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Sheath Broadening in Imploding Z-pinches due to Large-bandwidth Rayleigh-Taylor Instability

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Abstract

The magnetic Rayleigh-Taylor (RT) instability has been predicted[1] and observed[2] to cause break-up of the plasma sheath in imploding z-pinches. In this work we show that, for the type of density profile encountered in strongly-radiating pinches, instability at very short wavelengths grows to the non-linear stage and seeds progressively longer wavelengths. The result is a self-similar broadening of the sheath as found for mix layers in fluid RT unstable systems[3].

Introduction

The development of RT instability in imploding pinches is intimately tied to the evolution of the density profile, as can easily be seen from the expression for the growth rate of localized modes, $\gamma = \sqrt{g \cdot \nabla \ln \rho}$ where g is acceleration and ρ is the density. From this it is apparent that the steepness of the density profile (where the acceleration and gradient are in the same direction) controls the severity of the instability. For annular pinches with radially-inward acceleration, the unstable region is in the outer portion of the plasma sheath where the density decreases with radius.

If the imploding plasma sheath remains at low pressure ($P \ll B^2/8\pi$) due to radiative cooling and if, for the moment, we ignore the effects of instability, the density profile will develop a sharp outer edge[1]. This behavior has been found in 1D radiation-MHD simulations of pinches and can be understood on simple physical grounds. Consider a time during the implosion at which the current has diffused through the edge plasma, and assume constant resistivity for simplicity. The current density in the edge will then be roughly uniform. If the scales we are considering are small compared to the pinch radius, the edge region will also have nearly uniform magnetic field (or a field that increases with radius). The JxB force density will then be roughly uniform, or radially increasing in this region. If the pressure is low, then the plasma responds to the JxB force by accelerating, with much greater acceleration in the low density portions of the plasma. The differential acceleration leads to steepening of an initially-diffuse profile, with the process continuing until the profile is so steep that, even though the pressure is low, the pressure gradient is of order the JxB force. We can relax the constant resistivity assumption, which increases the effect for plasma liners, i.e., low density plasma will ohmically heat faster and radiatively cool more slowly, so it will tend to become hotter, and in the Spitzer regime, a better conductor. The low density plasma will then carry more current and undergo even greater differential acceleration. At sufficiently low density, the plasma may not behave Spitzer-like due to anomalous resistance (e.g. from lower-hybrid drift instability), and the steepening effect will be relaxed. In the following, we neglect this effect, i.e., assume it occurs at negligibly small density.

Within a few Alfven transit times for the edge (<< implosion time, typically), a state is reached where the density falls to zero on a spatial scale $\delta = c_s^2/g$ where c_s = sound speed. For imploding loads approximately a centimeter in radius driven by the Saturn accelerator at Sandia National Laboratory[4], g is of order 10^{15} cm/sec^2 so $\delta \sim 10\ \mu m \times (ZT_e+T_i)/A = 75\ \mu m$

for a sheath with $T_e = T_i = 50$ eV, Z=5, A=40. For annular loads, the total sheath thickness is given by the resistive skin depth which is about a millimeter for the Saturn case, so the sheath tends to a profile with an outer edge an order of magnitude sharper. Recent linear theory has shown that these profiles are unstable to a surface mode that is not stabilized by resistivity[2]. There is also a simple physical picture for the absence of resistive stabilization. Consider a sharp boundary plasma sheath undergoing magnetic acceleration. If the sheath is very resistive, the current is initially uniformly distributed, however if we perturb the surface of the sheath the current density is necessarily higher where the surface curves inward and lower in the outwardly curved regions to satisfy current conservation. The higher current density region experiences greater JxB force and so drives even further inward with the converse true for the low current density region. The growth rate turns out to be the same as if the plasma were a perfect conductor. The surface modes can therefore become unstable at short wavelengths, $\lambda = 2\pi/k <<$ magnetic skin depth and even $\lambda << \delta$, with growth rate $\gamma=(kg)^{1/2}$. At very short wavelengths the mode may be stabilized by viscosity or finite gyroradius effects. For the example parameters above, and at a typical Saturn accelerating field strength of 4×10^6 Gauss, the ion gyroradius is $r_i = 2.3$ μm. Finite gyroradius effects can enter when $kr_i \sim 1$, or at wavelengths of 10 - 15 μm. Viscosity limits are of similar magnitude. Modes with $\lambda = 15$ μm will exponentiate of order 100 times during the implosion, indicating strong nonlinearity should develop.

