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SHELL MODEL CALCULATIONS OF ELECTROMAGNETIC OBSERVABLES OF NUCLEI IN THE sd-SHELL REGION*

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The nucleus occupies a rather unique position in the physical world because it is a system which 1) must be treated within the framework of quantum mechanics, 2) it exhibits collective motion, and 3) it is conceivable that it can be treated as a "relatively" few-body system so that one might talk of a microscopic model in which the coordinates of all the constituent nucleons are treated explicitly. Thus, a dominant question in nuclear theory remains -- is it possible to describe the observed collective nuclear phenomena in terms of a truly microscopic model? All theoretical investigations of this question start in some way from a nuclear shell model. In recent years, shell model[1] technology has increased dramatically so that it is now possible to carry out large systematic investigations of nuclear structure in terms of large shell model bases. This is particularly true for light nuclei with mass less than 40, i.e. nuclei in the p-shell[2] and the sd-shell.[3] In this paper I would like to review the application of the shell model to the study of electromagnetic decay processes. I will first briefly outline what I mean by shell model calculation. I will then discuss the various effective operators which enter into the calculations of electromagnetic properties. Then I will review the existing calculations of such properties and how the results correlate with experimental data. I will try to give some idea of the sensitivity

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of these calculations to the "ingredients" of the shell model calculation.

Let me begin with a discussion of what I mean by a shell model calculation. We assume that nuclei consist of nucleons, neutrons and protons, which interact through two-body forces only. Thus, we assume the exact Hamiltonian is

$$H = \sum_i T_i + \sum_{i<j} v_{ij}.$$

An exact solution of this Hamiltonian is unavailable for more than the two-particle system, so some approximation is necessary. We assume that much of the effect of the two-body interaction can be absorbed in a one-body central potential, so that to a first-order approximation, the particles move independently in a central field. Mathematically, we add and subtract a central field $U_o(r)$ to the Hamiltonian, i.e.

$$H = \sum_i T_i + U_o(r_i) + \sum_{i<j} V_{ij} - \sum_i U_o(r) \equiv H_0 + H_{12}$$

and we assume $H_{12} = V_{ij} - U_o(r_i)$ is weaker and can be treated in perturbation theory. This is done by expanding our wave functions in terms of eigenstates of H_0 and then diagonalizing H_{12} in this space. The space of eigenstates of H_0 is an infinite Hilbert space, so we must truncate. We put all the nucleons in the lowest available orbits. We assume that most of the particles go to fill up the lowest available orbits to form an inert core, and that the remaining particles can occupy only the lowest few unfilled shells. Thus, the problem is reduced to diagonalizing an effective interaction in the space of a few particles distributed over a few orbits. The resulting eigenvalues are the nuclear energy levels, and the eigenfunctions are correlated with nuclear wave functions. As an example, the nucleus ^{20}Ne is often treated as four

particles in the sd shell outside an inert ^{16}O core. There are active particles only in the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ orbits. The matrix elements of the effective Hamiltonian in this space is completely specified when we know all the three one-body matrix elements $\langle j|H_o|j\rangle$, (H_o is assumed to be diagonal), and the 63 two-body matrix elements $\langle j_1 j_2 JT|H_{12}|j_3 j_4 JT\rangle$. Thus, all the physics is in the choice of the model space and the choice of these 66 numbers, the interaction matrix elements. After that, the calculation is one of crank turning. We now have very powerful cranks. With the recent development of shell-model codes based on the Lanczos algorithm by Whitehead,[1] it is now possible to do shell-model calculations of all nuclei in the sd shell with $8 < N,Z < 20$. This means that matrices which are as large as 5000×5000 in a (J,T)-coupled representation can be calculated, and accurate estimates of the lowest eigenvalues of this matrix can be obtained in reasonable time.

There has been a considerable maturation of the interpretation of shell-model calculations in recent years. The line of reasoning prevalent now is as follows. There exists a true Hamiltonian H, for which the eigenvalues are true nuclear energies E, and the eigenvectors ψ^E, are nuclear wave functions, i.e.

$$H|\psi_i^{E_i}\rangle = E_i|\psi_i^{E_i}\rangle.$$

For various physical observables there are related Hermitian operators O, such that matrix elements of the form

$$\langle\psi_i^{E_i}|O|\psi_j^{E_j}\rangle$$

can be used to calculate experimentally measured quantities. H and O are defined in infinite Hilbert spaces. Is it possible to find effective

operators $\mathcal{H}$ and $\mathcal{O}$ defined in a small space with the following properties:

$$\mathcal{H}\,|\chi_i^{E_i}\rangle = E_i|\psi_i^{E_i}\rangle$$

$$\langle\chi_i^{E_i}|\mathcal{O}|\chi_j^{E_j}\rangle = \langle\psi_i^{E_i}|O|\psi_j^{E_j}\rangle ?$$

