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Abstract.
We present a model for gamma ray bursts based on the compression of neutron stars in close binary systems. Our general relativistic hydrodynamic computer simulations of close neutron star binaries have found that as the orbit shrinks the density of the neutron stars rises. This compressional effect has been estimated to produce thermal energies in the neutron stars of the order of magnitude 10^{52} to 10^{53} ergs on a timescale of a few seconds. This is a possible source of energy for gamma-ray bursts. The hot neutron stars will emit neutrino pairs which will partially recombine to form an electron positron pair plasma. The pair plasma will recombine after expansion to produce photons which closely mimic the characteristics of gamma-ray bursts.

Introduction

To model gamma-ray bursts we have made hydrodynamic calculations of neutron star binaries (NSBs). These calculations predict a compression and heating of the neutron stars (NSs) prior to merger. The resultant heating of the NSs from these calculations was then fed into a computer program modeling the hydrodynamics and neutrino diffusion inside a NS. These transport calculations indicated substantial electron-positron (e^+e^-) pair production. This resulting production of the e^+e^- plasma was then modeled using a hydrodynamic code from which the gamma-ray signal was extracted.

Neutron Star Binary Calculations

Wilson *et al* [1, 2, 3, 4] have developed a general relativistic hydrodynamic computer program in three spatial dimensions to study NSBs. It assumes that the spatial part of the metric is conformally flat. At each time step the Einstein constraint equations are solved. This means that at each time step we have a valid solution to Einstein's equations. The hydrodynamic equations are then used to advance the fluid variables in the metric space just computed. The principal approximation in this scheme is the neglect of gravitational radiation. The gravitational radiation is taken into account at the quadrupole level. In this approximation it is found to be quite small, $\frac{\dot{J}}{\omega J} \sim 10^{-4}$ where J is the angular momentum and ω is the angular frequency.

The key result observed in these calculations is that the closer the stars come together the higher the stellar density. In these hydrodynamic calculations the matter is taken to have zero temperature. The origin of the compression can be attributed to two properties of the equations. First, the source terms for the metric potentials have contributions from the kinetic energy of the matter. For example, the three-space conformal factor ϕ is given by

$$\nabla^2\phi = 2\pi\phi^5\left[D + E + \frac{U^2}{2}(D + E + P\sqrt{1+U^2}) + \frac{K^2}{16\pi}\right] \tag{1}$$

where D is the coordinate density of baryons and E is the coordinate density of internal energy. P is the pressure and K^2 is the square of the extrinsic curvature tensor. U^2 is the squared magnitude of the spatial components of the four velocity. Second, the accelerations due to the metric curvature terms can be put in the form:

$$\frac{dP}{dr} \cong -(D + E + P\sqrt{1+U^2})\frac{(1+2U^2)}{\sqrt{1+U^2}}\frac{d\alpha}{dr} \tag{2}$$

for the case of stationary motion. α is the metric lapse function which is analogous to the Newtonian gravitational potential and r is the radial distance from the center of each star. This force enhancement factor only occurs in circular motion. Circular motion can not be transformed away by a simple coordinate boost.

In Mathews and Wilson[2] the thermal energy that would arise from the compression discussed above was calculated. The energy of compression would first appear as radial oscillations in the stars, but it is argued that non-simple swirling fluid motions observed in the hydrodynamic calculations would interact with the radial motions to produce shock waves. In the above paper the generation of magnetic fields due to the non-simple swirling fluid motion was calculated. The e-folding time for the magnetic field was found to be one millisecond. The equipartition field is 10^{17} gauss. Magnetic field reconnection should also help convert fluid motions into thermal energy. In Figure 1 the thermal energy available in a pair of orbiting neutron stars is presented as a function of the squared orbital velocity U^2. Note that this energy release is a $(\frac{v}{c})^4$ effect. The right hand termini of the curves of Figure 1 arise from either the stars having spiraled in to the last stable orbit or the stars having collapsed on to themselves to form black holes. Figure 1 is for a particular matter equation of state. From calculations of other equations of state we estimate that for two stars the energy available ranges from (0.3 to 1.0) $\times 10^{53}$ ergs.

