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# INTERMEDIATE STRUCTURE IN THE $^{238}U$ NEUTRON CAPTURE CROSS SECTION*

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Recent measurements of the $^{238}U$ neutron capture cross section show large fluctuations in the unresolved resonance region. To test whether or not the observed long-range fluctuation of the neutron capture represent departures from the compound nuclear model, the Wald-Wolfowitz runs and correlation tests were applied to the $^{238}U$ neutron capture data obtained at ORELA. The Wald-Wolfowitz runs test deals with the statistic, R, which is the number of unbroken sequences of data points above or below a given reference line. This statistic is to be compared with the expected value of runs $E(R) \pm \sigma(R)$ arising from randomly distributed data. In the correlation test we have computed the first serial correlation coefficient of the data as well as its expected value and variance for a set of random data. In both tests one computes the probability, P, for the given statistical entity to depart from its expected value by more than $\varepsilon$ standard deviations. Both tests confirm the presence of intermediate structure between 5 and 100 keV. The range of the structure far exceeds the width of the experimental resolution and level widths.

($^{238}U$ capture cross section; intermediate structure; doorway states)

## Introduction

There is considerable experimental evidence of large fluctuations of the $^{238}U(n,\gamma)$ cross section in the unresolved resonance region.[1,2] Similar structure was first reported by Kopsh, Cierjacks, and Kirouac[3] in the total $^{238}U$ neutron cross section between .5 and 4.35 MeV. These observed fluctuations are wider than the sharp resonances associated with the compound nucleus levels and narrower than the broad structure due to the energy dependence of the penetration coefficients. They represent departures from the statistical nuclear model in localized energy regions, hence forming an intermediate structure, rather unexpected in the framework of the compound nucleus model.

Intermediate structure in the fission cross section of various fissile nuclei is by now well known,[4-7] as well as its interpretation[8,9] in terms of Strutinsky's double humped barrier.[10]

Although subthreshold fission yields have been found in the case of the $^{238}U$ nucleus,[11] The fact that both the capture and total $^{238}U$ cross sections exhibit intermediate structure leads one to hypothesize that the observed structure is indeed due to nuclear dynamics phenomena in the entrance (neutron) channels, such as "doorway" states.

The problem of extracting the intermediate structure from the observed cross section has been given considerable attention in the past.[13] James[14] and Moore[15] were the first to introduce the use of nonparametric statistics[16] for the detection of intermediate structure in the fissile nuclei. Extensive studies in this field have been also performed by Baudinet-Robinet and Mahaux.[17] We present here the results of some statistical tests performed on the $^{238}U(n,\gamma)$ cross section in which the Wald-Wolfowitz[16] nonparametric statistics were utilized.

## Statistical Treatment of the $^{238}U$ Capture Cross Section

The Wald-Wolfowitz[16] runs and correlation tests were applied to the $^{238}U$ neutron capture data obtained at ORELA.[2] The Wald-Wolfowitz runs test deals with the statistic, R, which is the number of unbroken sequences of data points above or below a given reference line. This test evaluates the expected number of runs E(R) from randomly distributed data, as well as the corresponding variance $\sigma^2(R)$. It is then shown that the ratio

$$\varepsilon_R = [|R - E(R)| - 1/2]/\sigma(R)$$

follows a normal probability distribution. The evaluation of the test proceeds by computing the probability, $P(\varepsilon_R)$, that a ratio equal or larger than $\varepsilon_R$ could arise from the sampling of random data.

The Wald-Wolfowitz correlation test is based on the computation of the usual autocorrelation function of the data with some modification, so that "end effects" are mitigated. Wald and Wolfowitz also give the distribution of the correlation function for random data. The end result of this test is the ratio $\varepsilon_C$, between the particular serial correlation coefficient obtained from the actual data and its value for a set of random data, as well as the probability $P(\varepsilon_C)$.

