrac{1}{2}-t_z^1(i))\ \varepsilon_n\ r_i^{\,2}\ Y^2\ (\Omega_i) \right] \tag{2a}$$

$$= \sum_{i=1}^{A} \left[\frac{1}{2}\ (1+\varepsilon_p+\varepsilon_n)\ r_i^{\,2}\ Y^2\ (\Omega_i) + t_z^1(r)\ (1+\varepsilon_p-\varepsilon_n)\ r_i^{\,2}\ Y^2\ (\Omega_i) \right] \tag{2b}$$

Here $t_z(i)$ is $+\frac{1}{2}$ for protons. In the second form (2b), the operator is expressed in terms of isoscalar and isovector components.

The use of effective charges in shell model calculations is a failure only if one interprets shell model wave functions and operators as true wave functions and operators. In the phenomenological sense, I stated that we could consider the model a success if we could find simple effective operators and simple effective wave functions such that a large body of experimental data could be correlated with the calculations. I further stated that if there were a theory whereby these simple operators could be derived from bare or exact operators, we could claim some success in deriving a microscopic theory. Dr. Brussaard will speak on this latter point later. I will quickly review the phenomenological situation now.

Usually one uses effective charges $\varepsilon = 0.5e$ for both the neutrons and protons in the s-d shell. Most measured E2-observables in the s-d shell which are describable with the shell model are dominated[1)] by the isoscalar

part of the effective operator (2b). Thus, within the state-independent effective operator approximation used here, only the sum $(\epsilon_p+\epsilon_n)$ is really defined by most of the experimental data. As I'll show here, the isoscalar charge with $\epsilon_p+\epsilon_n = 1.0$ is consistent with much of the data available on E2-observables. As a first illustration, Table 2 summarizes the strongest

Table 2. Selected Strong B(E2)-Transitions to Ground States in s-d Shell Nuclei. In all calculations, an added effective charge of 0.5e is included in E2-operators for both neutrons and protons.

			$B(E2)_{i-f}$ e^2fm^4	
Nucleus	J_i,T_i	J_f,T_f	Expt.[a]	Theory[a]
^{17}F	1/2,1/2	5/2,1/2	64±2	38
^{17}O	1/2,1/2	5/2,1/2	6±0.1	4
^{18}F	3,0	1,0	15±2	15
^{18}O	2,1	0,1	7±1	3
^{19}F	5/2,1/2	1/2,1/2	21±1	19
^{20}Ne	2,0	0,0	57±8	48
^{21}Ne	5/2,1/2	3/2,1/2	63±13	81
^{22}Na	4,0	3,0	94±15	101
^{22}Ne	2,1	0,1	40±3, 66±12	55
^{35}Cl	5/2,1/2	3/2,1/2	78±2	74
^{36}Ar	2,0	0,0	59	59
^{38}Ar	2,1	0,1	38±9	28

[a]Observed and calculated values compiled for nuclei with $A \leq 22$ from Ref. 1, and for nuclei with $A \geq 34$ from Ref. 2.

transition measured to the ground state in nuclei with $A \leq 22$ or $A \geq 34$ and the calculated values for the same transitions. These calculations are all made in complete $(sd)^{A-16}$ models, with effective charges 0.5 for both neutrons and protons. The structure calculations are done with "realistic" interactions, and are described in detail in Refs. 1 and 2. We see that the qualitative features of the data are quite well reproduced. There are discrepancies with ^{17}O, ^{17}F, and ^{18}O. All the rest of the data are consistent with the calculations. The discrepancies could reflect problems related to the fact that the $1/2^+$ state in ^{17}F is unbound, and/or to deformed state

admixtures which might be important close to the closed shells. More on this later. For relative E2-transition rates between states within ground state rotational bands, the shell model predictions are in good agreement with experiment[1]. Interband transitions are observed to be weak, and they are calculated to be weak, although absolute values for weak transitions are in error by orders of magnitude. In the middle of the shell, the interband transition rates are sensitive to the interaction[11], as discussed for ^{26}Mg above.

The systematic behavior for quadrupole moments for s-d shell nuclei is summarized in Table 3. This table is extracted from a recent preprint by

Table 3. Calculated and Observed Quadrupole Moments in s-d Shell Nuclei.

	Q(e fm^2)			Q(e fm^2)	
Nucleus	Expt.[a]	Theory[a]	Nucleus	Expt.[a]	Theory[a]
^{17}O	-2.6±0.9	- 2.6	^{18}O	-16±1[b]	- 2.0
^{19}F	11.2±2	- 9.2	^{20}Ne	-23±3	-14.3
^{21}Ne	+9.3±1.0	+10.3	^{22}Ne	-17.0±3	-13.6
^{23}Na	+11±2	+10.4	^{24}Mg	-21±3	-16.0
^{25}Mg	22	+17.8	^{26}Mg	-14±4	-11.9
^{27}Al	15.3±0.3	13.8	^{28}Si	+16±3	+14.3
^{33}S	-5.5±0.5	- 7.1	^{32}S	-19±4	-13.6
^{35}S	+4.0±0.4	+ 7.8	^{34}S	6±6	10.0
^{35}Cl	-7.9±0.1	- 9.1			
^{36}Cl	-1.7±0.2	- 0.9			
^{37}Cl	-6.2±0.6	- 8.4			
^{39}K	5.5±0.4	7.7			

[a]Information taken from Ref. 22.

[b]Ref. 45.

Wildenthal and Schwalm[22]. All measured quadrupole moments for s-d shell nuclei are listed there, as well as shell model calculations for all these moments. All calculations assume 0.5e effective charges for ε_p and ε_n. The left half of the table includes all ground state moments as measured by hyperfine interaction methods. The right half summarizes all moments of

first excited $J^{\pi}=2^{+}$ states in even-even nuclei as measured by reorientation techniques. The calculations for $A \leq 22$ and $A \geq 34$ are in complete $(sd)^{A-16}$ spaces. The calculations in masses to $23 \leq A \leq 33$ are made in truncated spaces. A variety of typical effective interactions were used, as described in the articles referenced in Table 3. Again the systematic behavior of the moments is reproduced. To the extent that the sign of the moment reflects the shape of the nucleus, we see that the shape is correctly reproduced in all cases, and by and large the magnitudes are in good agreement.