In order to estimate the time scale for the evolution of the binary system we use the formula for the emission of gravitational waves given by post Newtonian calculations. The gravity wave orbital energy loss formula then leads to a thermal energy time dependence given by

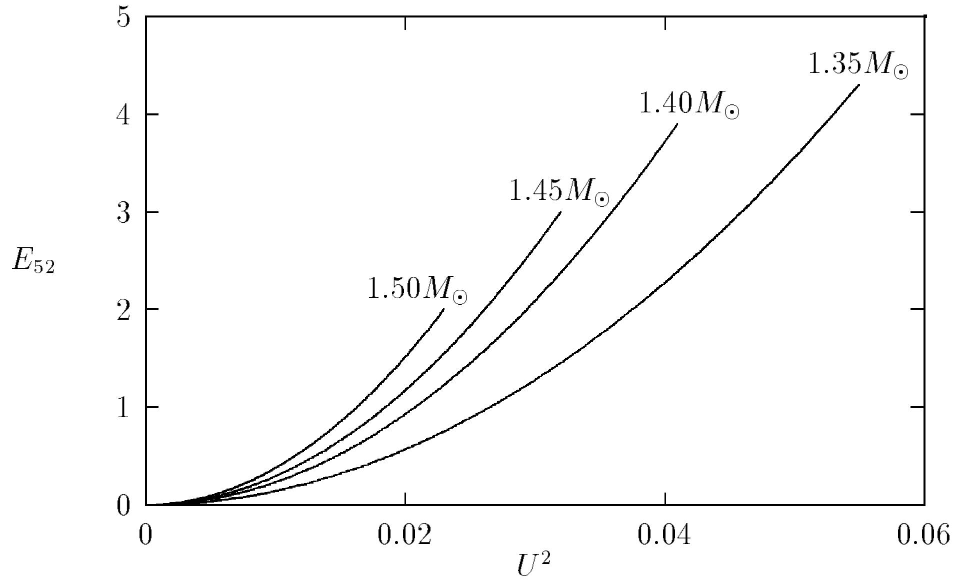

Figure 1. Released gravitational energy E_{52} ($\times 10^{52}$ ergs) as a function of U^2 for a range of star masses.

$$E_{th}(t) = \frac{E_{th}^0}{[1-(64/5)(Mf)^{5/3}ft]^{1/4}} \tag{3}$$

where $t = 0$ is the time at which the orbital frequency f is reached. Note that $t < 0$ for this formula. Figure 2 shows the time evolution of the energies and neutrino luminosity. The luminosity has been estimated by a simple neutrino diffusion calculation[2].

In summary, we observe from the neutron star binary hydrodynamic calculations that thermal energies up to 10^{53} ergs are potentialy available in the stars. This thermal energy would lead to central temperatures of the order of 50 MeV and luminosities of 10^{53} ergs/sec. before the system collapses. The emission of neutrinos will lead, via neutrino-antineutrino ($\nu\overline{\nu}$) annilation, to the production of an e^+e^- pair plasma outside the neutron stars.

In order to estimate the properties of the e^+e^- plasma we put a thermal energy of 10^{52} ergs into a 1.45 solar mass neutron star and followed the stellar evolution through the Mayle-Wilson supernova computer program[5]. The calculation was only run for 10 milliseconds. We found that the efficiency of pair production to be 3% and the entropy of the plasma to be 10^{10} per baryon. The radii of neutron stars in binaries at late times are about 9 km while the last stable orbit about such a star is at radius 12 km. The strong gravitational field will strongly bend the neutrino trajectories just above the stellar surface. The bending is estimated to increase the $\nu\overline{\nu}$ annilation rate by about a factor of 3. Thus we expect the energy of the e^+e^- pair plasma to be in the range 10^{51} to 10^{52} ergs. The high entropy of the plasma means that little baryonic matter is ablated from the stellar surface.

