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**Neural Network Modeling of Pulsed-Laser Weld Pool Shapes  
in Aluminum Alloy Welds**

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## Abstract

A model was developed to predict the weld pool shape in pulsed Nd:YAG laser welds of aluminum alloy 5754. The model utilized neural network analysis to relate the weld process conditions to four pool shape parameters: penetration, width, width at half-penetration, and cross-sectional area. The model development involved the identification of the input (process) variables, the desired output (shape) variables, and the optimal neural network architecture. The latter was influenced by the number of defined inputs and outputs as well as the amount of data that was available for training the network. After appropriate training, the "best" network was identified and was used to predict the weld shape. A routine to convert the shape parameters into predicted weld profiles was also developed. This routine was based on the actual experimental weld profiles and did not impose an artificial analytical function to describe the weld profile. The neural network model was tested on experimental welds. The model predictions were excellent. It was found that the predicted shapes were within the experimental variations that were found along the length of the welds (due to the pulsed nature of the weld power) and the reproducibility of welds made under nominally identical conditions.

## Introduction

The weld pool shape is critically important in terms of determining the quality of a weld. The depth of penetration, in particular, is often the most important feature that governs the integrity of a weld. Over the last two decades, many fundamental studies have tried to develop models that predict the weld pool shape from first principles<sup>1-8</sup>. These models have become increasingly sophisticated over the years and have been very useful in providing a better, more fundamental understanding of the factors that affect the weld pool shape. However, as the models have become more advanced, they have also become more complicated. Consequently, although they are better able to consider the many factors that influence weld pool development

and the final weld pool shape, they are still not totally accurate, are often difficult to use, and normally require extensive computing time. Thus, they are not particularly amenable to in-process applications such as control loops where simplicity and rapid response time are required. For the use of models in real-time process applications, the ability to make instantaneous predictions is desirable and often essential.

One possible solution for providing real-time predictions of weld pool shape (as well as other weld attributes such as cracking propensity, properties, etc.) is the utilization of neural network models. These models are empirically based but they can be quite sophisticated while still maintaining the essential feature of rapid response time. Several recent papers have addressed the issue of predicting weld shape with neural networks in arc-welding<sup>9-11</sup> and laser spot-welding<sup>12,13</sup>. The present paper describes the application of neural network modeling to the problem of predicting weld pool shape in pulsed Nd:YAG laser aluminum welds. The approach that is presented is quite general and can be applied to any welding process, provided the proper data for training the neural network are available. The present study shows that good accuracy can be achieved with the use of neural networks, without requiring an extensive data set for training the neural network.

## Neural Networks

A very simple description of the concept behind neural networks is given below. There is extensive literature on the theory behind neural networks. The reader is referred to other publications for more details<sup>14,15</sup>. Neural networks are modeled after the learning process in the human brain. A network structure consists of interconnected layers of nodes; the nodes include input and output nodes as well as internal, hidden nodes. These nodes are "connected" to each other so that the value of one node will affect the value of another. The relative influence that a given node has on another one is specified by the "weight" that is assigned to each connection. A schematic diagram of a simple neural network is shown in Figure 1. There are three layers in the

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Output Layer

Hidden Layer

Input Layer

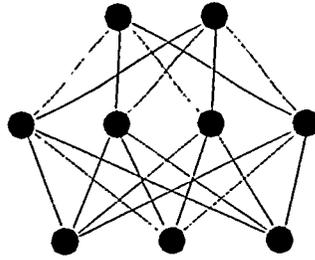


Figure 1: Schematic diagram showing the multiple layer structure of a neural network and the inter-connectivity between the nodes of the network.

diagram. In the example of Figure 1, the input layer has three nodes, representing three input variables, while the output layer consists of two nodes, corresponding to two output variables. In addition, one hidden layer with four nodes is shown in the diagram. For example, when applied to weld shape modeling, the input nodes may correspond to weld process conditions such as welding speed, power, and material thickness while the output nodes may represent weld pool shape parameters such as width and penetration depth. The neural network is trained by introducing a training set based on experimental data for inputs and corresponding outputs. A training routine is then carried out in which outputs are predicted and these are compared with the true outputs. Starting with a simple initial configuration, the weights are continuously adjusted by an optimization process to yield better, more accurate predictions. Through this learning process of many iterations, a complicated set of empirical relationships between input and output variables may be developed. Eventually, with minimal influence from the user, the network "learns" a scheme in which outputs are associated with the inputs. In the present analysis, a feed-forward network with a back-propagation learning scheme was utilized<sup>15</sup>.