There are two points worth amplifying here. The first has to do with the moments measured by the reorientation technique. As discussed by Wildenthal and Schwalm, these numbers have been considered somewhat suspect. In early calculations[23], in more severely truncated model spaces, the trend seemed to be that the ground state moments could be fit with one effective charge, while the excited state moments, measured with the reorientation technique, required an effective charge roughly 30% larger than the effective charge needed to fit the ground state moments. Further, if one assumes the low-lying states in ^{20}Ne can be described by an adiabatic rotational model, the observed $B(E2)_{2-0}$ and 2^{+} quadrupole moment cannot be described in terms of <u>one</u> intrinsic quadrupole moment. The calculated quadrupole moments in Table 3 were determined from shell models in more complete spaces than were used in the earlier work described above. I don't believe the numbers listed in this table offer strong evidence for any discrepancy. This statement is made on the condition that I be allowed to ignore the results for ^{18}O and ^{20}Ne. I'll return to these below. The ^{22}Ne number is almost within experimental error of the measured value. The calculated values for A = 23-33 are all in somewhat truncated spaces. For all but one of these cases (^{33}S), the calculated moments for <u>both</u> the ground state moments and the excited state moments are too small, and the deviations are quite similar for those nuclei which are close together in mass. The truncation of the space almost certainly reduces the quadrupole deformation. I strongly suspect that complete $(sd)^{A-16}$ space calculations will lead to good agreement for both ground state moments and excited state moments with one effective charge.

I ignored the results for ^{18}O and ^{20}Ne. The large quadrupole moment of ^{18}O is probably a manifestation of the importance of core-excited deformed states. Such an argument has been proposed[24] to explain an even larger

anomaly in ^{42}Ca. There the calculated quadrupole moment in a conventional shell model is +1.0e fm^2, while the measured one is -19e fm^2. If these deformed-state effects are so important in ^{18}O, there exists the possibility that some of the discrepancy for the ^{20}Ne quadrupole moment has a similar origin.

It is interesting to note the importance of the large space configuration mixing on these s-d shell quadrupole moments. de Kock and Glaudemans[25] have calculated a number of s-d shell quadrupole moments in an independent particle model (i.e., for A = 28, the space is $d_{5/2}^{A-16}$, and for $A \geq 33$, the space is $d_{5/2}^{12}, s_{1/2}^{4}, d_{3/2}^{A-32}$). For all the cases where measured moments were available to them, with the exception of ^{21}Ne, the correct sign was predicted for the moment by this model. They determined the necessary effective charge for this truncated space to reproduce the observed magnitude. In Fig. 4, a plot of effective charge vs. A is shown. The effective charge peaks in the middle of the shell, where the truncation is most extreme. As I have discussed above, this plot would be a much straighter line for results from shell models in complete $(sd)^{A-16}$ spaces.

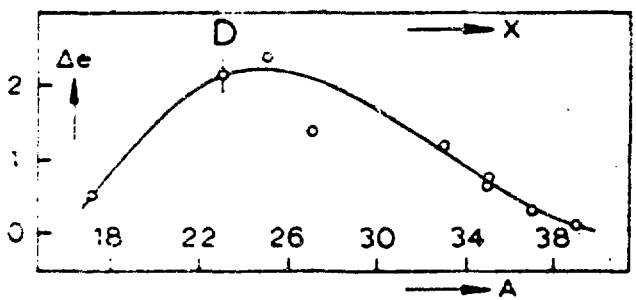

Fig. 4. Plot of Effective Charge in s-d Shell Needed to Reproduce Observed Quadrupole Moments in Simplest Independent Particle Shell Model Space ($\varepsilon_n = \Delta\varepsilon e$ $\varepsilon_p = 1 + \Delta\varepsilon e$).

We move now to E2-observables in the f-p shell. The situation here appears to be much less neat than in the s-d shell. Let us assume that we know nothing about the effective charge, and let us determine this number directly from the data on E2-transition rates in the calcium isotopes. In Table 4 I have listed the effective charges one would need[18] to fit each of several strong E2-transitions between low-lying states in ^{42}Ca, ^{43}Ca, and ^{44}Ca. This number is obtained by calculating the B(E2)-values for these transitions in a shell model space of $(fp)^{40-A}$ states using the Kuo-Brown f-p shell interaction, and assuming an effective charge of 1.0. The needed effective charge is then the square root of the ratio of the experimental

Table 4. Empirical Effective Charges for Transitions Between Low-Lying of Light Calcium Isotopes.

Nucleus	J_i	J_f	$\varepsilon_n = \sqrt{\frac{B(E2)_{obs}}{B(E2)_{calc}}}$ [a]
^{42}Ca	2	0	1.8
	4	2	1.9
	6	4	0.7
^{43}Ca	5/2	7/2	1.2
	3/2	7/2	1.6
	11/2	7/2	1.6
^{44}Ca	2	0	1.3,1.8

[a]Taken from Ref. 14.

B(E2) to the shell model B(E2). For comparison, the effective charge for the 2-0 transition in ^{18}O determined in this way is 0.7e. In Table 4 we see that in all but one case, the empirical effective charge is 2-3 times as large in the f-p shell as in the s-d shell. If we define the effective charge as arising from one-particle one-hole core-polarization processes, it is unlikely that any effective operator calculation can lead to simple effective operators for use in "conventional" f-p shell model calculations which will reproduce this behavior. This strong enhancement almost certainly reflects contributions due to complicated many-particle many-hole deformed states. Brown, et al.[26] have attempted to analyze all the data in the f-p shell nuclei near ^{40}Ca and ^{48}Ca and ^{56}Ni in terms of one set of neutron and proton effective charges. They first estimate the contributions of deformed states in A=40-42 nuclei from schematic calculations similar to the Brown-Green calculations[46], and then fit effective charges to the transition strength not accounted for by the deformed states. They find that effective charges $\varepsilon_p \approx 0.2e$ and $\varepsilon_n \approx 0.6e$ describe a number of transitions in these nuclei. Such results are consistent with perturbation theory calculations of the core-polarization effective charge in this region. The calculated B(E2) values in ^{42}Sc and ^{43}Sc which result from $(fp)^{A-40}$ shell model calculations with the Kuo-Brown interaction and with effective charges $\varepsilon_p = 0.2e$ and $\varepsilon_n = 0.5e$ and in which deformation effects are not accounted

for are summarized in Table 5 for those cases where measured B(E2) values

Table 5. Calculated and Observed B(E2) Values in $^{42,43}Sc$.

Nucleus	$2J_i, 2T_i$	$2J_f, 2T_f$	$B(E2)_{i-f}$ e^2 fm^4 Expt.[a]	(ε_p=0.2, ε_n=0.5) Theory[a]
^{42}Sc	6,0	2,0	35	23
	10,0	14,0	20-25	21
	4,2	0,2	72	19
^{43}Sc	3,1	7,1	67	62
	11,1	7,1	123	20
	15,1	11,1	49	23
	19,1	15,1	27	15

[a]Information taken from Ref. 14.

are available. In this table only two of seven calculated transition rates are in reasonable agreement with measured values. Assuming that the experimental values are correct, it is obvious that the shell model is a failure in this region for such calculations as these, in significant contrast to the s-d shell. In Table 6 the calculated and measured quadrupole moments

Table 6. Calculated and Observed Quadrupole Moments in Light f-p Shell Nuclei.