The $^{238}U(n,\gamma)$ cross section obtained at ORELA (40 m flight station) was averaged over energy intervals varying from 600 eV up to 3 keV in the region between 5 and 100 keV. The lower value of 600 eV is about six times the total experimental and Doppler resolution of the measurement at 100 keV. These data were then fitted to an average cross section (with s-wave and p-wave contributions) according to the expression

$$\langle\sqrt{E}\,\sigma_\gamma(E)\rangle = g_1 \frac{A_0}{\langle\Gamma_0(E_n)\rangle} R^\gamma_{0n}(E_n) \div \left(\frac{\rho^2}{1+\rho^2}\right)$$

$$\times A_1\left[g_1 \frac{R^\gamma_{1n}(E_n)}{\langle\Gamma_1(E_n)\rangle} + g_2 \frac{R^\gamma_{2n}(E_n)}{\langle\Gamma_2(E_n)\rangle}\right] \qquad (2)$$

where

$$A_0 = (4.125 \times 10^6)\langle\Gamma_\gamma\rangle S_0$$
($S_0$ = s-wave strength function)

$$A_1 = (4.125 \times 10^6)\langle\Gamma_\gamma\rangle S_1$$
($S_1$ = p-wave strength function)

$$\langle\Gamma_0(E_n)\rangle = \langle\Gamma_\gamma\rangle + (E_n)^{1/2}\langle\Gamma^0_{0n}\rangle$$

$$\langle\Gamma_1(E_n)\rangle = \langle\Gamma_\gamma\rangle + \left(\frac{\rho^2}{1+\rho^2}\right)(E_n)^{1/2}\langle\Gamma^0_{1n}\rangle$$

$$\langle\Gamma_2(E_n)\rangle = \langle\Gamma_\gamma\rangle + \left(\frac{\rho^2}{1+\rho^2}\right)(E_n)^{1/2}\langle\Gamma^0_{2n}\rangle$$

$$\rho = 1.8473 \times 10^{-3}(E_n)^{1/2}, \text{ and}$$

$g_0$, $g_1$, $g_2$ = statistical weight factors.

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Table 1. Input Nuclear Parameters for the Average Cross Section Calculation and the Resulting s-wave and p-wave Strength Functions.

| J | ℓ | $g_J$ | $\Gamma^0_{0n}$ (a) (eV) | $\Gamma^0_{1n}$ (a) (eV) | $\Gamma^0_{2n}$ (a) (eV) | Strength Function ($\times 10^{-4}$) |
|---|---|---|---|---|---|---|
| 1/2 | 0 | 1 | .0021 | - | - | 1.31 |
| 1/2 | 1 | 1 | - | .00298 | - | 1.72 |
| 3/2 | 1 | 2 | - | - | .00164 | 1.72 |

[a]ENDF/B-IV values (ref 19) with $\langle\Gamma_\gamma\rangle$ = .0235 eV.

The average radiation width, $\langle\Gamma_\gamma\rangle$, and the neutron widths for the s-wave and p-wave components, $\langle\Gamma_{0n}\rangle$, $\langle\Gamma_{1n}\rangle$, and $\langle\Gamma_{2n}\rangle$, are given in Table 1. The statistical fluctuation factors $R^\gamma_{0n}$, $R^\gamma_{1n}$, and $R^\gamma_{2n}$ (ref 18) were computed at each neutron energy. The values obtained for the $S_0$ and $S_1$ strength functions are also given in Table 1. These values are higher than the ENDF/B-IV[19] values of $S_0 = 1.05 \times 10^{-4}$ and $S_1 = 1.57 \times 10^{-4}$ reflecting the higher values for the $^{238}U$ capture cross section obtained at ORELA and the assumed $\langle\Gamma_\gamma\rangle$=.0235 eV.