Simulations

We have done 2D MHD simulations to study the nonlinear development of the modes, starting with a sheath in hydrodynamic equilibrium with uniform, steady acceleration and resistive equilibrium (uniform current density). The curvature of the sheath is neglected for simplicity. The model profiles are given by:

$$T = const.$$

$$\rho(x) = \rho_0\left(1+\frac{\delta}{\Delta}-\frac{x}{\Delta}-\left(1+\frac{\delta}{\Delta}\right)e^{-\frac{x}{\delta}}\right)$$

$$B = B_0\left(1-\frac{x}{\Delta}\right)$$

$$\Delta = \frac{B_0^{\,2}}{4\pi\rho_0 g} \qquad x = r_{sheath} - r$$

These profiles become invalid at $x = \Delta$, reflecting the fact that sheath solutions are inherently time dependent. They are useful, however, for studying modes with wavelengths short compared to Δ since we can vary the resistivity without affecting the unperturbed profile and mode growth occurs more rapidly than the rate of change of Δ for a time-dependent sheath solution. A square 70 x 70 mesh is used with reflecting boundary conditions in z. The code takes a Lagrangian step and the mesh is rezoned to the initial mesh each cycle, so the calculation is effectively Eulerian. The linear growth of short wavelength surface modes is observed with growth rate independent of resistivity, in agreement with the analytic theory[2]. The temperature and resistivity are held fixed, with the dimensionless resistivity appearing in linear theory, $\mu = \left(\eta c^2/4\pi\right) g/c_s v_A^{\,2}$, set to a range of values ($v_A = B_0/\sqrt{4\pi\rho_0}$). Typically $\mu \sim 1$ for Saturn implosions, so we have done simulations with $\mu = 0.1, 1.0, 10.$ to cover the relevant range. The value of $\delta/\Delta = 0.1$ was held fixed for all simulations, and the equilibrium is initially perturbed with a random velocity perturbation of peak magnitude $10^{-3}c_s$.

The non-linear development is analogous to fluid RT behavior[3]. Short wavelength modes saturate when the amplitude is a fraction of the wavelength, and drive longer wavelength modes. These, in turn, drive even longer wavelengths until the scale is of order the size of the simulation. The characteristic perturbation scale and depth of the "mix layer" where plasma and accelerating magnetic field become intermixed is roughly proportional to gt^2, with little memory of initial conditions. Variation of the resistivity over 2 orders of magnitude about the typical value for Saturn had small effect on the scaling. Figure 1 shows results for the case with $\mu=1$.

In Fig. 2, the "mix depth" is plotted for the case $\mu=1$. The mix depth is defined (for numerical convenience) as the distance one would have to "dig" into the unperturbed profile in order to produce the mass displaced to $x<0$ by the turbulent flow. Comparison with the z-averaged profiles indicates that significant deviations from the original profile extend about a factor 1.7 times the mix depth defined in this way. With this multiplier, the mix penetration depth for the light fluid (vacuum magnetic field) into the heavy fluid (plasma) scales as .026 gt^2, in comparison with .042 gt^2 found for 2D simulations of classically RT unstable, sharp boundary fluids[3]. Calculations with $\mu= 0.1$ and 10. give the same value for the mix penetration depth to accuracy similar to that shown in Fig. 2. It will be important to extend these calculations to the case where the simulation domain and mix region extend to scales larger than the sheath edge scale height, δ. Given that at long scales the system more closely resembles the classically unstable, sharp-boundary case, it seems likely that the penetration depth scaling as gt^2 will persist.