Nucleus	J	Q(e fm^2) Observed[a]	(ε_p=0.2, ε_n=0.5) Calculated[a]
^{42}Ca	2^+	-19±8	+1
^{43}Sc	7/2	-26	-16
^{44}Sc	2	-10	-6
^{44}Sc	6	-19	-16

[a]Ref. 14.

for f-p shell nuclei with $A \leq 44$ are summarized (where again ε_p = 0.2e and ε_n = 0.5e). The picture is not any better here. (Remember that the B(E2) is quadratic in the E2 matrix element while the quadrupole moment is only

linear.) The ^{42}Ca number was discussed briefly above. It appears that almost the entire moment of the first 2^+ state in ^{42}Ca comes from core-excited deformed-state configurations. There is still a significant discrepancy for the ^{43}Sc moment, while the results begin to appear reasonable in ^{44}Sc.

Next consider M1-observables. There is extensive data available on magnetic moments of ground states (and a few excited states) in s-d and f-p shell nuclei. The measured magnetic moments for s-d shell nuclei with $A \leq 22$ and $A \geq 34$ are compared with calculated values in Table 7. In these

Table 7. Calculated and Observed Magnetic Moments in s-d and f-p Shell Nuclei.

	$M(\mu_N)$			$M(\mu_N)$	
Nucleus	Expt.[a]	Theory[a]	Nucleus	Expt.[b]	Theory[b]
^{17}F	+4.72	+4.79	^{41}Ca	-1.91	-1.59
^{17}O	-1.89	-1.91	^{42}Ca	-2.88	-3.0,-2.52
^{18}F	+2.86	+2.88	^{43}Ca	-1.67	-1.32
^{19}F	+2.63	+2.87	^{43}Sc	4.80	4.61
^{19}F	+3.69	+3.55	^{43}Sc	3.18	3.15
^{20}F	+2.09	+1.92	^{44}Sc	2.60	2.56
^{21}Ne	-0.66	-0.90	^{44}Sc	3.64	3.88
^{22}Na	+1.75	+1.82			
^{35}Cl	+0.82	+0.68			
^{35}S	±1.0	+1.21			
^{36}Cl	+1.28	+1.55			
^{37}Ar	+0.95	+1.43			
^{37}Cl	+0.68	+0.59			
^{38}K	1.37	+1.24			
^{39}K	+0.39	+0.13			

[a]Information on nuclei with $A \leq 22$ taken from Ref. 1; information on nuclei with $A \geq 34$ taken from Ref. 2.

[b]Information taken from Ref. 14.

calculations, the operator is assumed to be the free nucleon M1 operator. For the A=17 nuclei, the measured value agrees very well with the calculated values, implying that the bare operator is the "correct" effective operator.

It turns out that this is really the result of more or less fortuitous cancellation of higher-order corrections. In the mass 39 system, the proton hole moment deviates from the Schmidt value by almost 0.3 μ_n, so that here the higher-order renormalization is a little more significant. The agreement between shell model and experiment is qualitatively good throughout the s-d shell. It is quantitatively somewhat better at the lower end of the shell than at the upper end of the shell. This is consistent with the fact that renormalization effects are stronger in A = 39 than in A = 17. In those cases where these moments were calculated with different interactions[1,2], there was no great sensitivity to the interaction evidenced. The measured magnetic moments in the light f-p shell nuclei are also summarized in Table 7, and also shown there are the moments calculated in conventional shell models[18], wherein the M1 operator is assumed to be the bare operator. There is again a fairly significant deviation in the ^{41}Ca case. This same discrepancy appears in the low-spin states in ^{43}Sc and ^{43}Ca, while it disappears for the high-spin states in ^{42}Ca and ^{43}Sc. This might imply that the deformed state effects are diminished in the high-spin states.

There is not nearly as much experimental data available on B(M1) values in the s-d shell or the light f-p shell as is the case for B(E2) values. For more or less well-known reasons[1], isoscalar M1 transitions are extremely weak in these regions. The calculated values of isoscalar transitions involve strong cancellation effects, and are relatively sensitive to the interaction. Some of the most extensive investigations of M1 transitions in the s-d shell have been reported by the Utrecht group[22]. These calculations are in the middle of the s-d shell. They use s-d shell wave functions in a severely truncated $(sd)^{A-16}$ space, and they treat the various matrix elements of the effective M1 operator, $\langle j_1 \| M1 \| j_2 \rangle$ as free parameters which they adjust to fit observed transition rates. They find significantly different effective operators for nuclei which differ only by one or two mass units. Thus, they did not satisfy the "simple"-effective operator condition. This result surely reflects the fact that they work in a very truncated space. In general the M1 operator is sensitive to excitation of particles of states from orbits with $J = \ell + 1/2$ to orbits with $j = \ell - 1/2$, so that the calculated B(M1) values are sensitive to truncation within the shell.

M1 transitions with $\Delta T=1$ are strong. There are only three or four cases where observed $\Delta T=1$ M1 transitions can be compared with large shell model calculations. In ^{20}Ne, two such transitions are accurately described in a calculation[59] in which the bare M1 operator is used. In the f-p shell, two such transitions are observed in ^{42}Ca. There are serious discrepancies between the observed values, and those calculated[18] with the bare M1 operator in an $(fp)^2$ shell model.

The major successes of the shell model have been for energy levels, spectroscopic factors, E2 observables in the s-d shell, and magnetic moments. The general experience has been that these observables are not very sensitive to the particular effective interaction which is used (with the exceptions already noted for the middle of the s-d shell). There has been a fair amount of interest and activity in the study of two-nucleon transfer reactions, with the hope that the analysis of such reactions would be more sensitive to two-particle correlations in the nuclear wave functions, and thus sensitive to the two-body effective Hamiltonian which is used. I would next like to review briefly the record so far for shell model analyses of some two-nucleon transfer reactions. I think the general results can be summarized by saying that the uncertainties in the DWBA analyses are much more important in this case than in the one-nucleon transfer case, so that we are not yet at the point where too much can be learned about shell model wave functions from such reactions. (For a rather complete discussion of spectroscopy with two-nucleon transfer reactions, see Ref. 29.)