Thus, the eigenvalues of $\mathcal{H}$ in the small space exactly equal the eigenvalues of H in the large space, and the matrix elements of $\mathcal{O}$ in the small space equal the matrix elements of 0 evaluated with the related eigenfunction in the large space. If it is possible to find relatively simple operators $\mathcal{H}$ and $\mathcal{O}$, (e.g. involving relatively few parameters, few-body operators, etc.) such that a large quantity of experimental information can be correlated, then one has a useful phenomenological model. In this sense, the shell model has achieved considerable success, which is extensively documented. It is obvious that much of this success results from the fact that we deal with matrix elements, and thus with overlaps of wave functions. A trivial example of this would be if one were interested merely in the direct overlap of two wave functions. If the wave functions are appropriately orthonormalized, the overlaps are one, no matter how complex the wave functions are. When one evaluates the matrix element of a one-body operator, in an N-particle system, he overlaps the part of the wave function involving N-1 of the particles, and again much of the complexity of the wave functions is swept away in this overlap. Physical observables only tell of the relation of two wave functions, and tells nothing about the details of individual wave functions.

As I said, the shell model has been a phenomenological success. Thus, there are simple operators which do a good job of correlating nuclear phenomena. One might start talking of a "realistic" microscopic calculation

if one could calculate the appropriate operators relevant to a given model space starting from the known "real" one- and two-body operators. The successes in this direction are much more unclear. Until now, low-order perturbation theory has been used. There are mathematical problems associated with questions of convergence. I'll make a few more comments on this matter with regard to effective charges in a few minutes.

Most of my discussion of actual calculations today will center on the so-called sd-shell nuclei, with A = 18-38. In all cases, an inert ^{16}O core is assumed, and the one-body energy operator is taken from the observed spectrum of ^{17}O. Most of the results will be for cases where the sd shell effective interaction is the realistic interaction of Kuo,[4] or a modification of it due to Preedom and Wildenthal.[5] The sd shell is probably the most extensively studied region insofar as large shell-model calculations are concerned. From the theoretical point of view, as I said above it is now possible to do complete $(sd)^n$ calculations for all sd-shell nuclei. Experimentally, it is known that many of the nuclei in the lower half of the shell exhibit rotational structure. By this I mean that there are rotational bands evidenced in the energy level spectra which have approximately J(J+1) spacing and the observed electromagnetic properties, particularly E2 transitions, can be described by simple rotational models. It is thus an ideal testing ground for seeing if a microscopic model can display collective features.

Traditionally one concentrates on comparisons of the shell model energy spectra with observed spectra. I will only briefly talk of such comparisons here. The most interesting recent developments here have to do with the relative positions of rotational bands. When the realistic interactions of Kuo[4] and Kuo-Brown[6] have been used in the past, the following systematic pattern is found. Where there are two known rotational bands in a nucleus,

the "realistic" shell-model calculations also display two rotational bands, but all the states of the excited bands are consistently too low with regard to the states in the ground state band. Wildenthal[5] has tried to modify the Kuo interaction empirically to alleviate this problem. He has treated as free parameters a subset of the total of 63 two-body matrix elements as calculated by Kuo, and adjusted these free parameters to give a best fit to over 100 levels in nuclei with A = 18-22. There are several cases in these nuclei where there are two rotational bands. When this interaction, referred to as the Preedom-Wildenthal (PW) interaction, is used to calculate[7] properties of a number of nuclei with A < 32, it is found that most of the difficulties with these excited bands are largely removed. It is also found[7] that the ground state binding energies as calculated with the Preedom-Wildenthal interaction are in substantial agreement with experiment, in contrast to the Kuo interaction. The main qualitative difference between the Kuo and PW interactions is that the strength of the average interaction between $d_{5/2}$ and $s_{1/2}$ particles, and between $d_{5/2}$ and $d_{3/2}$ particles is reduced by about 0.5 MeV in the PW interaction. This has a distinct effect on the purity of the rotational bands, as I'll discuss below. As a typical example of the agreement of theory-experiment for energy levels, the observed and calculated spectrum of ^{25}Mg is shown in Fig. 1. Column a is the observed spectrum, column b the spectrum calculated with Preedom-Wildenthal, and column c is the spectrum of the excited K=1/2 band as calculated with the Kuo interaction. The $1/2^+$ member of this band has been arbitrarily shifted to agree with the observed position of this state. Relative to this $1/2^+$ state, the calculated $5/2^+$ state would be at about 2 MeV. We see that the spectrum calculated with the PW interaction is in quite good agreement for all positive parity states up to 5.0 MeV.

Our main interest here is on electromagnetic observables, so let's move on to them. Since we are dealing with only positive parity orbits, with $\ell \leq 2$, the only possible electromagnetic observables are E0, M1, E2, M3, and E4 observables. There is not yet enough information available on E0, M3, and E4 observables to have any systematic comparisons with shell-model calculations, so we will concentrate here on M1 and E2 observables.