### Experimental Conditions

Autogenous, pulsed Nd:YAG laser welds were made on 3-mm-thick sheet of aluminum alloy 5754 and 2-mm-thick sheet of alloy 6111. Although the neural network analysis considered all the data for both alloys, only the results for alloy 5754 will be presented here due to space limitations. The complete results will be published elsewhere<sup>16</sup>. A range of welding conditions was examined and the parameters are listed in Table 1. All welds were made at approximately 4 pulses/mm in order to insure sufficient overlap of the pulses. The average power was varied from 50 to 250 W to include a wide range of power levels and corresponding pool shapes and sizes without reaching full penetration. The aim was to cover typical welding conditions used in practice. In all cases, the laser beam was focused on the top surface. The welds were sectioned and transverse cross-sections were examined metallographically. The first ten conditions in Table 1 (along with ten conditions for alloy 6111) were used for training the neural network. Five transverse views were analyzed

Table 1: Laser welding conditions for alloy 5754.

ID	Weld Speed (mm/sec)	Pulse Energy (Joules)	Average Power (Watts)	Pulse Duration (msec)
5-1-1	6.38	4.1	101	2.2
5-1-2	6.38	2	51	2.2
5-1-3	6.38	2.9	74	2.2
5-2	10.2	5.5	203	2.2
5-3	10.2	4.1	165	2.2
5-4	10.2	3	125	2.2
5-5	10.2	3.5	158	2.2
5-6	2.55	11.3	123	7.5
5-7	3.83	13.2	196	7.5
5-8	3.83	9.5	190	7.5
5-1-R	6.38	4.0	100	2.2
5-2-R	10.2	5.0	200	2.2
5-5-R	10.2	3.95	158	2.2
5-1-N	5	5	100	2.2
5-2-N	5	7.5	150	2.2
5-3-N	5	9.05	181	2.2
5-4-N	3	8.33	100	2.2
5-5-N	3	6.25	75	2.2
5-6-N	3	4.17	50	2.2
5-7-N	7.65	8.13	244	2.2
5-8-N	7.65	6.67	200	2.2
5-9-N	7.65	5	150	2.2
5-10-N	7.65	3.33	100	2.2

for each weld condition to compensate for the variation in weld profile shape due to the pulsed nature of the welding process. Later, thirteen additional welds (with "R" and "N" suffixes in Table 1) were made and these were used to test the neural network predictions. Of these thirteen conditions in the second round of welds, three welds ("R" suffix, Table 1) were made under the same nominal conditions as three of the welds in the first round. This allowed for an evaluation of the reproducibility of the welds. The remaining ten welds in the second round ("N" suffix,

Table 1) were used to test the neural network prediction capabilities as these welds were made under conditions that were different from those used to train the network.

### Network Development

Initially, twenty different welds were made for the two different alloys (and thicknesses) and these were used as the training set for the neural network<sup>16</sup>. This is not a very extensive training data set. When identifying the neural network structure in terms of the number of hidden layers and hidden nodes, the size of the training set must be taken into account. If training data for an unlimited number of welding conditions were available, then the optimum neural network structure is likely to be quite complicated, with many hidden nodes and perhaps even several hidden layers. However, in the present case, with the small number of conditions that were examined, it was determined that only a very simple neural network structure was justified<sup>16</sup>. Otherwise, the number of adjustable parameters in the network (the weights associated with each of the hidden nodes) would be greater than the number of training-set data points and the neural network would be over-specified. Under such conditions, the neural network could “learn” and “memorize” very accurately but its predictability would be poor. Thus, only one hidden layer with two nodes was used. A more detailed analysis justifying this choice of network architecture is provided elsewhere<sup>16</sup>. The input nodes corresponded to the variable process parameters and the output nodes consisted of pool shape parameters, as described in the next section. The final neural network structure is shown in Figure 2.