Let us first review the sources of sensitivity and uncertainty. If it is assumed that the two-particle transfer proceeds by a direct process, and if we believe the reaction can be described by the DWBA, then the cross section for the reaction is proportional to

$$\sum_{J,T}\left|\sum_{j_1,j_2} \beta_{j_1 j_2} \langle\psi_{A+2}\|(a^\dagger_{j_1} \times a^\dagger_{j_2})^{J,T}\|\psi_A\rangle\right|^2$$

where $a^\dagger_j$ is the usual single particle creation operator. The reaction mechanism information is contained in $\beta_{j_1 j_2}$, while the information on nuclear structure is contained in the matrix element. Note in particular that while the sum is incoherent in the transferred angular momentum and isotopic spin (J,T) it is coherent in contributions from transfers to

different pairs of single particle orbits. Because of this coherence, there is expected to be sensitivity to the wave functions. This sensitivity is enhanced for the following reason. In two-particle transfers with mass 3 and 4 particles, the reaction is dominated by the transfer of pairs of particles which are in relative s-states with respect to their centers-of-mass. Thus, the cross section is in part proportional to the probability that the pair j_1, j_2 are in a relative s-state. This probability is obtained by transforming the wave function of the pair from j-j to L-S coupling through the usual 9-j coefficient, and then from single particle coordinates to relative and center-of-mass coordinates through a Talmi transformation. Thus, the cross section becomes proportional, in part, to factors of the form

$$\begin{bmatrix} \ell_1 & \frac{1}{2} & j_1 \\ \ell_2 & \frac{1}{2} & j_1 \\ L & S & J \end{bmatrix} \langle n_1 0_1, n_2 \ell_2; L | n\ell = 0, NL; L \rangle$$

These coefficients are rather strongly dependent on the ℓ's and j's involved. For instance, this coefficient for the $(1p_{3/2})^2$ configuration is three times[28] as big as the coefficient for the $(0f_{7/2})^2$ configuration. When this term is squared, the cross section for $p^2_{3/2}$ is an order of magnitude stronger than for $f^2_{7/2}$. Similarly in the s-d shell, the s^2 configuration is highly favored over the d^2 configurations. It is a hope that this sensitivity can be useful in studying various wave functions with the analysis of two-nucleon transfer reactions. The angular distribution for the typical direct two-nucleon transfer process is often a very clear signature of the coupled angular momentum of the particle pairs, and is relatively insensitive to the individual j's of the pair. This signature, plus rather strong selection rules on ΔS and ΔT have made the two-nucleon transfer process very useful in spectroscopy for determining spins and parities. It also makes the most distinctive features of the observed cross sections relatively useless insofar as studying the relations between various model wave functions.

Thus, the reaction can be useful in shell model studies only to the extent that one can expect to predict absolute or relative cross sections. One must first ask about the accuracy of the DWBA in analyzing such reactions. The traditional approach has been to calculate the structure part

of the cross section with shell model wave functions. One then replaces the oscillator wave functions for the transferred pair with some appropriate wave functions such as those of a Woods-Saxon well. The choice of the strength of the well is important. The most popular approach at present seems to be to assume that each transferred particle is bound with one-half the separation energy of the transferred pair of particles. A second conventional approximation is to assume that the interaction between the transferred pair and the remaining particles in the projectile is a delta function in the centers-of-mass of the pair and of the residual particles, the so-called zero-range approximation. These approximations have received considerable attention.

Bayman and Feng[30] have investigated the accuracy of the zero-range approximation. They find that the relative cross sections predicted for the $^{40}Ca(t,p)^{42}Ca$ reaction to states with different J-values can differ by as much as 40-80% in the zero-range approximation as compared with more exact finite range calculations. They find that in a finite range calculation, wherein some ground state correlations are included in a pairing approximation, and in which the binding energy prescription described above is used, the absolute cross section is predicted to be roughly 1/3 of the observed cross section.

The question of the two-nucleon overlap function and the resulting form factor has been studied in some detail recently, primarily by Ibarra[31]. He has studied an approximate method for including effects due to one- and two-particles in the continuum on the two-particle transfer form factor. His results suggest that the more exact form factor for the $^{40}Ca(t,p)^{42}Ca$ reaction can lead to a factor of two difference in the calculated cross section as compared to the binding energy prescription conventionally used.

From these investigations, one can say that the use of the zero-range approximation and the well-depth prescription introduce uncertainties in calculated cross sections of roughly a factor of two to three for strong direct transitions.

There are important effects in these reactions due to ground state correlations that are not contained in conventional shell model calculations. Vary, et al.[32] have studied the $^{208}Pb(p,t)^{206}Pb$ reaction. They use a two-particle RPA formalism to calculate the effects of ground state

correlations in this reaction. Their general results are that with respect to a conventional two-hole shell model calculation of ^{206}Pb, the ground state correlations enhance the transition to the ^{206}Pb ground state by a factor of two, they enhance the transition to the first 2^+ by 40%, and they have relatively little effect on other transitions. For identical-particle two-particle transfers, correlations among valence particles introduced by the pairing part of the residual interaction concentrate the two-particle transfer strength into the ground states, much as the quadrupole strength is concentrated in the lowest 2^+ state by the quadrupole-quadrupole part of the residual interaction. The RPA results imply that much of the strength added to a model space by introducing core excitation goes mainly to the ground state. Similar results are reported by Bayman and Hintz[33] in studies of the calcium isotopes.

There are, of course, the usual uncertainties due to optical model ambiguities. In addition to these DWBA uncertainties there is the question of the importance of various two-step non-direct processes in these two-nucleon transfer reactions[29]. I'll cite an example of such effects in a few minutes.

Bearing in mind all these uncertainties, what is the record so far on shell model predictions of two-nucleon transfer reactions? Let me first show examples of two cases where the shell model is expected to fare well, the lead region and the calcium isotopes.

The reactions ^{208}Pb(p,t)^{206}Pb and ^{206}Pb(p,t)^{204}Pb have been studied[35]. Shell model wave functions have been generated for the relevant nuclei in a model space involving the six lowest neutron hole orbits in ^{208}Pb. A "realistic" interaction developed by Kuo and Herling[36] was used for the effective Hamiltonian. The predictions of this model for energies and various other observables are in above average agreement with experiment. The two-particle spectroscopic factors calculated with these same wave functions have been used in a standard DWBA (i.e. zero-range approximation and well-depth prescription) analysis of the observed cross sections. The results of this analysis for transitions to the low-lying $J=0^+$, 2^+, and 4^+ states in ^{206}Pb and ^{204}Pb are summarized in Fig. 5. The figure shows relative cross sections where for each J, the cross section is normalized to the strength of the transition to the lowest state of that J in the ^{208}Pb(p,t)^{206}Pb reaction. With this normalization, we see that the qualitative features of

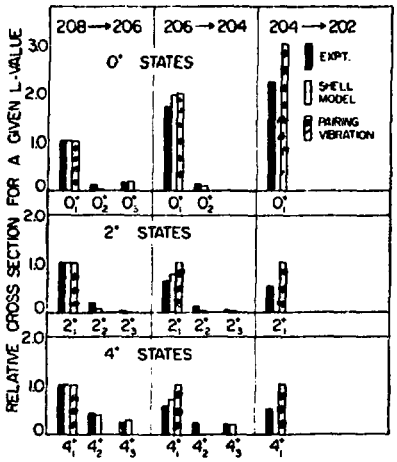

Fig. 5. Observed and Calculated Relative Cross Sections for $^{208}Pb(p,t)^{206}Pb$, $^{206}Pb(p,t)^{204}Pb$ and $^{204}Pb(p,t)^{202}Pb$ Reactions.

the relative cross sections are accurately reproduced. In these figures we see also the predictions of the simplest pairing-vibration model. The gross features of the data are also reproduced by this model. The pairing-vibration model can only predict the strength to the lowest state of each spin. For these states, where there is a deviation between the experiment and theory, the shell model is in better agreement than the pairing-vibration model.