Conclusions

The 2D resistive magnetohydrodynamic calculations reported here show that density profiles expected for imploding, radiating pinches are unstable to RT modes at very short wavelengths. The short wavelengths grow to nonlinear amplitudes at early times, then stimulate longer wavelength modes resulting in self-similar growth of a mix region between plasma and the accelerating field. The behavior is similar to that of classically-unstable fluids and appears to be insensitive to the resistivity.

Since the implosion distance also scales as gt^2, one might expect the turbulently-broadened sheath width at the time of stagnation to scale proportional the initial pinch radius, e.g. sheath width ~ stagnation radius ~ 0.1 x initial radius. One complicating factor occurs when the RT growth is sufficiently large that low density "bubbles" break completely through the sheath and rapidly implode to the axis. In general, simulations including the complete geometry, self-consistent circuit, energy, magnetic field and radiation transport are required to account for the bubble-breaking and other complexities. The present work may provide a guide for establishing physically-reasonable initial perturbation levels for more complete calculations. Such calculations usually lack the resolution to follow the turbulent cascade from microscopic scales early in time to the macroscopic scale flows that are well resolved.

[1] T.W. Hussey, N.F. Roderick, and D.A. Kloc, J. Appl. Phys. **51**, (1980) 1452.
[2] J.H. Hammer, et.al., Phys. Plasmas **3**, (1996) 2063.
[3] D.L. Youngs, Physica **12D**, (1984) 32.
[4] J.H. Hammer, S. Maxon, M. Tabak, K. Estabrook, J. L. Eddleman, C.W. Hartman, A. Toor , G. B. Zimmerman and J.S. De Groot, Bull. Am. Phys. Soc. 39, (1994) 1605.

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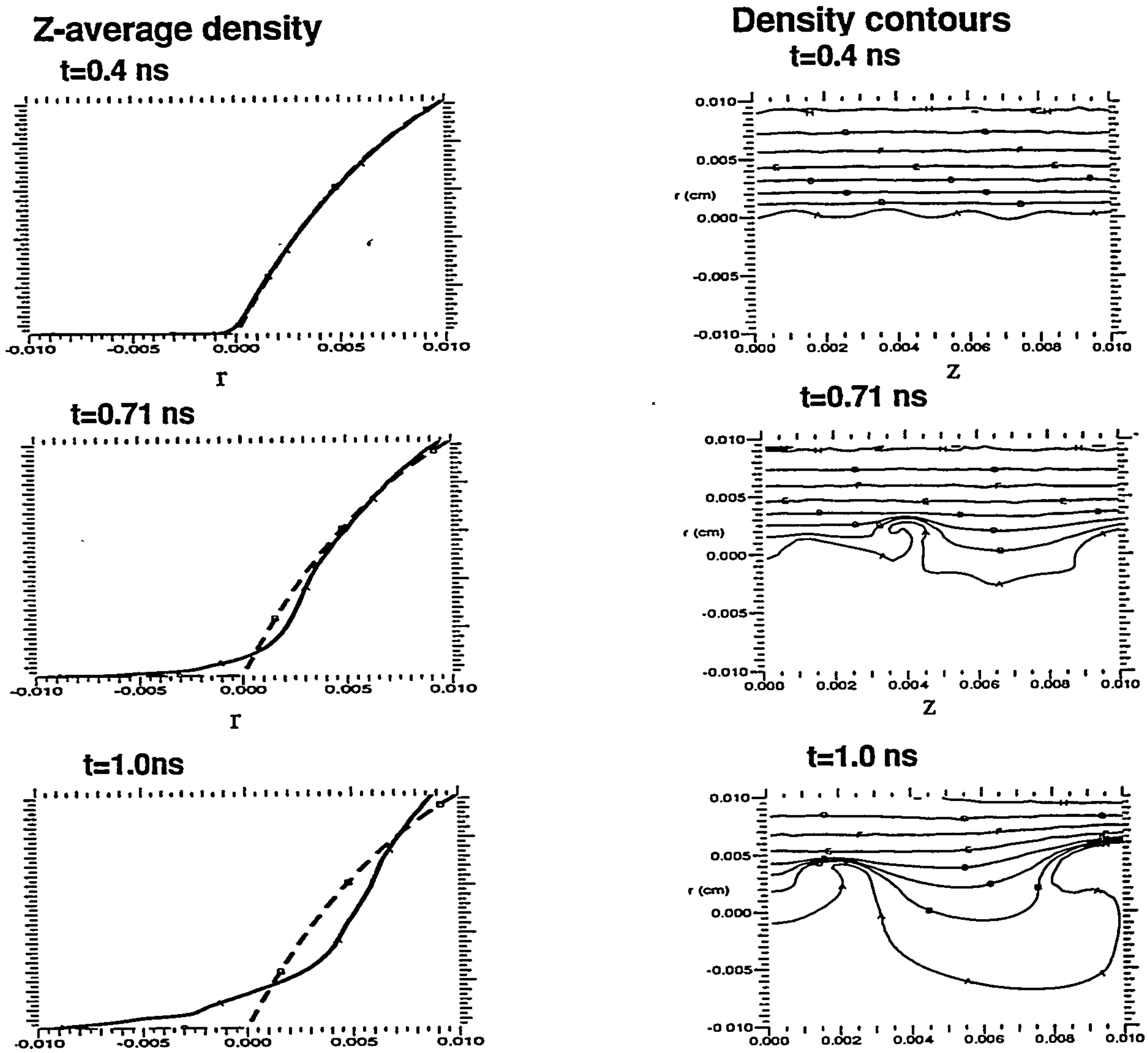
Z-average density
t=0.4 ns
t=0.71 ns
t=1.0ns
-0.010
-0.005
0.000
0.005
0.010
r
Density contours
t=0.4 ns
t=0.71 ns
t=1.0 ns
r (cm)
0.000
0.002
0.004
0.006
0.008
0.010
Z