Let us start first with E2 observables. The "bare" operator for E2 observables in the long-wave-length approximation is

$$O^{E2} = \sum_{i=1}^{A} (1/2+t_z)\ e\ r_i^{\ 2}\ Y^2(\Omega_i)$$

where e is the proton charge and $t_z = +1/2$ for protons. For such an operator, our shell model for ^{17}O says there are no E2 decays in ^{17}O, i.e. it is a pure neutron state. Experimentally, there is a sizeable electric quadrupole moment[8] for the ground state, and a strong $1/2^+ \rightarrow 5/2^+$ E2 transition.[9] The conventional step at this point is to say that the neutron polarizes the ^{16}O core, in the process of which protons are excited out of the core, and these can decay. This effect is usually approximated by assigning an effective charge to the neutron, and since the proton can also polarize the core, it has an effective charge also. Then, in this approximation, the effective operator is

$$\begin{aligned} O^{E2} &= e\sum_{i=1}^{A} (1/2-t_z)\varepsilon_n\ \ r_i^{\ 2}\ Y_2(r_i) + (1/2+t_z)(1+\varepsilon_p)\ \ r_i^{\ 2}\ Y^2(r_i) \\ &= e\sum_{i=1}^{A} 1/2(1+\varepsilon_p+\varepsilon_n)r_i^{\ 2}\ Y^2(r_i) + \sum_{i=1}^{A} t_z(i)(1+\varepsilon_p-\varepsilon_n)r_i^{\ 2}\ Y^2(r_i) \end{aligned}$$

where in the last line I simply rewrite the operator in terms of isoscalar and isovector terms. One method of determining these effective charges is

to choose them to fit the observed transition rates in the single-particle nuclei, such as ^{17}O and ^{17}F. The calculated E2 transition rate eventually is reduced down to a coefficient times the radial integrals of the form

$$\varepsilon_{n,p} \int r^2 dr\, R_{n\ell}(r)\, r^2\, R_{n'\ell'}(r)$$

where $\varepsilon_{n,p}$ is the effective neutron or proton charge, whichever is appropriate. Thus the value of the effective charge depends on the radial integral. It depends, for instance, on whether you use oscillator wave functions or Woods-Saxon wave functions. There can be some sensitivity to this choice of wave function, particularly if one is dealing with single-particle states that are very loosely bound. For reasonably tightly bound particles, the differences in choice of single-particle wave functions gives 10-15% difference[10] in the radial integrals. Thus, to be precise, the effective charge is a function of the choice of single-particle wave function. In practice, for the type of comparisons I'll illustrate in this talk, which deal primarily with relative values, this ambiguity is not significant. There is one more point that should be stressed here. We have made a state-independent effective charge approximation. Thus, in the sd shell, we say that no matter what single-particle orbits are involved, and no matter what is the energy of the transition involved, the effective charge is the same. There is no a priori reason for this to be so.

If we use oscillator wave functions with an appropriate size parameter, and if we make this state-independent effective charge assumption, what does experiment tell us? I've summarized the situation in Table 1 for ^{17}O, ^{18}O, ^{18}Ne, ^{18}F, ^{42}Ca, ^{206}Pb, and ^{210}Po. These all represent nuclei which are one- and two-particle or hole systems in the shell model. For the two-particle systems, the wave functions are the results of diagonalizing

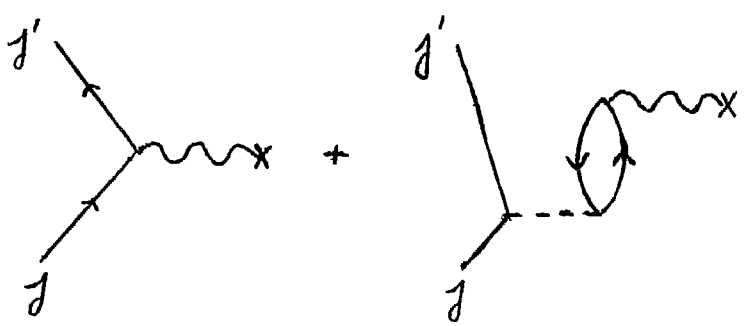
"realistic" interactions with experimental single-particle energies. In this table I list the observed transition strengths in these nuclei, and I list the effective charge which is needed in the simple shell model to reproduce these transitions. The ^{18}F transition shown is an isoscalar transition, so only the isoscalar effective charge is determined by this transition, which tells us the sum of the neutron and proton effective charges. These comparisons do not offer much support to the state-independent effective charge concept. In the ^{17}F case, the initial state is the very loosely bound $1/2^+$ state. In this case the harmonic oscillator approximation is very questionable. In the mass-18 nuclei, the isoscalar effective charge is only half of the isoscalar charge suggested by the ^{18}Ne and ^{18}O results. Finally, the mass-18 effective charges are significantly different from the mass-17 charges. In the case of ^{42}Ca I have listed three transitions between the lowest states of J = 0, 2, 4, and 6. These are simple two-neutron states in the shell model. The neutrons carry much larger effective charges here than in the sd shell, and there is strong evidence for a state dependence.