The overall normalization required to fit the 0^+ states is roughly twice that required to fit the states with higher spins. This presumably reflects the significance of ground state correlations. It is consistent with the factors discussed by Vary, et al.[32] for such effects.

Very similar analyses of the (p,t) and (t,p) reactions[37] on the calcium isotopes have been performed. In this case the shell model calculations are in a slightly truncated $(fp)^{A-40}$ space. The (t,p) reaction has been studied on the targets $^{40,42,44,46,48}Ca$. In Table 8, the results of a conventional DWBA analysis of the $^{44}Ca(t,p)^{46}Ca$ reaction are summarized. These results are fairly typical of the results for the $^{40,42,44}Ca$ experiments. In this table, Y represents the sum of the observed and/or predicted cross sections at a selected set of 5 angles. The experimental and calculated Y's are separately normalized to 100 for the ground state transitions. The results are similar to what one finds for single-nucleon transfer spectroscopic factors. In the experimentally observed cross sections, there is a significant splitting of the strength to states at a relatively high energy. In the calculation, there is strength in one or two high-lying states that occurs at about the same energy as the observed strength. The column labeled R is the ratio of the theoretical Y value to the observed Y value. Four of these five ratios are roughly 2. Thus, if we doubled the calculated ground state cross section, the theory-experiment

Table 8. Calculated and Observed Relative Cross Sections for $^{44}Ca(t,p)^{46}Ca$ Reaction. Y is sum of cross sections at 12 forward angles. ΣY represents total strength to several states listed in that column.

	Expt.			Theory			
J	E(MeV)	Y	ΣY	E(MeV)	Y	ΣY	R
0	0	100		0	100		
2	1.35	8		1.44	14		1.8
4	2.57	2		2.63	4		2
6	(2.98)			2.99	2		
2	3.02	6					
	3.64	3		4.15	89		
	3.86	3		5.15	50		
	4.43	20					
	4.99	6	38			139	36
4	4.74	20		4.74	55		2.2
	(5.53)	5	25				
0	5.32	14					
	5.60	55		5.21	183		
	5.63	37		5.49	26		
	6.05	9	115			209	1.8

comparison would be much more favorable. This is again consistent with the result that the shell model calculations underestimate the ground state pairing correlation by a factor of about 2. Analyses analogous to these for the $^{46,48}Ca(t,p)^{48,50}Ca$ reactions are not so satisfactory. The shell model calculations for these nuclei are in a significantly truncated $(fp)^{A-40}$ space, and this may account for some of the difficulty. Thus, from these two "good" shell model regions, we have a consistent picture. Pairing correlations are underestimated in the model ground state wave functions, while the relative properties of the excited states are well described by the theory, better than the known inaccuracies of the DWBA might suggest.

What is the situation in the s-d and f-p shell? In the s-d shell, much of the experimental work[38)] has been on ^3He induced reactions for targets in the middle of the shell. The emphasis in these experiments has been to test the nuclear wave functions. In an early investigation by Hardy and Towner[39)] the sensitivity of the cross section to the wave function was clearly evidenced. Before comparison with the two-nucleon transfer experiments, the signs of some of the matrix elements of the effective interaction

were undetermined by the shell model calculations in question. It turned out that the observed cross sections were in serious disagreement with the calculation with one sign assumed for one matrix element, but there was substantial agreement with the other sign.

Unfortunately, for the nuclei involved in most s-d shell two-particle transfer experiments, full s-d shell model wave functions were not available for the analysis, so there is an ambiguity in whether to assign discrepancies in theory-experiment comparisons to DWBA problems, or wave function deficiencies. In Table 9, I have summarized the results of Nann, et al.[40)]

Table 9. Calculated and Observed[a] Cross Sections for $^{29}Si(p,t)^{27}Si$ and $^{29}Si(p,^{3}He)^{27}Al$ Reactions. The meanings of ZOSM, FOSM, and MSDI are explained in detail in Ref. 40.

	$\sigma_{exp}/\sigma_{theory}$					
	ZOSM		FOSM		MSDI	
J_f	$(p,^{3}He)$	(p,t)	$(p,^{3}He)$	(p,t)	$(p,^{3}He)$	(p,t)
5/2	0.36	1.25	0.45	3.10	0.58	1.25
1/2	1.10	0.19	0.80	0.35	1.05	0.75
3/2	2.10	0.38	2.10	0.65	1.30	0.75
7/2	1.50	0.48	1.50	0.70	0.90	0.60
5/2					0.65	4.10

[a]Ref. 40.

on the $^{29}Si(p,^{3}He)^{27}Al$ and $^{29}Si(p,t)^{27}Si$ reactions. These results are typical of mid s-d shell results now in the literature. In the $(p,^{3}He)$ case there can be $\Delta S=0$, $\Delta T=1$, and $\Delta S=1, \Delta T=0$ transitions possible, and the relative strengths of these are a function of the still-ambiguous spin-isospin exchange character of the nucleon-nucleon interaction used in the reaction analysis. The authors present arguments for a certain value of this exchange component, and then compare observed and calculated cross sections with various sets of wave functions. The wave functions vary from pure independent particle wave functions to wave functions obtained from diagonalizing an effective interaction in a truncated space of $(sd)^{A-16}$ configurations. The calculations were made in the zero-range approximation, and the usual binding energy prescription for the form factor was used. The results are summarized in Table 9 for five low-lying states. ZOSM is the

independent particle model (i.e. $^{29}Si = (d^{12}_{5/2}, s_{1/2})$, etc.). MSDI is a more complete shell model calculation. These results are for the case where the spin-isospin exchange character was chosen to maximize the theory-experiment agreement. The points I wish to emphasize here are that 1) there is only rough agreement for the states shown here, 2) the results are sensitive to the set of wave functions used, and 3) the results are within limits of uncertainties of DWBA. For other states not shown here, the agreement is not so good. The results for the (p,^{3}He) reaction to analogs of the states shown in this table are not nearly as good as for the (p,t). The (p,^{3}He) results are obviously more sensitive to the spin-isospin-exchange in the residual interaction. Finally, the "best" wave functions here are still in a highly truncated basis. The situation at best is ambiguous. There has been some work recently on two-particle transfer on s-d shell nuclei where "complete" s-d shell wave functions are available, so that some of the ambiguity is reduced. The most recent results[41] are for the $^{18}O(^3He,p)^{20}F$ reaction, which results are summarized in Table 10. The analysis is a standard one,

Table 10. Calculated and Observed Cross Sections[a] in the $^{18}O(^3He,p)^{20}F$ Reaction.