Once the optimal neural network architecture was identified, the final neural network was trained and the output was used to predict actual weld pool shapes. The overall procedure is shown in Figure 3. A commercially available software program (NeuralWorks Professional II/PLUS<sup>TM17</sup>) was used to carry out the neural network analysis. A back-propagation learning scheme

with a sigmoidal transfer function was used<sup>16</sup>. Several hundred thousand iterations were made during the learning process to identify the final neural network. Various learning parameters were tested in an effort to evaluate their effect on the final network accuracy. It was found that the learning parameters had little influence on the learning ability of the network. In addition, the starting point (“seed” number) for the learning process was varied randomly in order to identify a “best network”.

The accuracy of the network was evaluated by two means. First, comparable nets were created with the same architecture and same random seed number but with only 19 of the 20 training points. The 20<sup>th</sup> point was used as a blind test point. This was repeated for each of the 10 conditions for alloy 5754 in the first round of welds. In this way, a quantitative estimate of the accuracy of the network could be established. A second accuracy test was made by visually comparing the predicted weld pool shapes with the second set of thirteen welds. Both of these tests indicated that the predicted weld pool shapes were reasonably accurate and typically within the variation found along each weld and among duplicate welds made under similar conditions. It should be noted that an absolute best-network is never found since further learning or use of a more extended set of initial seed numbers is always going to produce some marginal improvement. Therefore, from a practical perspective, some limits on the learning process must be applied. In the current study, the networks were evaluated every 10,000 iterations and if no improvement was found after 20 consecutive checks (200,000 iterations), then the training was terminated.

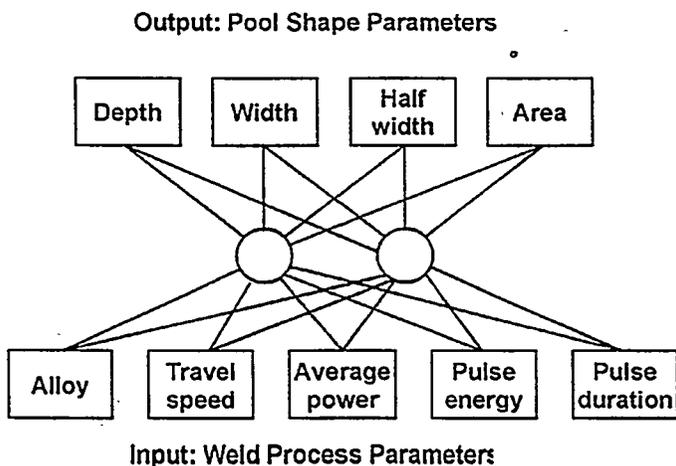


Figure 2: Neural network structure for pool shape prediction.

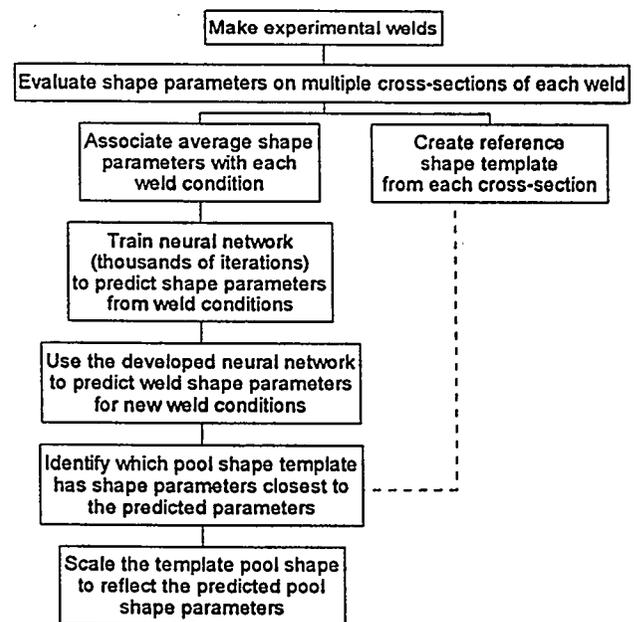


Figure 3: Flow chart showing the sequence of operations to produce the training data set, to train the neural network, and to predict weld pool shape with the developed network.