There is one more qualitative feature in the data worth mentioning here. If the effective charge does arise from a core-polarization process, then the simplest interpretation would suggest that the neutron effective charge would be larger than the proton effective charge. This is because the neutron can interact with the protons in the core through the strong T=0 components of the nucleon-nucleon force as well as the T=1 force, while the protons can interact only through the T=1 force. Thus, neutrons should excite protons more strongly than protons. Experimentally, the proton charges are larger than the neutron charges. It would be extremely useful to have good data on B(E2) values in ^{19}O and ^{20}O from (d,pγ) and (t,pγ) reaction experiments. These could cast direct light on the neutron effective charge

question.

Thus, this naive interpretation of the data does not strongly support the simple state-independent, nucleus-independent effective charge. Most certainly the difficulty lies with the interpretation of the A=17 and 18 (and A=41 and 42) systems as simple one- and two-particle systems outside closed shells. There are important core-excitation contributions to the low-lying levels of these nuclei that cannot be treated as simple perturbations insofar as electromagnetic decay processes are concerned. It is one of the unfortunate ironies that those nuclei which are considered to be simple in the shell model picture are in fact among the most complex. Thus, much of the "realistic" interaction game has been played in the A=17 and 18 region, and most approaches have used perturbation theory to handle core effects. It is a good possibility that these core effects become truly perturbative in the middle of shells, when the shell model simplicity is lost. At the bottom of Table 1 I've listed effective charges in the lead region, where the core may be more stable. Here the neutron effective charge <u>is</u> larger than the proton, and the state dependence for the polonium transitions is not so severe.

There is an extensive literature on the calculation of effective charges due to core polarization.[2] These calculations are also not without ambiguity. Basically they involve evaluating diagrams of the form



The first term simply represents the interaction of a particle with the electromagnetic field. The second process involves exciting a hole-particle state in the core, and this hole-particle state decays by a gamma emission. The treatment of this hole-particle state is a critical one. One could use an oscillator basis and perturbation theory, one could first diagonalize in a hole-particle space, and then use perturbation theory in the resulting space (Tamm-Dancoff), or one could include effects on the hole-particle states due to ground state correlations through the RPA formalism. There are questions of what renormalizations should be included in the TDA or RPA, what single-particle energies for the hole and particle states, what interaction, etc. Because of these uncertainties, it is difficult to draw any conclusions as to the success of realistic "effective"-charge calculations. This difficulty is obviously magnified by the fact that no one really knows what the experimental numbers are either.

To this point, then, the picture for the E2 effective operator is seemingly very complex. As I'll show now, the major shell configuration mixing shell-model calculations give a much more simple picture. There are now calculations of E2 observables throughout the sd shell in model spaces which are very near to being complete. In Table 2, I have compiled a listing of E2 transitions from the first excited state to the ground state for those cases where there is experimental information available, and where the analogous numbers have been calculated in the complete $(sd)^{A-16}$ space. Thus, the cases are restricted to $A \leq 22$ and $A \geq 34$. In all the calculations, effective charges of 0.5 were assumed for both the neutrons and the protons, and the oscillator parameter was determined from the relation $\hbar\omega = 41\ A^{-1/3}$ MeV. We see that this systematic data is very accurately reproduced by the shell-model calculations. The significant discrepancies are for the nuclei close

to the ^{16}O core. In table 3, I have listed calculated and measured quadrupole moments in the sd shell. In this case, the values are included for all nuclei with $16 \leq A \leq 40$ for which moments are measured. Thus, the models for $23 \leq A \leq 33$ are in somewhat truncated spaces. Again, constant effective charges of $\varepsilon_p = \varepsilon_n = 0.5e$ are used. In all cases the signs of the moments are correctly calculated, and the calculated magnitudes are generally in good agreement with measurement. The major discrepancies are for ^{18}O and ^{20}Ne. In the middle of the shell, the calculated numbers are a little smaller than the measured ones but this is almost certainly a truncation effect. The effects of configuration mixing are quite important in these calculations. In Fig. 2, I show a figure from the work of deKoch and Glaudemans.[25] They assumed the simplest independent particle models for nuclei with $A \leq 28$ $(d_{5/2}^{A-16})$ and $A \geq 32$ $(d_{3/2}^{A-32})$, with a good seniority approximation. Within these models they determined the size of the effective charges (with $\varepsilon_p = \varepsilon_n$) which was needed to reproduce the observed quadrupole moment. As we see, the charge needed in the middle of the shell is extremely large. The results in Table 3 show that with configuration mixing this effective charge is constant, so that when we take in an adequate space, the effective shell model operator does become "simple".