J	$E_{observed}$ (MeV)	E_{calc} (MeV)	$10^3 \times \sigma_{exp}/\sigma_{DWBA}$
2	0	0	0.51
3	0.66	0.61	0.83
4	0.82	1.21	0.36
1	1.06	0.92	1.03
5	1.82	2.29	0.48
2	2.05	1.76	1.15
1	3.49	2.47	0.69

[a]Ref. 41.

similar to those described above. Considerable information can be gathered from this reaction on spins and parities from angular distributions, and from relative contributions of different L-transfer. The shell model apparently predicts correctly which transitions are dominated by one L-transfer, and which transitions involve several L-transfers. Table 10 shows data on states which are believed to be accurately described as $(sd)^4$

states. The absolute cross sections to these states are all roughly comparable, to within an order of magnitude. The relative cross sections for these transitions are reproduced to within a factor of two or three.

As I mentioned above, in all the cases I've mentioned so far, the two-nucleon transfer process is assumed to be a one-step direct transfer. There is at least one striking case in the s-d shell where two-step processes are extremely important. At the Argonne two-nucleon transfer conference[29], the Michigan State group reported on studies of the $^{26}Mg(p,t)^{24}Mg$ and $^{24}Mg(p,t)^{22}Mg$ reactions. In the $^{24}Mg(p,t)^{22}Mg$ reaction, the shell model predicted there to be no strength to the first 2^+ state in ^{22}Mg, while experimentally the state was populated with a cross section quite comparable to cross sections to other excited states in ^{22}Mg. A recent analysis[42] in terms of a two-step inelastic excitation, $^{24}Mg(p,p')^{24}Mg(2^+) \rightarrow {}^{24}Mg(2^+)(p,t)^{22}Mg(2^+)$, is found to account for almost the entire cross section. This thus adds another complication which cannot be ignored.

The situation in the f-p shell is similar to that in the s-d shell. There has been very little detailed analysis of two-particle transfer with full f-p shell wave functions. There are data on the $^{42}Ca(^3He,p)^{44}Sc$ reaction which has been analyzed in terms of pure $f^n_{7/2}$ wave functions, with generally unsatisfactory results. There is one possibly interesting systematic effect with regard to $J^\pi=1^+$ states in f-p shell nuclei as summarized by Hansen at the Argonne Conference[29]. The DWBA prediction of strength to 1^+ states is consistently too small by a factor of 2-3 relative to the strength to 5^+ states. This might suggest some important $(1^+,T=0)$ correlations similar to the $(0^+,T=1)$ correlations that are so important for two-neutron transfers.

In summary then for the two-particle transfer processes in so-called "good" shell model nuclei, there is a potential for testing the two-particle correlations in the wave functions, but DWBA uncertainties still mask such effects. It does not seem to be a field where the structure and the reaction theorist can be quite so independent as has been the case in general for one-nucleon transfer reactions, but it does seem to be a field ripe for exploitation by a collaborative effort of structure and reaction theory.

In the remaining few minutes, I'd like to make a few comments on shell model calculations of nuclei near closed shells in which the model spaces explicitly include multi-particle multi-hole core-excited configurations.

It is well known that the so-called closed shells are not closed. In nuclei near closed shells there is a class of states reasonably well described by the conventional shell model; there are states whose parity precludes their being described in conventional shell model spaces, and finally there are low-lying normal parity states which cannot be described by conventional shell model calculations. Such states are usually referred to as deformed states. A major step in understanding the structure of these states in closed- and near-closed shell nuclei was the work of Brown and Green[46]. They studied the structure of ^{16}O in terms of a three intrinsic state space. They included the normal closed shell spherical state, and two states constructed from single-particle states of a deformed potential, a two-particle two-hole state and a four-particle four-hole state. They correlated much of the observed data on energy levels and E2-observables in ^{16}O with such a model. It was a natural question to ask if one could reproduce the features of such a calculation by diagonalizing an effective two-body Hamiltonian in a space of spherical orbits. An affirmative answer was found by Zuker, Buck, and McGrory[47] (ZBM) and by Zuker[48]. In their model an inert ^{12}C core was assumed and the $p_{1/2}$, $d_{5/2}$, and $s_{1/2}$ orbits were included in the model space. They found a phenomenological two-body interaction which gave substantial agreement with the observed features of nuclei around ^{16}O when single-particle energies similar to the energies of states in ^{13}C were used. The same model was successfully extended[52] to the description of all nuclei in the mass range A = 16-20. Thus, one model describes many features of many nuclei, so that one could call the phenomenological model a success. Recently, a similar calculation[53] was made in a model space in which the $d_{3/2}$ orbit was added to the model space used in ZBM, and the effective interaction was derived for this space from the Reid nucleon-nucleon potential. Although the inclusion of the $d_{3/2}$ orbit did not significantly increase the degree of phenomenological success, this calculation equalled the success of the previous calculation, so that a step was taken towards making the calculations "realistic". I might digress to emphasize that this is as far as anyone has progressed towards a "realistic" calculation of the excited spectrum of a "closed-shell" nucleus. An outstanding failure of nuclear theory is that no realistic calculation of ^{16}O in terms of 16 active particles has been in any way successful in describing the first excited state as a four-particle four-hole state. This is discussed extensively by

Irvine, et al.[44]. The crucial ingredients in the ^{12}C core calculations are: 1) the choice of the single-particle spectrum wherein the s-d shell orbits are only 3 MeV above the $p_{1/2}$ orbit, and 2) the inclusion of both the $d_{5/2}$ and $s_{1/2}$ orbits. The small single-particle spacing is crucial to obtaining a low-lying four-particle four-hole state. The inclusion of both the $d_{5/2}$ and $s_{1/2}$ orbits is essential if the multi-particle states are to display any quadrupole deformations as experiment suggests they must.

There are observed features of low-lying states in nuclei around ^{16}O which are not satisfactorily described by the core-excited models, and which I believe result from shortcomings in describing the relative admixture of so-called spherical and deformed states in one eigenstate. I'll cite two examples. The first has to do with the $^{18}O(p,t)^{16}O$ reaction. This reaction has been studied in detail by Adelberger, et al.[49]. In Table 11, I have

Table 11. Normalizations of DWBA, N_{DWBA}, Required to fit Observed Cross Sections in $^{18}O(p,t)^{16}O$ Reactions to Low-Lying Normal Parity T=0 States.

	Excitation		N_{DWBA}	
J^{π}	Expt.[a]	Theory[a]	ZBM[a]	COEXISTENCE[a]
0^+	0	0	4	4[b]
0^+	6.06	6.19	4	72
2^+	6.92	7.39	15	
2^+	9.83	10.27	3600	
4^+	10.34	10.66	2	
0^+	11.30	10.68	6	
2^+	11.49	11.77	280	
0^+	12.02	12.48	5	

[a]Ref. 49.