The fit obtained from Eq. (2) is shown in Fig. 1 (with some statistical error flags at selected points). Also shown in the figure is the mock-up $^{238}U(n,\gamma)$ cross section computed by Monte-Carlo techniques on the basis of the ENDF/B-IV[19] average resonance parameters and s-wave strength function. The mock-up cross section data were also fitted by Eq. (2) with only the s-wave contribution. Inspection of Fig. 1 already reveals quite a deal of structure of the cross section about its average value for the experimental data, as compared with the "good" fit in the case of the capture cross section computed on the basis of the statistical nucleus model.

The average cross section fit was utilized as the reference line for the Wald-Wolfowitz runs test. For the correlation test the data were divided through by the average fit to remove the correlation bias introduced by the energy dependence of the s-wave and p-wave penetration factors.

The results of the statistical tests are given in Table 2. Both tests show that there is a probability of less than $10^{-5}$ for the average ratio $\langle\epsilon_c\rangle$ = 5.1 and less than $10^{-4}$ for the average ratio $\langle\epsilon_R\rangle$ = 4.1 to occur from the sampling of random data. Essentially, the Wald-Wolfowitz nonparametric statistics test whether or not a given sample exhibits longer correlations than the expected from a set of random data. When applied to the mock-up cross section and averaged over energy intervals larger than the level widths, the results of Table 2 merely indicate that the compound nuclear model behaves from the viewpoint of correlation length similarly to a set of random data. Hence, the present results are to be interpreted as showing the presence of long-range correlations which represent departures from the compound nucleus model.

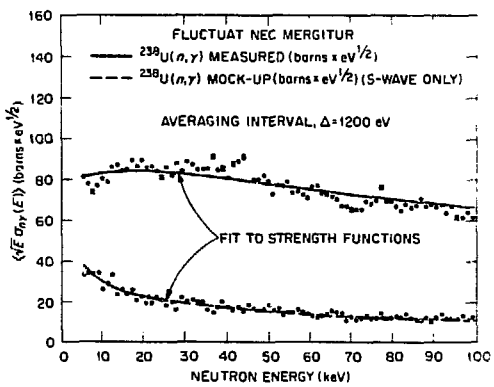


Fig. 1. Fit of the $^{238}U(n,\gamma)$ Cross Section Measured and Mocked-up to the strength functions.

Table 2. Results of the Wald-Wolfowitz Correlation and Runs Tests for the Measured and Mockup $^{238}U$ Capture Cross Section

| Width (keV) | Capture (Measured) $\epsilon_c$ | $P(\epsilon_c)$ | $\epsilon_R$ | $P(\epsilon_R)$ | (Capture (Mockup) $\epsilon_c$ | $P(\epsilon_c)$ | $\epsilon_R$ | $P(\epsilon_R)$ |
|---|---|---|---|---|---|---|---|---|
| .6 | 5.9 | $<10^{-5}$ | 4.5 | $<10^{-5}$ | .52 | .60 | .03 | .98 |
| .9 | 6.1 | $<10^{-5}$ | 3.9 | $2.7\times10^{-3}$ | .20 | .60 | .36 | .34 |
| 1.0 | 5.9 | $<10^{-5}$ | 4.6 | $<10^{-5}$ | .37 | .71 | .68 | .50 |
| 1.2 | 6.2 | $<10^{-5}$ | 5.6 | " | .65 | .52 | .34 | .73 |
| 1.5 | 5.3 | $<10^{-5}$ | 4.8 | " | .31 | .76 | .002 | .99 |
| 1.8 | 2.2 | $2.7\times10^{-2}$ | 3.6 | $<2.7\times10^{-3}$ | .97 | .33 | 1.11 | .27 |
| 2.0 | 4.9 | $<10^{-5}$ | 3.8 | " | .22 | .82 | 1.2 | .23 |
| 3.0 | 3.9 | $2.7\times10^{-3}$ | 1.8 | $7.2\times10^{-2}$ | 1.1 | .27 | .006 | .99 |