## Weld Pool Shape Characterization

In order to predict weld pool profiles, it is first necessary to identify parameters that characterize the weld pool shape. One approach is to describe the cross-section profile in terms of an analytical function. However, this is complicated by several factors. First, the experimental cross-sections included a wide range of shapes, from shallow half-ellipses to deep and narrow welds. In addition, the weld profiles often included inflection points that could be difficult to describe by simple geometric functions. Some typical weld pool cross-sections are shown in Figure 4. Third, the number of parameters that could be used to describe the weld pool shape had to be limited. This was because there was a limited amount of data available for training, and it was not appropriate or justifiable to develop a model with a large number of adjustable parameters. Instead, it was desirable to keep the number of parameters that were used to describe the weld pool geometry to a minimum. Finally, the use of an analytical function to describe the weld pool shape was avoided because the choice of

the function was somewhat arbitrary. Instead, physical parameters relating to the actual weld pool shape were used.

The four parameters describing the actual shape of the weld pool cross-section were penetration depth, width (at the top of the weld), width at half penetration (referred to as "half-width"), and total area. These four parameters were evaluated from the experimental weld pool cross-sections, as shown schematically in Figure 4. Since pulsed-laser welds were examined, the weld pool shape was not constant along the length of the weld but, rather, fluctuated as a result of the pulsing power source. Five weld cross-sections were analyzed from each weld to account for the variability in pool cross-sections due to the pulsing power source. The four pool shape parameters were measured on each of these five cross-sections and the average values were used in the training of the neural network.

The top surface of the welds was often highly irregular and variable, as shown in Figure 4. This presented a problem when ascertaining the area of the welds. It was decided to use the actual weld cross-section areas, without artificially cutting off the protuberances on the top surface. However, when using the output from the neural network model to construct a predicted weld pool cross-section, a flat top surface was imposed.

The output from the neural network model consisted of the four weld profile parameters, penetration depth, width, half-width, and area. It was necessary to convert these four parameters into an actual weld profile. This was accomplished by using the experimental weld profiles as templates. The output shape parameters from the neural network were compared to the entire set of experimental weld pool profile parameters and the closest match was identified. Then, the corresponding experimental weld profile was scaled appropriately so that the final profile corresponded to the predicted penetration and width parameters. In this way, the predicted weld shape resembled the experimental weld cross-sections and there was no need to impose an arbitrary analytical function to describe the complicated profiles. The weld profile template library was relatively extensive because all five cross-sections that were taken from each weld were utilized, although only welds from the first round were included.

As discussed in greater detail later, the shape of the second set of welds ("R" and "N" labels in Table 1) showed some features that were different from the first set. The second set tended to be deeper and the fusion zone boundaries were steeper near the top of the weld, even under the same nominal conditions. This implies that, in addition to the four parameters listed in Table 1, there was at least one other parameter that had an influence on the weld pool shape, and this parameter was not necessarily held constant. Without using this additional parameter as an input in the neural network, scatter in the training data set is introduced and the ability of the neural network to fit the data is compromised. Furthermore, the pool profiles from the second set of welds were not included in the template library. This limited the degree to which the neural network could accurately predict the pool profiles from the later round of welds. Ideally, the neural network training set should have included some data from each of the two sets of welds; development of such an improved network is being considered for future work.