As I said, there are significant discrepancies for ^{18}O and ^{20}Ne. I suspect that these discrepancies are due to the importance of core-excitation effects. This is almost certainly true for ^{18}O, where the measured moment is eight times the shell model prediction. A similar large discrepancy exists in ^{42}Ca, the fp shell analog of ^{18}O. There both the sign and magnitude of the quadrupole moment of the first 2^+ state are in disagreement with the shell model. The data around ^{42}Ca has been qualitatively accounted for by Towsley, et al.[11] with a model in which 50% of this lowest 2^+ is a deformed

4-particle 2-hole state. If a similar effect is important in ^{18}O, it is not unreasonable that a similar effect is significant in ^{20}Ne.

The shell model has achieved moderate-to-good success in describing relative B(E2) values for transitions between low-lying states in a given nucleus. As an example which I think may be fairly called "representative", the measured B(E2) values for transitions among states in the lowest two rotational bands in ^{25}Mg are listed in Table 4. Also in this table are given the values calculated for these transitions in the complete $(sd)^9$ space by Cole, et al.[12] Two interactions are used, one the Kuo realistic interaction designed for this model space and the other the PW interaction which is described briefly above. Remember that the PW interaction does a much better job of describing the relative positions of the rotational bands in the energy spectrum of ^{25}Mg. The upper part of Table 4 lists in-band transitions, while the last four transitions listed are inter-band transitions. Insofar as the B(E2) values are concerned, we see that there is quite satisfactory agreement for both in-band and inter-band transitions with the PW interaction. For the Kuo interaction, the calculated B(E2) values for in-band transitions are generally quite similar to the PW numbers, but the inter-band B(E2) values are quite different for the two interactions. It is encouraging that the interaction which better reproduces the energy levels also give a better description of the B(E2) values. This sensitivity of the calculation is a feature that has appeared clearly only with the really large space shell-model calculations made possible with the Glasgow program! The results[13] in ^{26}Mg are quite similar to those given here for ^{25}Mg, i.e. energy levels and electromagnetic properties are both better reproduced with the PW interaction than with the Kuo interaction. The sensitivity is most striking with regard to inter-band transitions. These are generally small and hard to

measure, but they are potentially quite useful.

Thus, in summary, the phenomenological shell model is quite successful in describing many observed E2 quantities with a very simple effective charge approximation. The results suggest that calculations of effective charges from core-polarization processes should lead to an isoscalar effective charge $(1+\varepsilon_p+\varepsilon_n) \approx 1.0$, and there is no strong empirical evidence for state dependence.

Let us move next to M1 observables. The bare M1 operator is

$$M1 = \sum_{i=1}^{A} \{1/2 \quad [\bar{\ell}(i) + (g_p+g_n)\bar{s}(i)] + t_z'(i) \quad [\bar{\ell}(i) + [g_p-g_n]\bar{s}(i)]\}$$

$$= 0.50\ \bar{J} + 0.38\ \bar{S} + \sum_{i=1}^{A} t_z'(i) \quad [\bar{\ell}(i) + 9.40\ \bar{s}(i)]$$

where $g_p = 5.58\ M_o$ and $g_n = -3.82\ M_o$.

For this operator, a particle in an orbit $j = \ell \pm 1/2$ can decay only to another orbit $j' = \ell \pm 1/2$, i.e. to itself or to its spin-orbit partner. Because all the tensors in the bare M1 operator are vectors, the following selection rule exists: there are no M1 transitions between systems of identical particles in the same j-orbit, i.e. j^n configurations of protons only, or of neutrons only. The isoscalar part of the operator makes no contribution to decays between S=0 states. The isovector term dominates the isoscalar term where both processes are possible. It is generally found in shell-model calculations that matrix elements for the M1 operator are small between low-lying states. They are thus more sensitive than the model space and the effective Hamiltonian which is used than is the case with the E2 observables.

As one uses an effective charge for E2 calculations, one should in principle use an effective M1 operator in truncated shell-model calculations.

There has been a long history of investigations[14] of renormalization of the M1 operator. Suffice for our purposes to say that the renormalization of the M1 operator in the shell model we use for sd-shell nuclei is expected to be small. There is, of course, direct experimental evidence on this from the magnetic moments of the ^{17}O and ^{17}F ground states. The measured values of these moments are included in Table 5, as are the values calculated with the bare M1 operators. For these two nuclei, there is essentially exact agreement as is well known. All measured magnetic moments of sd-shell nuclei with $A \leq 22$ and $A \geq 34$ are listed in Table 5, as are values calculated in complete $(sd)^{A-16}$ model spaces with reasonable effective interactions, and for which calculations of the bare M1 operator are used. The calculated results for nuclei with $A \leq 22$ are in excellent agreement with measurement. Arguments can be made[15] to show that these results imply that there can be no significant quenching of the isovector part of the bare M1 operator when used as an effective operator for these calculations. There are some relatively large discrepancies for the heavier nuclei. Thus, both effective operator theory and the results for magnetic moments indicate that the bare M1 operator is the appropriate effective M1 operator in the sd shell, at least for the lighter nuclei.