[b]N_{DWBA} for ground state transition in coexistence model arbitrarily normalized to 4.

summarized the observed relative cross sections to low-lying levels in ^{16}O,

and also the relative cross sections calculated in a conventional DWBA analysis with the form factor calculated from the ZBM wave functions. The data on the strengths to the $J=0^+$ states was used to demonstrate a failure of the Brown-Green coexistence model, and a success of the more complex shell model calculation. However, I think when one also includes the data on 2^+ and 4^+ states, it is obvious that there are problems with the core-excitation model as well as with the coexistence model. It seems unlikely that form factor and optical model uncertainties can account for these discrepancies. A second example of possible difficulty is shown in Table 12.

Table 12. Calculated and Observed E2-Observables in $^{16,17,18}O$.

			B(E2) e^2fm^4 ($\varepsilon_p=\varepsilon_n=0.35e$)	
Nucleus	J_i,T_i	J_f,T_f	Expt.	Theory
^{16}O	$2_1,0_1$	$0_1,0$	5[a]	7
		$0_2,0$	41[a], 100[b]	40
	$2_2,0$	$0_1,0$	0.1[a]	0.1
	$4_1,0$	$2_1,0$	110[a]	37
^{17}O	5/2,1/2	5/2,1/2	-3[a] Q(e fm^2)	-2.8
	1/2,1/2	5/2,1/2	6[a]	9
^{18}O	$2_1,1$	0,1	10[c]	12
	$2_1,1$	$2_1,1$	-16[c] Q(e fm^2)	[illegible]

[a]Ref. 43.

[b]Ref. 44.

[c]Ref. 45.

This shows some E2-observables in the $^{16,17,18}O$ isotopes. In these calculations the Zuker[48)] effective interaction is used, harmonic oscillator single-particle wave functions are used, and an effective charge of 0.35 was assumed for both neutrons and protons. The value of 0.35 was chosen so as to roughly reproduce the quadrupole moment in ^{17}O. Since this space is much different from the conventional $(sd)^n$ space discussed above, a comparison of the effective charges in the two spaces is of limited significance. These

calculations have been repeated with Woods-Saxon wave functions with very similar results. There is some uncertainty in the experimental value of the $2_1^+ \rightarrow 0_2^+$ transition in ^{16}O. The value of 100 is mentioned without reference. In ^{16}O the strong transitions are between states in the rotational band based on the 0^+ state at 6 MeV. The enhancement of these transitions are reproduced in the model. If the value of 100 for the $2_1^+ \rightarrow 0_2^+$ is correct, then the effective charge for the deformed band must be more than double the value used here. If the value 41 is correct, then the observed values imply a strong state dependence of the effective charge for transitions within the excited 4p-4h band. The calculated values for the ^{17}O observables are acceptable (obviously so for the quadrupole moment, to which the effective charge was roughly normalized). Perhaps the most interesting discrepancy is for the quadrupole moment in ^{18}O. This is the first excited 2^+ state in ^{18}O, and it is generally described as a spherical shell model state. The observed value is about three times the calculated value. This large observed value is almost certainly due to admixtures of deformed states. Perhaps the most serious deficiency of the model space used in these calculations is the omission of $p_{3/2}$ orbit excitations. The four-hole state $(p_{1/2})^{-4}$ is a spherical state, and so the four "holes" of the four-particle four-hole state in the ^{16}O calculation make no contribution to deformation. The added deformation possible with the inclusion of $p_{3/2}$ excitation could be significant in these E2-observable calculations. The inclusion of this orbit could also be very significant in the analysis of the $^{18}O(p,t)^{16}O$ reaction to the 2^+ states, as discussed by Ohanian[49].

A next obvious step would be to extend these core-excitation calculations to the ^{40}Ca core region. Zuker[54] has previously discussed preliminary steps in this direction, as have Federman and Pittel[55]. In both cases the model space is restricted to $d_{3/2}$ holes and $f_{7/2}$ particles. The calculations reported so far have shed some light on the gross qualitative structure of nuclei in this mass region. Surely the restriction to the $d_{3/2}$-$f_{7/2}$ space is too severe. The amount of deformation achievable in such a model space is severely restricted. The importance of the $p_{3/2}$ and $s_{1/2}$ orbits in achieving deformation in the s-d shell and the f-p shell has been discussed previously[50]. These two low-lying orbits are also of extreme importance in any analysis of multi-nucleon transfer reactions, as discussed above. While practical necessities require the use of a severely truncated space, the

experimental data suggest that deformation admixtures are larger in the Ca region than in the oxygen region. This is suggested by the following qualitative results:

1. The abnormally large effective charges in the calcium isotopes already pointed to above.

2. The analysis of Towsley, et al.[24] of the ^{42}Ca 2_1^+ quadrupole moment suggests that the first 2^+ state (the shell model 2^+ state) is 50% deformed.

3. If the ^{42}Ca ground state were a two-particle state, and the 3.35 MeV state in ^{40}Ca were a 4p-4h state, there would be no intensity to the excited state in the ^{42}Ca(p,t)^{40}Ca reaction. But, experimentally[49], the first excited 0^+ state at 3.35 MeV is populated with 25% of the intensity of the ground state transition, as compared to 7% for the analogous comparison of the ^{18}O(p,t)^{16}O cross sections.

4. On the other hand, in the ^{38}Ar(^{3}He,n)^{40}Ca reaction[56], the 3.35 MeV state is not populated. This is consistent with the simple picture given in item 3 above.

5. The spectroscopic factors extracted[57] from experimental cross sections in the DWBA for the ^{41}Ca(d,p)^{42}Ca reaction suggest that deformation effects are significantly underestimated in the Gerace-Green[58] calculation of ^{41}Ca and ^{42}Ca. These calculations are analogous to the Brown and Green ^{16}O calculations.

Thus, in just these few nuclei there are a number of intriguing experimental results demanding interpretation, and I expect the $(f_{7/2},d_{3/2})$ model will not successfully meet this challenge. Ways must be found to include other important parts of the space in such a calculation if one is to approach a microscopic understanding of the light f-p shell nuclei. Perhaps Andres Zuker will give us some assistance in charting this path in the next talk.

I would like to acknowledge many useful conversations with B. H. Wildenthal on various aspects of this talk.