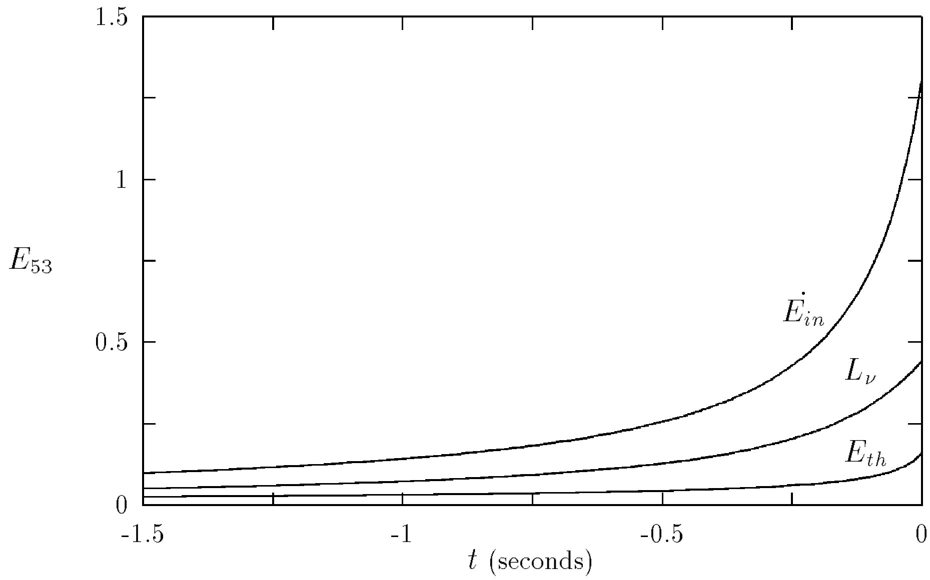

Figure 2. Estimated neutrino luminosity L_ν ($\times 10^{53}$ ergs/sec), the total accumulated internal energy E_{th} ($\times 10^{53}$ ergs) and the rate of gravitational energy release $\dot{E}_{in}$ ($\times 10^{53}$ ergs/sec), for a 1.40 $M_\odot$ star. Time $t = 0$ is when orbital or stellar instability is reached.

Pair Plasma Hydrodynamics

Having roughly defined the initial parameters of the hot e^+e^- pair wind blowing off the surface of a NS, we wish to follow its evolution and characterize the observable gamma-ray emission. To study this a spherically symmetric, special relativistic hydrodynamic computer code is employed to track the flow of baryons, e^+e^- pairs and photons deposited at the surface of an isolated NS.

The fluid is modeled by the following special relativistic hydrodynamic equations:

$$\frac{\partial D}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D V^r) \tag{4}$$

$$\frac{\partial E}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E V^r) - P\left[\frac{\partial \gamma}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \gamma V^r)\right] \tag{5}$$

$$\frac{\partial S_r}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 S_r V^r) - \frac{\partial P}{\partial r} \tag{6}$$

where D and E are the coordinate densities of baryonic and thermal energy (e^+e^- and photons) respectively, and S_r is the radial coordinate momentum density. The radial covariant component of the 4-velocity U_r, , , Lorentz factor γ and the radial coordinate velocity V^r are defined by

$$S_r \equiv (D + , E)U_r, \quad , \equiv 1 + \frac{P\gamma}{E} \quad \gamma \equiv \sqrt{1 + U_r^2}, \quad V^r \equiv \frac{U^r}{\gamma} = \frac{U_r}{\gamma}. \tag{7}$$

To track the e^+e^- pairs we define a pair equation. The observed pair annihilation rate must be corrected for relativistic effects; specifically time dilation will slow down

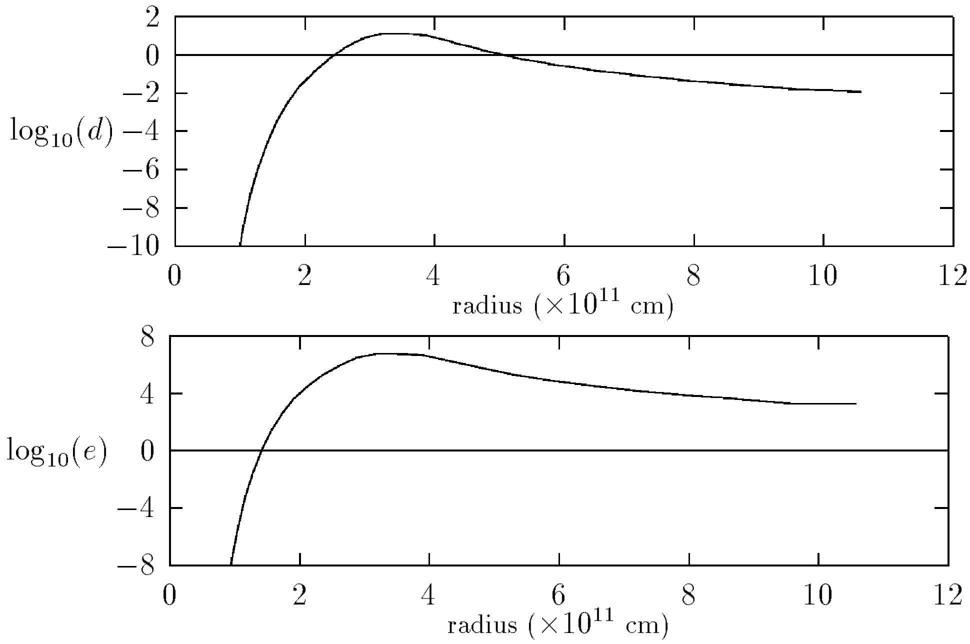

Figure 3. (Top) Proper baryon density profile. **(Bottom)** Proper internal energy density profile.