In recent years, there has been a significant increase in the amount of data available on M1 transitions in sd-shell nuclei. It is thus possible to start some systematic analysis of the ability of the shell model to account for this data. As I said, these B(M1) values are generally fairly small, especially for isoscalar transitions. This results in extreme sensitivity to the calculation. Let us concentrate here on isovector allowed M1 transitions which are not so small. As I will show now, the theory-comparison for such transitions is still very confusing. Even for these "relatively strong"

transitions there is sensitivity to the effective Hamiltonian which is used. Before this sensitivity is useful, much more careful measurements must be made. If one can accept the data as it exists, there is some suggestion that the isovector M1 operator should be quenched, in contradiction to the implications of magnetic moment data.

Let us look first at this systematic data. In Table 6 are listed a reasonably large selection of the available data or isovector allowed M1 transitions between low-lying, mostly bound, states in sd-shell nuclei. For some cases, the B(M1)'s were calculated with various effective Hamiltonians. In those cases, the numbers in the calculated column indicate the range of the values calculated with the different Hamiltonians. We see first that there is a correlation in the calculated and observed results between relatively weak and relatively strong B(M1) values. For the strongest transitions there is a consistent trend that the calculated value is too large compared to the observed value. The main exception to this is for the two transitions in ^{20}Ne. This indication that the isovector M1 operator should be quenched is difficult to reconcile with the results for the magnetic moments. For the case of the heavier nuclei, it is worthwhile pointing out that the ^{39}K transition is absolutely forbidden with the bare operator. Since this measured strength is comparable with the values observed for the other transitions in these heavier nuclei, it is hard to say much about the theory-experiment comparison there.

In Table 4 are listed the measured B(M1) values in Weisskopf units in ^{25}Mg. I show these numbers to show: 1) a case where the relative B(M1) values are well accounted for by the shell model, the Preedom-Wildenthal model in this case, 2) a case where there is no obvious need to quench the isovector operator, and 3) to show again the sensitivity of the shell-model

calculations to differences in the PW and Kuo interactions. Again, in those cases where there are significant differences in the calculations, the PW interaction is more successful.

As I stated above, the data on these B(M1) values, as well as some B(E2) values, is subject to significant experimental uncertainties. One example, hopefully not typical, is shown in Table 7. Here are given the measured B(E2) and B(M1) values for transitions between states in the ground-state band of ^{21}Ne. There are two sets of measured values. The calculated values for these quantities are from a complete $(sd)^5$ shell-model calculation in which the Kuo realistic effective interaction was used, the bare M1 operator was used, and effective charges of $\varepsilon_p = \varepsilon_n = 0.5e$ were used in the E2 operator. Not indicated in this table is a disagreement between the two experimental groups involved as to the lifetimes of the states involved. The published lifetimes of Pronko[16] and of Bailey, et al.[17] differ by factors of roughly 4-6. The Pronko results were reanalyzed by Bailey, et al. with an "improved" method. The resultant lifetimes still disagree with those "measured" by Bailey, et al. by a factor of two to three. The calculated B(M1) and B(E2) values are consistently intermediate to the two sets of experimental numbers. If one is an optimist, he could say that the relative B(M1) and B(E2) values may well be in quite good agreement with the "correct" experimental values. It would obviously be useful to resolve these discrepancies.

Thus, I think we are left with an ambiguous situation for the B(M1) operator. There is some indication that a renormalization of the isovector M1 operator is needed, but as long as the experimental situation is as unclear as it appears to be, one cannot argue very strongly for this.

I have tried to show here that there is reason to believe that much of the systematic features of data on electromagnetic properties of light sd-shell nuclei can be described by the phenomenological shell model with very simple effective operators. It appears now that the large shell-model calculations of these observables in the middle of the shell may provide a useful test of the various effective interactions. This can only be true if the reliability of the experimental numbers is more firmly established.

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Figure Captions

Figure 1. Calculated and observed spectrum of ^{25}Mg. Column a is observed positive parity spectrum; column b is spectrum calculated in $(sd)^9$ space with Preedom-Wildenthal interaction; column c is spectrum calculated in $(sd)^9$ space with Kuo interaction. The information in this figure is from Ref. 12.

Figure 2. Plot of effective charge, Δe, which is added to bare neutron and proton charges in order to reproduce observed quadrupole moments in simplest shell model spaces in sd shells, i.e. $d_{5/2}^{A-16}$ for $A \leq 28$, and $d_{3/2}^{A-32}$ for $A \geq 34$.

Table 1. Effective Charges in One- and Two-Particle Nuclei as Determined Directly from the Observed Transitions.