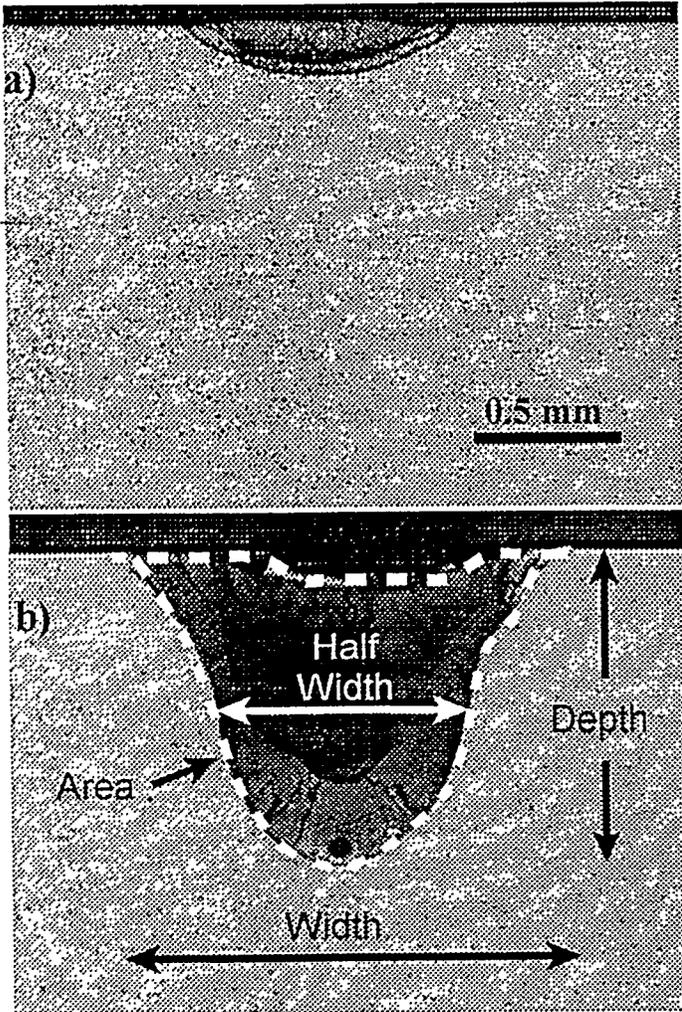


Figure 4: Cross-section micrographs of welds (a) 5-5 and (b) 5-7 (see Table 1) showing the range in pool shapes that was observed. Also shown in (b) are the four pool shape parameters that were used to describe the cross-section profile. The same scale marker applies to both figures.

## Results and Discussion

First, it is informative to examine the reproducibility of the weld profiles in the laser welds. This can be assessed in two ways. In Figure 5, five weld pool cross-sections from a typical weld are superimposed. Clearly, the weld pool shape is not exactly constant along the length of the weld. This kind of superposition provides some guidance as to the variation in pool shape that can be expected within the same weld. The range in weld pool cross-sections also provides a basis for assessing the accuracy of the predicted pool profiles.

The second method to assess the reproducibility of the welds is to compare welds made under the same nominal conditions. Figure 6 shows the weld pool profiles for three such repeat welds (5-1-R, 5-2-R, 5-5-R) compared to the original welds (5-1-1, 5-2, 5-5, respectively). Although good reproducibility is shown in Figure 6 for welds 5-1-1 and 5-1-R, the reproducibility is noticeably worse for welds 5-5 and 5-5-R. There is a tendency for the "R" welds to be deeper than the corresponding welds in the initial set. Several factors may contribute to the lack of reproducibility. First, the conditions were not exactly the same (see Table 1). Second, focusing the laser on the top surface was done visually and this may not be completely reproducible. Third, the mirrors were slightly different because a defect in the laser mirror was corrected between runs. Finally, different devices were used to measure the average power and so the values may not be exactly the same. Although these differences may have compromised the exact duplication of the weld conditions, the resultant differences may be comparable to those found in everyday practice when trying to reproduce previous weld conditions. Therefore, the comparisons are considered to be representative of reproducibility in typical welding environments.

The entire series of predicted weld pool shapes are shown in Figure 7 for all 13 of the welds in the second round of tests, along with the experimental profiles. The thick weld pool outline is the predicted one while the thinner outlines represent the five experimental cross-sections for each weld. In general, the predictions compare favorably with the experimental profiles. There does not seem to be any consistent error in the predictions in terms of over-predicting or under-predicting the weld pool profiles, although some shape discrepancies exist, as discussed below. Considering the experimental reproducibility, as revealed in Figures 5 and 6, the accuracy of the predictions is within the experimental variations observed.