the apparent pair annihilation process for a fast moving fluid. Thus we take a continuity equation analogous to Equation (4) and add a second term to account for annihilation:

$$\frac{\partial N_{pairs}}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 N_{pairs} V^r) + \overline{\sigma v} N_{pairs}(N^0_{pairs}(T) - N_{pairs})/\gamma^2 \tag{8}$$

where the coordinate pair number density is N_{pairs} and $\overline{\sigma v}$ is the mean pair annihilation cross-section times the thermal velocity. Although $\overline{\sigma v}$ depends on T, it varies little in the temperature range of interest and thus can be taken to be a constant. $N^0_{pairs}(T) = n^0_{pairs}(T)\gamma$, where $n^0_{pairs}(T)$ is the proper equilibrium e^+e^- pair density at temperature T given by the appropriate Fermi integral with a chemical potential of zero.

The total proper energy equation, including photons and e^+e^- pairs, is

$$e_{tot} = aT^4 + e_{pairs} \tag{9}$$

where coordinate energy in Equation (5) is related to proper energy by $E = e_{tot}\gamma$ and e_{pairs} is the appropriate zero chemical potential Fermi integral normalized to give the proper e^+e^- pair density $n_{pair} = N_{pairs}/\gamma$ as determined by Equation (8).

To model the energy deposition at the surface of a NS we inject baryon and pair-photon energy densities into the innermost zone (at $r = 10^6$ cm) of the computer code at a rate determined by the time derivative of the heating energy given in Equation (3) In the results presented here we have injected a total energy of 10^{51} ergs, consistent with available energy estimated in NSB calculations above. Since the entropy per baryon of the wind is quite high we take the rate of injection of baryons as $\dot{D} = 10^{-10}\dot{E}$.

The hydrodynamic equations are evolved, allowing the plasma to expand. Once the system becomes transparent to Thompson scattering, ($\int N_{pair}(r)\sigma_T dr \sim 1$ where σ_T

is the Thompson cross-section) then we assume no further scattering, the calculation is stopped and the photon gas is analyzed to determine the photon signal. Figure 3 shows typical proper density and energy profiles at the end of a calculation.

Observables

We find that the photons and e^+e^- pairs appear to decouple at virtually the same time throughout the entire photon-e^+e^- pair plasma (when the cloud has reached a radius $\sim 10^{12}$ cm), thus we take this event to be instantaneous and to occur when the cloud becomes optically thin to Thompson scattering. We then look at two observables, the time integrated number spectrum $N(\epsilon)$ and the total energy received as a function of observer time $\varepsilon(t)$.

The Spectrum

To get the spectrum, as mentioned above, we assume that the e^+e^- pairs and photons are equilibrated to the same T when they decouple. Thus the photons in the fluid frame (denoted with a prime: $'$) make up a Planck distribution with the form

$$u'_{\epsilon'}(T') \approx \frac{\epsilon'^3}{exp(\frac{\epsilon'}{T'}) - 1} \tag{10}$$

but $\frac{u_\epsilon}{\epsilon^3}$ is a relativistic invariant[7]. This implies $\frac{\epsilon}{T}$ is also a relativistic invariant. So a Planck distribution in an emitter's rest-frame with temperature T' will appear Planckian to a moving observer, but with boosted temperature $T = T'/(\gamma(1-v\cos\theta))$ where $v\cos\theta$ is the component of fluid velocity (c=1) directed toward the observer. Thus

$$u_\epsilon(\theta, v, T') \approx \frac{\epsilon^3}{exp(\gamma(1-v\cos\theta)\frac{\epsilon}{T'}) - 1} \tag{11}$$

gives the observed spectrum of a blackbody with rest-frame temperature T' moving at velocity v and angle θ with respect to the observer.