## Theoretical Interpretation

The fact that intermediate structure has been observed in both the $^{238}U$ capture and total cross sections strongly favors the "door-way" states hypothesis. These short-lived states at the entrance channel interact via a residual interaction with the compound nucleus states. It can be shown[20] that the (n,γ) T-matrix component is given by

$$T_{n\gamma} = 1 \sum_{\lambda} \frac{g_{\lambda n}\, \Gamma_{\lambda\gamma}^{1/2}}{D_{\lambda}(E) - \phi_{\lambda}(E)} \quad , \tag{3}$$

with the complex neutron width given by

$$g_{\lambda n} = \Gamma_{\lambda n}^{1/2}\left[1 - \sum_{\mu}\left(\frac{\Gamma_{\mu n}}{\Gamma_{\lambda n}}\right)^{1/2}\left(\frac{W_{\lambda\mu}}{D_{\mu}}\right)\right], \tag{4}$$

and

$$\phi_{\lambda}(E) = \sum_{\mu} \frac{|W_{\lambda\mu}|^2}{D_{\mu}} \quad , \tag{5}$$

where

$$D_{\lambda} = (E_{\lambda} - E) + \frac{i}{2}\Gamma_{\gamma} \quad , \tag{6}$$

$$D_{\mu} = (E_{\mu} - E) + \frac{i}{2}\Gamma_{\mu} \quad , \tag{7}$$

$$W_{\lambda\mu} = \text{residual interaction (eV)} \quad , \tag{8}$$

and the other symbols have the usual meaning. The subindex, λ, goes over the compound nucleus states, and the subindex, μ, classifies the "doorway" states.

On the basis of the previous results (valid only for $|W_{\lambda\mu}/D_{\mu}| \ll 1$) the average cross section is again given by Eq. (2) in which the s-wave and p-wave strength functions are multiplied by the energy-dependent "modulation" factors $M_0(E)$ and $M_1(E)$, respectively, where

$$M_{\ell}(E) = \left(1 + \sum_{\mu} |D_{\mu}|^{-4} A_{\mu}^{\ell}\right) \Big/ \left(1 + \sum_{\mu} |D_{\mu}|^{-2} B_{\mu}^{\ell}\right) \tag{9}$$

$$A_{\mu}^{\ell} = \frac{1}{2}\left[\Gamma_{\mu}^{(\ell)}\right]^2 \Gamma_{0\mu n}^{(\ell)} \left\langle \frac{|W_{\lambda\mu}|^2}{\Gamma_{0\lambda n}^{(\ell)}} \right\rangle_{\text{compound nucleus}} \quad (\text{eV}^4) \tag{10}$$

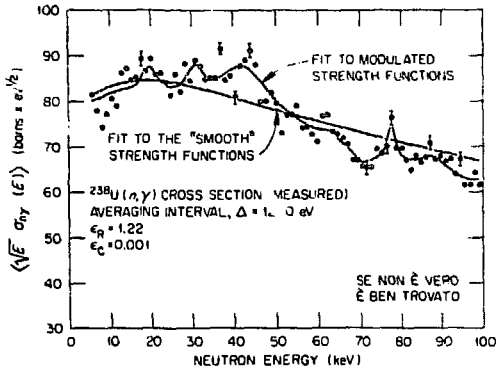


Fig. 2. Fit of the $^{238}U(n,\gamma)$ Cross Section Measured to the Modulated Strength Function.

$$B_{\mu}^{\ell} = \Gamma_{\mu}^{(\ell)} \left\langle \frac{|W_{\lambda\mu}|^2}{\Gamma_{\lambda}^{(\ell)}} \right\rangle_{\text{compound nucleus}} \quad (\text{eV}^2) \tag{11}$$

$$\ell = 0, 1 \quad ,$$

and the brackets indicate an average over the compound nucleus states. The neutron widths $\Gamma_{0\lambda n}^{(\ell)}$ and $\Gamma_{0\mu n}^{(\ell)}$ are reduced neutron widths and the total level widths $\Gamma_{\lambda}^{(\ell)}$ and $\Gamma_{\mu}^{(\ell)}$ are computed at the level energies. These modulation factors arise from the fact that the interaction between the compound nucleus states and the doorway states "modulates" both the neutron widths and the average level spacing.