Two discrepancies between the predicted and experimental weld pool shapes can be identified, and these are associated with differences in the experimental profiles between the first and second rounds of welds. The first welds were slightly asymmetrical about the centerline compared to the second set. This inherent asymmetry was carried over into the predicted profiles since the predicted shapes were based on the shapes in the template library, which contained profiles from only the first round. Also, the original set of welds were wider at the top of the weld, and had flanges on either side. These features were not as common in the second round of welds. However, since the predicted profiles were based on templates from the original

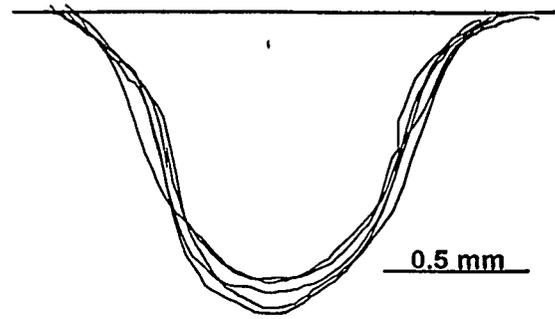


Figure 5: Typical variation in weld pool cross-section along the length of a single weld (5-7, Table 1). The top line is the nominal top surface of the sheet.

welds, these features appeared in the predictions. The presence of flanges in the predicted weld profiles was compensated by a tendency to under-predict the penetration depth, and in this way the predicted weld area was comparable to the experimental cross-section area. In theory, these discrepancies can be avoided by including the second set of welds into the training routine and also using the second set of weld profiles in the template library. This