Fig. 1

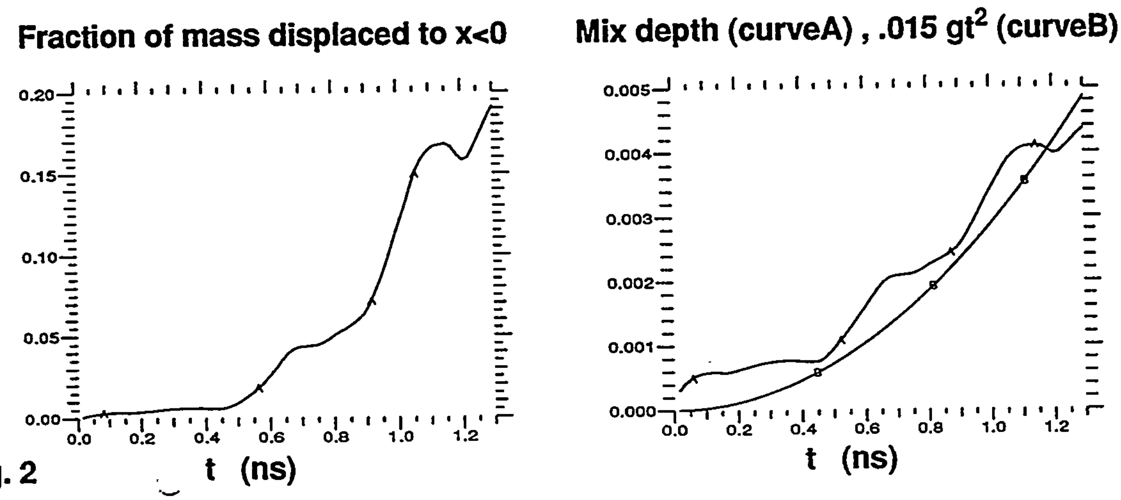
Fraction of mass displaced to x<0
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0
1.2
t (ns)
Mix depth (curveA) , .015 gt2 (curveB)
0.005
0.004
0.003
0.002
0.001
0.000
t (ns)

Fig. 2