Nucleus	J_i	J_f	Effective Charge
^{17}O	1/2	5/2	$\varepsilon_n = 0.6^a$
^{17}F	1/2	5/2	$\varepsilon_p = 1.0^a$
^{18}O	2	0	$\varepsilon_n = 0.8^a$
^{18}Ne	2	0	$\varepsilon_p = 1.4^a$
^{18}F	3	1	$\varepsilon_n+\varepsilon_p = 1.0^a$
^{42}Ca	2	0	1.8^a
^{42}Ca	4	2	1.9^a
^{42}Ca	6	4	0.7^a
^{206}Pb	2	9	1.0^b
^{210}Po	4	2	0.6^b
^{210}Po	6	4	0.8^b
^{210}Po	8	6	0.5^b

[a]Ref. 18.

[b]Ref. 19.

Table 2. Calculated and Observed Quadrupole Moments in sd-Shell Nuclei.

Nucleus	Q(e fm^2) Expt.[a]	Q(e fm^2) Theory[a]	Nucleus	Q(e fm^2) Expt.[a]	Q(e fm^2) Theory[a]
^{17}O	-2.6±0.9	- 2.6	^{18}O	-16±1[b]	- 2.0
^{19}F	11.2±2	- 9.2	^{20}Ne	-23±3	-14.3
^{21}Ne	+9.3±1.0	+10.3	^{22}Ne	-17.0±3	-13.6
^{23}Na	+11±2	+10.4	^{24}Mg	-21±3	-16.0
^{25}Mg	22	+17.8	^{26}Mg	-14±4	-11.9
^{27}Al	15.3±0.3	13.8	^{28}Si	+16±3	+14.3
^{33}S	-5.5±0.5	- 7.1	^{32}S	-19±4	-13.6
^{35}S	+4.0±0.4	+ 7.8	^{34}S	6±6	10.0
^{35}Cl	-7.9±0.1	- 9.1			
^{36}Cl	-1.7±0.2	- 0.9			
^{37}Cl	-6.2±0.6	- 8.4			
^{39}K	5.5±0.4	7.7			

[a] Information taken from Ref. 20.

[b] Ref. 21.

Table 3. Selected Strong B(E2) Transitions to Ground States in sd-Shell Nuclei. In all calculations, an added effective charge of 0.5e is included in E2 operators for both neutrons and protons.

Nucleus	J_i,T_i	J_f,T_f	$B(E2)_{i-f}$ e^2fm^4 Expt.[a]	Theory[a]
^{17}F	1/2,1/2	5/2,1/2	64±2	38
^{17}O	1/2,1/2	5/2,1/2	6±0.1	4
^{18}F	3,0	1,0	15±2	15
^{18}O	2,1	0,1	7±1	3
^{19}F	5/2,1/2	1/2,1/2	21±1	19
^{20}Ne	2,0	0,0	57±8	48
^{21}Ne	5/2,1/2	3/2,1/2	63±13	81
^{22}Na	4,0	3,0	94±15	101
^{22}Ne	2,1	0,1	40±3, 66±12	55
^{35}Cl	5/2,1/2	3/2,1/2	78±2	74
^{36}Ar	2,0	0,0	59	59
^{38}Ar	2,1	0,1	38±9	28

[a]Observed and calculated values compiled from Ref. 3.

Table 4. Calculated[a] and Observed[a] B(E2) and B(M1) Values in ^{25}Mg for States in Ground State K = $5/2^+$ and First Excited K=$1/2^+$ Bands. States in excited K=1/2 band have primes on J labels.

$2J_i$	$2J_f$	B(E2) e^2fm^4 [a]			B(M1) μ_o^2		
		Expt.	Kuo	P.W.	Expt.	Kuo	P.W.
7	5	140±35	113	117	0.6±0.1	----	0.5
9	5	40^{+136}_{-20}	10	46			
3′	1′	40±30	61	69	0.03	0.13	0.04
5′	1′	100±4	87	87			
5′	3′	20±4	32	25			
7′	3′	125±5	54	77			
9	7	55±30	77	43			
7′	5′	108^{+90}_{-52}	3	7	0.02	0.10	0.02
1′	5	2.3±.04	0.1	2.4			
3′	5	3.5±1.7	30	6			
5′	5	3.0±0.9	---	36	0.002	----	0.001
7′	5	0.9±0.2	28	1.6	0.0	0.05	0.00

[a]Data compiled from Ref. 12.

Table 5. Calculated and Observed Magnetic Moments in sd-Shell Nuclei.

Nucleus	$M(\mu_N)$ Expt.[a]	$M(\mu_N)$ Theory[a]
^{17}F	+4.72	+4.79
^{17}O	-1.89	-1.91
^{18}F	+2.86	+2.88
^{19}F	+2.63	+2.87
^{19}F	+3.69	+3.55
^{20}F	+2.09	+1.92
^{21}Ne	-0.66	-0.90
^{22}Na	+1.75	+1.82
^{35}Cl	+0.82	+0.68
^{35}S	±1.0	+1.21
^{36}Cl	+1.28	+1.55
^{37}Ar	+0.95	+1.43
^{37}Cl	+0.68	+0.59
^{38}K	1.37	+1.24
^{39}K	+0.39	+0.13

[a]Information taken from Ref. 3.