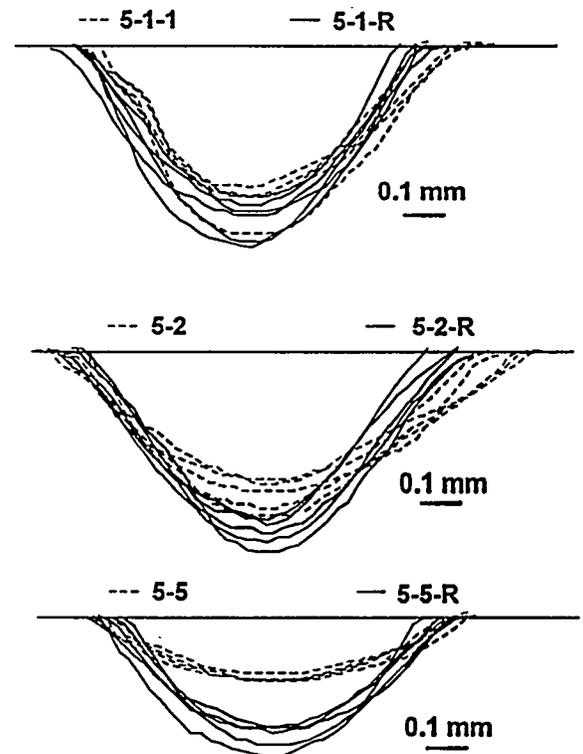
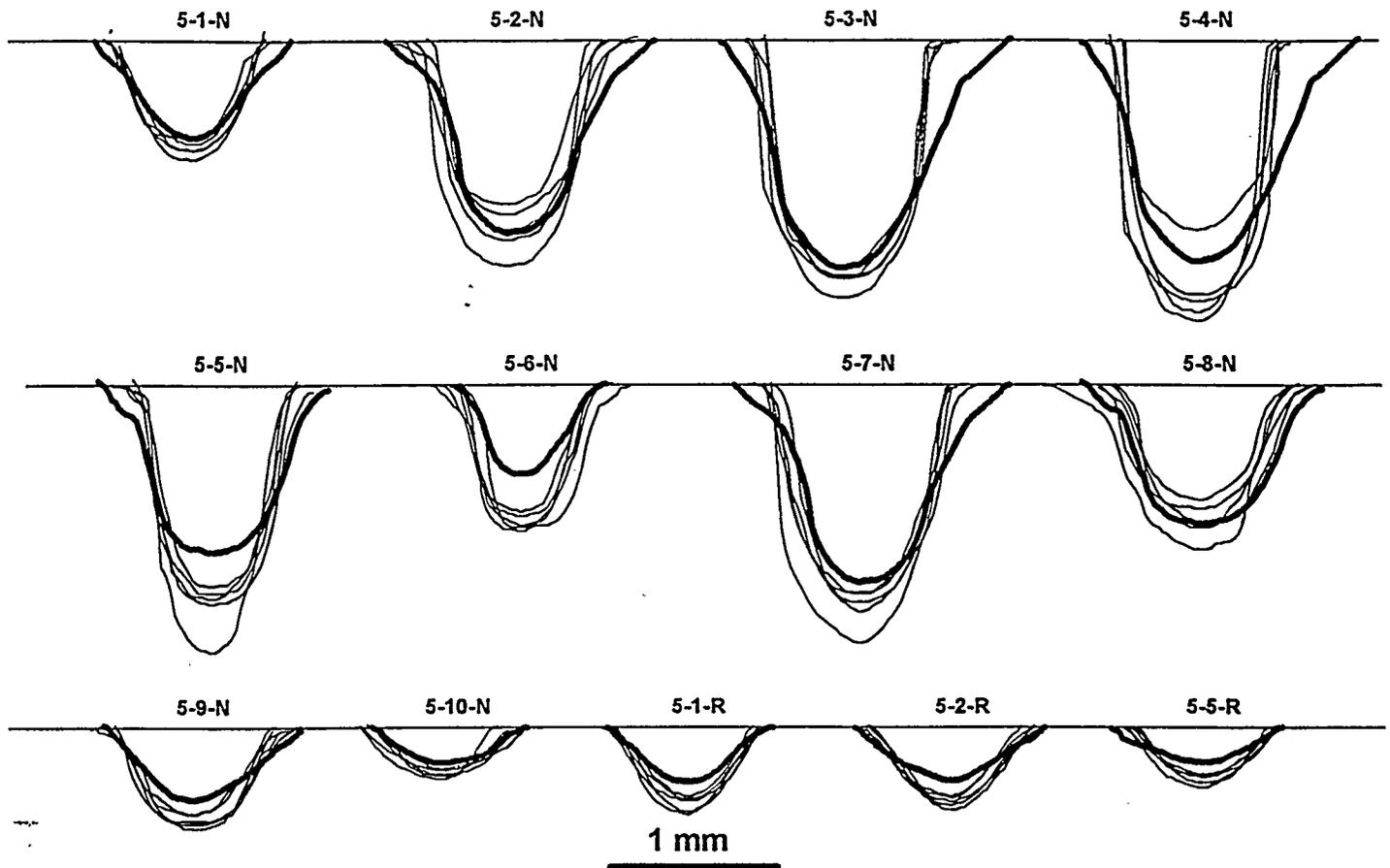


Figure 6: Superposition of weld profiles from two weld runs made under the same nominal conditions (refer to Table 1) for 5-1-1 and 5-1-R, 5-2 and 5-2-R, and 5-5 and 5-5-R.



**Figure 7:** Predicted weld pool shapes compared to experimental pool profiles. The heavy line is the predicted profile while the lighter lines are the five experimental profiles taken along the length of the weld. The same scale marker applies to all welds.

improvement is being examined in future development of the neural network.

Ultimately, the accuracy of the neural network predictions is controlled by the size of the training set of data that is available. Improved accuracy, if needed, could be achieved by using a larger training set. In the current case, this may not be warranted because of the limitations in the reproducibility of the welds.

The neural network model could be used in conjunction with other models to predict weld pool properties. If the weld profile is predicted by the network model, estimated thermal profiles can be superimposed on the weld profile, using the predicted fusion line as a known boundary condition. With the resultant spatial variation of thermal exposure, properties that are dependent on the thermal cycle can be estimated and a spatial variation of properties can be predicted.