In the present case we wish to calculate the spectrum from a spherical, relativistically expanding shell as seen by a distant observer. Since we know v and T' and radius R of the shell, we integrate over volume (and thus, for a shell, angle) with respect to the observer. We thus get the observed number spectrum $N_\epsilon = \int \frac{u_\epsilon \; dVolume}{\epsilon}$, per photon energy ϵ, per steradian, of a relativistically expanding spherical shell with radius R, thickness dR in cm, velocity v, Lorentz factor γ and fluid-frame temperature T' to be

$$N_\epsilon(v, T', R) = 4\pi R^2 dR \frac{\epsilon T'}{v\gamma} log\left[\frac{1 - exp[-\gamma\epsilon(1+v)/T']}{1 - exp[-\gamma\epsilon(1-v)/T']}\right] \tag{12}$$

which has a maximum at $\epsilon_{max} \cong 1.39\gamma T' \; eV$ for $\gamma \gg 1$. We may then sum this spectrum over all shells (the zones in our computer code) of our fireball to get the total spectrum shown in Figure 4. Since we a priori assume the photons are thermal, our spectrum has a high frequency exponential tail, but as seen in Figure 4, this spectrum is not thermal up through $\epsilon = 5$ MeV.

A key feature of this spectrum is that its peak is consistent with observation. It is interesting to note that, as seen in Figure 5, the bulk of the photons have a fluid

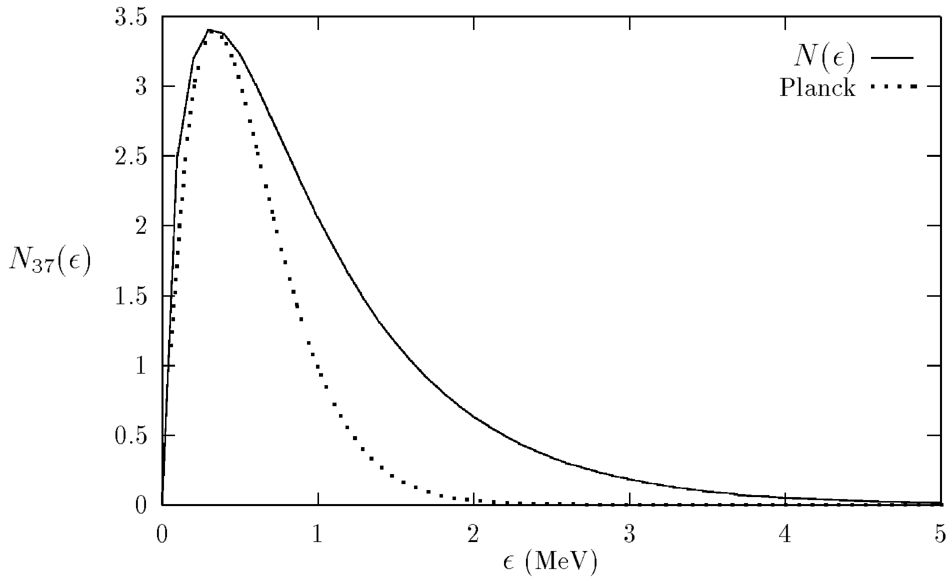

Figure 4. Photon number spectrum $N_{37}(\epsilon)$ $(\times 10^{37} photons/MeV/4\pi)$ from e^+e^- pair plasma. A reference Planck number spectrum fitted to the peak of $N(\epsilon)$ is shown to illustrate the non-thermal nature of $N(\epsilon)$.