List of References

1. E. C. Halbert, J. B. McGrory, B. H. Wildenthal, and S. P. Pandya, Advances in Nuclear Physics, Vol. 4 (M. Baranger and E. Vogt, eds.), Plenum Press, New York (1971).
2. B. H. Wildenthal, E. C. Halbert, J. B. McGrory, and T. T. S. Kuo, Phys. Rev. C4 (1971) 1266.
3. B. H. Wildenthal, J. B. McGrory, E. C. Halbert, and H. D. Graber, Phys. Rev. C4 (1971) 1708.
4. R. R. Whitehead and A. Watt, Phys. Lett. 41B (1972) 7.
5. B. J. Cole, A. Watt, and R. R. Whitehead, Phys. Lett. 49B (1974) 133.
6. B. J. Cole, R. R. Whitehead, and A. Watt, to be published.
7. B. J. Cole, A. Watt, and R. R. Whitehead, to be published.
8. J. B. French, E. C. Halbert, J. B. McGrory, and S. S. M. Wong, Advances in Nuclear Physics, Vol. 3 (M. Baranger and E. Vogt, eds.) Plenum Press, New York (1969).
9. R. R. Whitehead, Nucl. Phys. A182 (1972) 290.
10. M. Soyeur and A. Zuker, Phys. Lett. 41B (1972) 135.
11. P. W. M. Glaudemans, et al., contribution to this conference.
12. T. T. S. Kuo, Nucl. Phys. A103 (1967) 71.
13. B. M. Preedom and B. H. Wildenthal, Phys. Rev. C8 (1972) 1633.
14. J. B. McGrory, B. H. Wildenthal, and E. C. Halbert, Phys. Rev. C2 (1970) 186.
15. A. E. L. Dieperink and P. J. Brussaard, Nucl. Phys. A128 (1969) 34.
16. I. Talmi and I. Unna, Ann. Rev. of Nucl. Sci. 10 (1960) 353.
17. J. D. McCullen, B. Bayman, and L. Zamick, Phys. Rev. 134 (1964) B515.
18. J. B. McGrory, Phys. Rev. C8 (1973) 693.
19. R. Gross and I. Talmi, Phys. Lett. 44B (1974) 147.
20. G. T. Garvey and Y. Kelson, Rev. Mod. Phys. 41 (1969) 1.
21. C. Thibault and R. Klapisch, Phys. Rev. C6 (1972) 1509.
22. B. H. Wildenthal and D. Schwalm, to be published.
23. B. H. Wildenthal, J. B. McGrory, and P. W. M. Glaudemans, Phys. Rev. Lett. 26 (1970) 96.
24. C. W. Towsley, D. Cline, and R. N. Horoshko, Phys. Rev. Lett. 28 (1972) 368.
25. P. R. de Kock and P. W. M. Glaudemans, Phys. Lett. 34B (1971) 280.

26. B. A. Brown, A. Arima, and J. B. McGrory, to be published.
27. P. W. M. Glaudemans, P. M. Endt, and A. E. L. Dieperink, Ann. Phys. (N.Y.) 63 (1971) 134.
28. I. S. Towner and J. C. Hardy, Advances in Physics 18 (1969) 401.
29. Proceedings of the Symposium on Two-Nucleon Transfer and Pairing Excitations, Argonne National Laboratory, March 1972, ANL-PHY-1972H.
30. B. F. Bayman and D. H. Feng, Nucl. Phys. A205 (1973) 513.
31. R. H. Ibarra, Nucl. Phys. A211 (1973) 317.
32. J. Vary, R. J. Ascuitto, and J. N. Ginocchio, Nucl. Phys. A185 (1972) 349.
33. B. F. Bayman and N. Hintz, Phys. Rev. 172 (1968) 1113.
34. J. D. Garrett and O. Hansen, Nucl. Phys. A212 (1973) 600.
35. W. A. Lanford and J. B. McGrory, Phys. Lett. 45B (1973) 238.
36. T. T. S. Kuo and G. Herling, Nucl. Phys. A181 (1972) 113.
37. The DWBA calculations were made by B. S. Nilsson, to whom I am indebted for permission to quote unpublished information. The experimental data is from J. H. Bjerregaard, et al., Nucl. Phys. A103 (1967) 33.
38. For example, see H. Nann, et al., Nucl. Phys. A176 (1971) 553; H. Nann, Nucl. Phys. A192 (1972) 417; A198 (1973) 11; R. Meynadier, et al., Nucl. Phys. A161 (1971) 305.
39. I. S. Towner and J. C. Hardy, Advances in Physics 18 (1969) 401.
40. H. Nann, W. Benenson, W. A. Lanford, and B. H. Wildenthal, to be published.
41. D. J. Crozier and H. T. Fortune, to be published.
42. C. H. King, H. Nann, M. Shahabuddin, and B. H. Wildenthal, to be published.
43. S. J. Skorka, J. Hertel, and T. W. Retz-Schmidt, Nuclear Data 2 (1966) 347.
44. J. M. Irvine, C. D. Latorre, and V. F. E. Puchnell, Advances in Physics 20 (1971) 661.
45. A. M. Kleinfeld, K. P. Lieb, D. Werdecher, and U. Smilansky, Proceedings International Conference on Reactions Between Complex Nuclei, Vol. 1 (ed. by R. L. Robinson, F. K. McGowan, J. B. Ball, and J. H. Hamilton) North-Holland Publishing Co., Amsterdam, 1974, p. 27.
46. G. E. Brown and A. M. Green, Nucl. Phys. 85 (1966) 87.
47. A. P. Zuker, B. Buck, and J. B. McGrory, Phys. Rev. Lett. 21 (1968) 38.

48. A. P. Zuker, Phys. Rev. Lett. 23 (1969) 983.
49. E. G. Adelberger, P. T. Debevec, G. T. Garvey, and R. Ohanian, Phys. Rev. Lett. 29 (1972) 883; R. Ohanian, Ph.D. Thesis, Princeton University (1973).
50. K. H. Bhatt and J. B. McGrory, Phys. Rev. C3 (1971) 2293.
51. W. Schlegel, D. Schmitt, R. Santo, and F. Pühlhofer, Nucl. Phys. A153 (1970) 502.
52. J. B. McGrory and B. H. Wildenthal, Phys. Rev. C7 (1973) 974.
53. J. M. Irvine, G. S. Mani, V. Puchnell, A. Watt, and R. R. Whitehead, Phys. Lett. 44B (1973) 16.
54. A. P. Zuker, *Proceedings of Conference on* $f_{7/2}$ *Nuclei, Padua, 1971.*
55. P. Federman and S. Pittel, Phys. Rev. 186 (1969) 1106.
56. W. P. Alford, R. A. Lindgren, D. Elmore, and R. N. Boyd, Phys. Lett. 46B (1973) 356.
57. C. Ellegaard, J. R. Lien, O. Nathan, G. Sletten, F. Ingebreten, E. Osnes, P. O. Tjöm, O. Hansen, and R. Stock, Phys. Lett. 40B (1972) 641.
58. W. J. Gerace and A. M. Green, Nucl. Phys. A123 (1968) 241.
59. S. Maripuu and B. H. Wildenthal, Phys. Lett. 38B (1972) 464.