Once the neural network is developed, trends in behavior can be readily identified. By inputting fictitious process conditions, the variation in pool profile as a function of process parameters could be determined, as long as the conditions are within the range considered in the training data set. In this manner, an "ideal" experiment in which only selected parameters are allowed to vary can be conducted and the response in terms of weld pool shape can be followed. Also, with the developed network as a starting point, other process parameters such as focus plane

position can be included in the neural network model.

The criterion that was used to identify the best matching experimental pool profile with the predicted parameters gave equal weight to each of the four pool shape descriptors. If needed, this procedure could be readily altered if one or more of the pool shape parameters is more important than the others, and needs to be predicted with higher accuracy. For example, if the penetration depth is critical, and the width or area are less important, the pool shape prediction procedure could be modified to attach more weight to the depth when reconstructing the predicted weld pool cross-section.

Finally, the primary purpose of the application of neural networks to weld pool shape prediction was to achieve a method for rapid predictive capability. With the input of the process parameters, the time needed by the neural network to predict the output is a fraction of a second. Thus, the output is basically instantaneous, as was desired. Therefore, this methodology is ideally suited for quality control and process control applications.

### Limitations of the Analysis

There are several limitations to this approach for predicting weld pool shape as a function of process parameters. First, the initial development of the neural network can be tedious. This

development includes producing and analyzing the experimental data that is to be used for the training of the network, identification of the optimal network architecture, and testing the network for accuracy. However, once a network is developed, further learning and improvement should be easier because only perturbations on the existing network would be needed.

The current study did not consider full penetration welds. Such welds may introduce additional complications, including the choice of parameters used to describe the weld pool shape. Similarly, if additional parameters are important, such as reinforcement height or undercut, then correspondingly more data would need to be processed and be available to teach the neural network. With regard to the range of parameters for which the model is appropriate, one should not lose sight of the fact that the neural network model is empirically based and, as such, is only valid over the range of variables used in the learning process. Therefore, extrapolations to conditions outside the training range may be suspect.

Finally, it needs to be kept in mind that the neural network is process and material specific. Thus, the current neural network cannot be directly applied to other aluminum welding processes or to other alloys without assessment and modification. For example, if alloy composition has a strong impact on the weld pool shape, then alloy composition would have to be treated in detail as an input to the neural network model. Thus, in stainless steels where minor additions of surface active elements have a large impact on the final weld pool shape, the concentration levels of these critical components would have to be used as input parameters, and consequently, sufficient data to cover the range of possible compositions would have to be used to train the network.

With these limitations in mind, the neural network is, nonetheless, a powerful tool for predicting weld pool shape. Furthermore, the same technology can be readily used for predicting other weld features. The present study has clearly demonstrated that the neural network approach is a viable and useful method for predicting weld pool shape characteristics.

### Summary

A neural network analysis was successfully applied to predict the weld pool shape in pulsed Nd:YAG laser aluminum alloy welds. The predictions were within the experimental variation in pool shape that was found along the length of the welds as well as in duplicate welds made under the same nominal conditions. Variable process parameters included weld speed, average power, pulse energy, and pulse duration. The weld pool shape was described by four parameters, penetration, width, width at half-penetration, and weld area. The predicted weld shape parameters were converted into predicted weld profiles utilizing the actual experimental weld pool profiles as templates. In this way, predicted weld profiles resembled the actual welds and the use of artificial analytical functions to describe the overall profile was avoided. Although the neural network was developed specifically for the alloy and range of conditions that were investigated in this study, the feasibility of successfully predicting weld pool shapes by means of neural network analysis was clearly demonstrated.

Extension of the model to other alloys and other processes with different process parameters and shape (output) parameters is straightforward. This approach to predicting weld pool shapes allows for instantaneous results and therefore offers advantages in applications where real-time predictions are needed and computationally intensive predictions are too slow. The current model is available for testing at the following web site: "<http://www.engm01.ms.ornl.gov>".

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