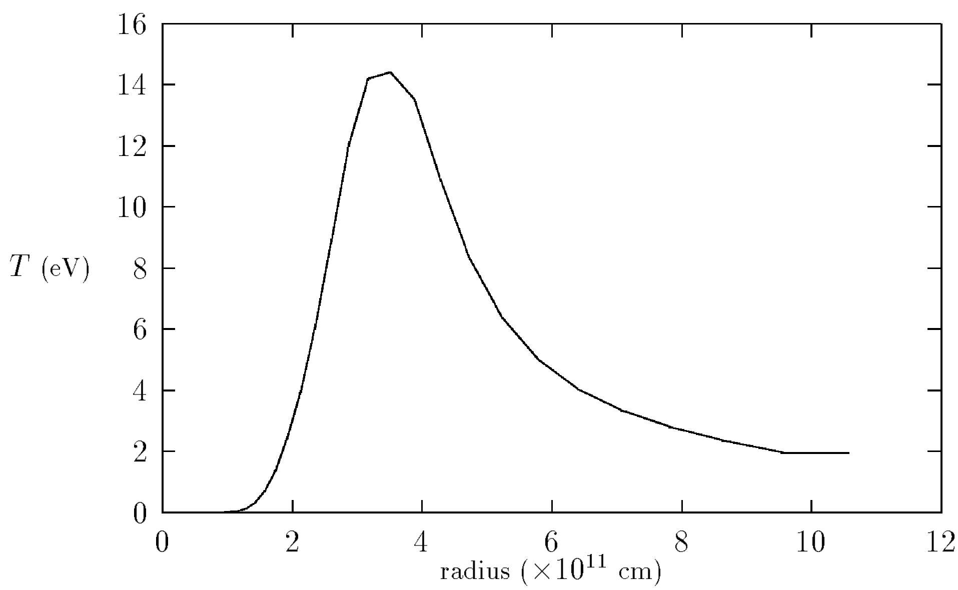

Figure 5. Photon gas temperature T' in the cloud.

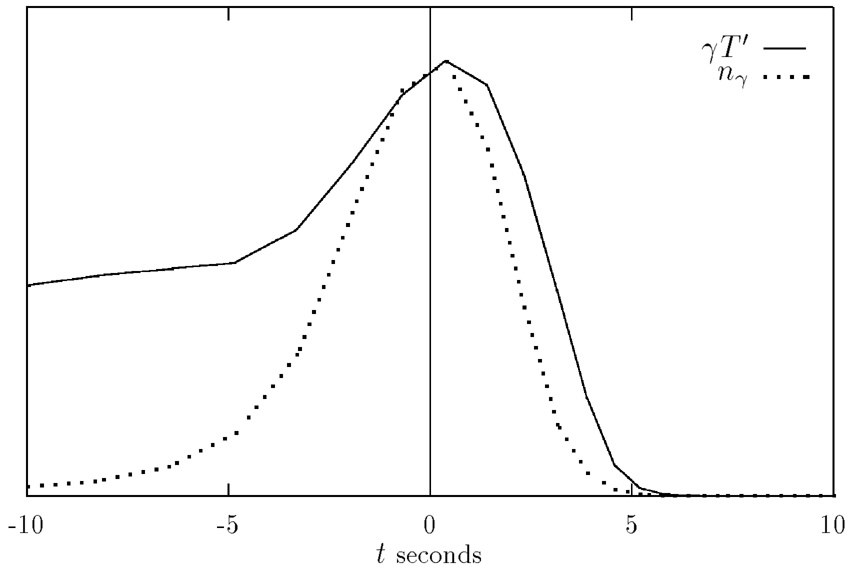

Figure 6. The product $\gamma T'$ as a function of simple shell arrival time compared to photon number n_γ. The temperature of the burst spectrum will rise and then fall over the bulk of the burst.

frame temperature T' of only $\epsilon \sim 5 - 15$ eV, but are Lorentz boosted by a γ gradually increasing from 3×10^4 at radius 3×10^{11} cm to 8×10^4 at radius 10^{12} cm. Thus our spectrum derives from a relativistic fluid motion.

To see how the spectrum changes over time, we have plotted $\gamma T'$ and n_γ versus shell time (light curve effects described in Equation(14) were not used). The temperature of the burst spectrum ($\sim \gamma T'$) will rise and then fall over the period of maximum photon emission.

The Light Curve

To acquire the observed light curve $\varepsilon(t)$ we again decompose the spherical plasma into concentric shells and consider two effects. First is the relative arrival time of the first light from each shell: light from outer shells will be observed before light from inner shells. Second is the shape of the light curve from a single shell.

Emission from moving stuff is beamed along the direction of travel within an angle $\theta \sim 1/\gamma$. The surface of simultaneity of a relativistically expanding spherical shell is an ellipsoid[6]. The time of intersection of an expanding ellipse and a fixed shell of radius R as a function of θ (i.e. the time at which emission from this intersection circle is received) is:

$$t = \frac{R\gamma^2}{v}(1+v)(1 - v\cos\theta) \cong R(1 + \theta^2\gamma^2) \qquad (13)$$

for $\theta \ll 1, \gamma \gg 1$. We find that, integrating our boosted Planck distribution of photons (Equation 11) over frequency, a relativistically expanding shell of radius R will have a time profile