A cavalier attempt has been made to extract the main features of the intermediate structure by fitting the average cross section of Eq. (2) with "modulated" strength functions. Although one expects both the s-wave and p-wave strength functions to be modulated, we arbitrarily changed only the s-wave contribution. The results of this procedure are shown in Fig. 2 together with the "smooth" strength function fit. The goodness of fit, expressed in terms of the chi-square test, improved by a factor close to 100 between the "smooth" and "modulated" fits. The modulated fit was then used as the reference line for the Wald-Wolfowitz runs test. For the correlation test, the data were divided by the modulated fit. The results of both statistical tests are given in Table 3 and indicate that the intermediate

Table 3. The Wald-Wolfowitz Runs and Correlation Tests for the $^{238}U(n,\gamma)$ Cross Section After Removal of the Intermediate Structure.

| Width (keV) | $\varepsilon_R$ | | | $\varepsilon_C$ | | |
|---|---|---|---|---|---|---|
| | Mock-up | Exp.[a] | Reduced[b] | Mock-up | Exp.[a] | Reduced[b] |
| .6 | .03 | 4.5 | .40 | .52 | 5.9 | .64 |
| .9 | .96 | 3.9 | .60 | .20 | 6.1 | .15 |
| 1.0 | .68 | 4.6 | .03 | .37 | 5.9 | .44 |
| 1.2 | .34 | 5.6 | .001 | .65 | 6.2 | 1.22 |
| 1.5 | .002 | 4.8 | .72 | .31 | 5.3 | .15 |
| 1.8 | 1.11 | 3.6 | 1.64 | .97 | 2.2 | .08 |
| 2.0 | 1.2 | 3.8 | .003 | .22 | 4.9 | .64 |
| 3.0 | .006 | 1.8 | .006 | 1.1 | 3.9 | 1.1 |

[a] Measured $^{238}U(n,\gamma)$ cross section.

[b] Measured $^{238}U(n,\gamma)$ cross section after removal of the intermediate structure.

structure had been adequately described. The assignment of the doorway levels to the s-wave strength function only prevents one from giving too much credibility to the level parameters obtained for the intermediate structure. The present results tend, however, to indicate an average level spacing of around 9 keV and reduced neutron widths averaging 40 eV.

## Conclusions

The two statistical tests applied to the measured $^{238}U(n,\gamma)$ cross sections give, with a high significance level, strong support to the presence of intermediate structure in the neutron capture of the $^{238}U$ nucleus. Although we cannot exclude the possibility for at least a fraction of this structure originating from subthreshold fission phenomena, the presence of large local fluctuations in the $^{238}U$ total cross section [3] gives support to the presence of doorway states in the reaction entrance channel.

From the theoretical results referred to here, one concludes that the intermediate structure [see Eq. (9)] is due not only to local enhancements of the reaction widths, but also to rapid fluctuations of the average level spacing.

The Wald-Wolfowitz correlation and runs tests are also valuable in connection with the fitting of the intermediate structure as they can be used as an "added" goodness of fit test.

As we have discussed earlier, not too much physical significance can be attached to the level locations and widths of the detected doorway levels. However, fits, like the ones described here, do afford a method of representing the average behavior of the $^{238}U$ capture cross section in the unresolved region. This may prove valuable for reactor calculations [21]

Studies similar to this should be applied to high resolution measurements of capture cross section in other elements such as thorium, gadolinium, and other nonfissile uranium isotopes, in order to ascertain whether there is a systematics that can be deduced regarding intermediate structure phenomena. In this regard one should mention the work of Macklin[22] who also showed the presence of intermediate structure in the gold capture cross section.

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