Table 6. Observed and Calculated Isovector B(M1) Values in sd-Shell Nuclei with A<22 and A>34

Nucleus	J_i,T_i	J_f,T_f	B(M1) M_o^2	
			Observed[a]	Calculated[b]
^{18}O	2,1	2,1	0.01±0.003	0.09→0.20
^{18}F	1,0	0,1	0.13±0.01	0.37→0.79
^{19}O	3/2,3/2	5/2,3/2	0.03±.002	0.00→0.05
^{20}Ne	0,2	1,1	2.50	2.38
	1,1	0,0	0.66	0.61
^{20}F	3,1	2,1	0.47±0.05	0.06→0.29
	4,1	3,1	0.10±0.02	0.10→0.24
	1,1	2,1	1.08±0.26	4.96→7.08
	2,1	3,1	0.51±0.20	4.16→4.30
^{22}Ne	1,1	0,1	.07± .01[c]	.04[c]
	1_2,1	0,1	.17± .03[c]	.34[c]
	1_3,1	0,1	.10± .02[c]	.09[c]
	1_4,1	0,1	.32± .03[c]	.74[c]
^{35}Cl	3/2,1/2	3/2,1/2	0.19±0.08	0.39→0.50
^{37}Ar	3/2,1/2	3/2,1/2	0.02±0.004	0.09→0.10
	3/2,1/2	3/2,1/2	0.06±0.02	0.06→0.09
^{39}K	1/2,1/2	3/2,1/2	0.03±0.01	ℓ-forbidden

[a]Ref. 22, except for ^{22}Ne.

[b]Ref. 3, except for ^{22}Ne.

[c]Ref. 23.

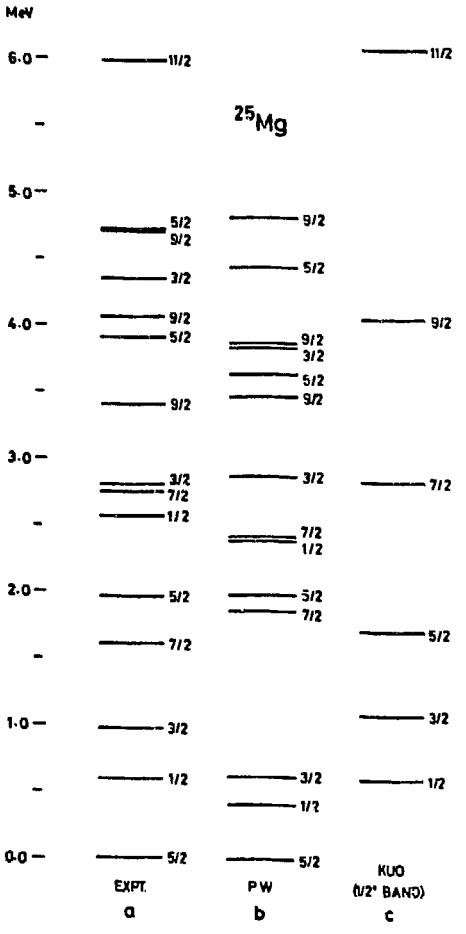
Table 7. Calculated and Measured B(M1) and B(E2) Values for ^{21}Ne, in Weisskopf units.

J_i	J_f	B(M1) Wu			B(E2) Wu		
		Expt.[a]	Expt.[b]	Calc.	Expt.[a]	Expt.[b]	Calc.[c]
5/2	3/2	0.21±0.03	0.03±0.01	.09	30±30	18±4	23
7/2	3/2				30±8	5±2	10
7/2	5/2	0.45±0.12	0.07±0.02	.16	44±30	7±3	17
9/2	5/2				36±7	6±2	13
9/2	7/2	0.49±0.09	0.12±0.03	.20	24±33	6±4	9
11/2	7/2				30±8	12±6	15
11/2	9/2	0.18±0.05	0.08±0.03	.20	9±11	4±3	8

[a]Ref. 17.

[b]Ref. 17.

[c]Ref. 3.


MeV
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^{25}Mg
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EXPT.
a
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PW
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KUO
(1/2⁺ BAND)
c

Fig. 1

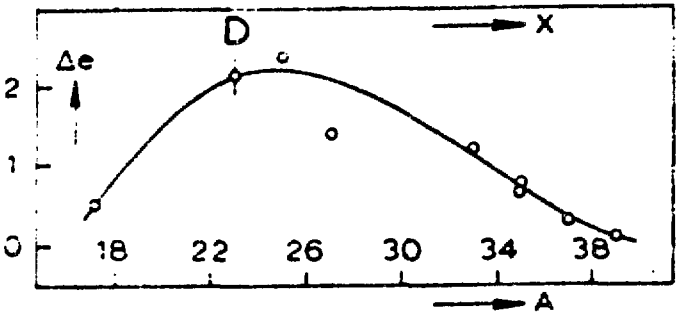

Fig. 2