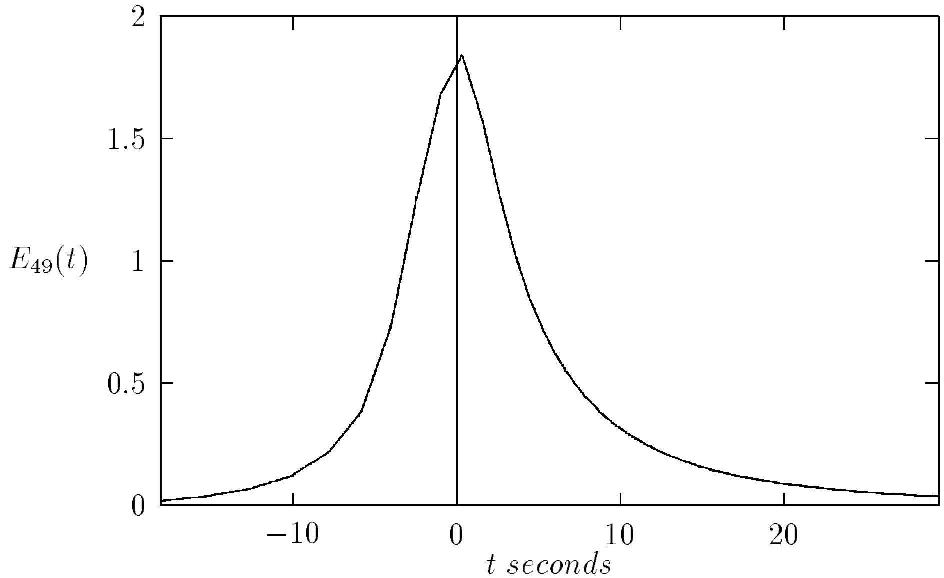

Figure 7. Light curve $\varepsilon_{49}(t)$ ($\times 10^{49} ergs/second/4\pi$)

$$\varepsilon(\tau, v, T', R) = 26\pi\gamma^2(1+v)^3(\frac{T'}{\tau})^4 R^2 dR \frac{d\tau}{v} \sim \frac{1}{\tau^4} \tag{14}$$

for $\tau > 1$ and where $\tau \equiv \frac{vt}{R}$. The final light curve is then constructed by summing the signal from all shells.

In Figure 7 we see an example of $\varepsilon(t)$ for NSB of equal star mass. Variation in the ratio of star mass in the NSB effects the relative compression and heating rate of each star, thus allowing a variety of GRB durations. This burst has a $T_{90} = 25$ seconds, where T_{90} is the time interval over which 90% of the energy is received.

1. Conclusion

We find that the photon signature that is plausibly generated from the compressional heating and e^+e^- pair emission of close NSBs matches that of observed gamma-ray bursts very well. In particular:

- An adequate amount of energy (10^{51-52} ergs) is yielded in gamma-rays to account for observed fluxes at cosmological distances (~ 1 Gpc).
- The peak of the photon spectrum is consistent with that of observations ($N_{peak} \sim$ 200 keV).
- Duration and shape of the light curve matches that of the most common "long" gamma-ray bursts; $T_{90} > 10$ seconds.

We note that there are no free parameters in this model, only the following physical parameters: the masses of the neutron stars, the equation of state of the neutron star matter and magnetic fields.

As seen in Figure 1 the energy available depends strongly on the neutron star mass. The low mass binaries will evolve much faster than Equation (3) would imply since more orbital energy will be lost by neutrinos than by gravity waves. The higher energy signals should have shorter time scales. Unequal star masses should lead to longer time signals since each star will evolve at a different time. Studies of the effect of different equations of state[2] indicate that stiffer matter will lead to more energy release.

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References

[1] Wilson, J R, Mathews, G J & Marronetti, P 1996 *Phys. Rev. D.* **54**, 1317

[2] Mathews, G J, & Wilson, J R 1997 *Astrophysical J.* **482**, 929—941

[3] Wilson, J R, & Mathews, G J 1997 submitted *Phys. Rev. D*

[4] Mathews, G J, & Wilson, J R 1997 submitted *Phys. Rev. Lett.*

[5] Wilson, J R, & Mayle, R W 1993 *Phys. Rep.* **227**, 97

[6] Fenimore, E E *et al* 1996 *Astrophysical J.* **473**, 998

[7] Rybicki G B, Lightman A P 1975 *Radiative Processes in Astrophysics* (USA: John Wiley & Sons, Inc.) p 146

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