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SEMI-LEPTONIC FORM-FACTORS FROM LATTICE QCD*

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ABSTRACT

We present results for semi-leptonic form-factors obtained on a statistical sample of 63 $32^3 \times 64$ lattices at $\beta = 6.0$ using quenched Wilson fermions. We find $f_+^{D\to Kl\nu}(q^2 = 0) = 0.73 \pm 0.06$, $A_2/A_1(D \to K^* l\nu) = 0.79 \pm 0.23$, $V/A_1(D_s \to \phi l\nu) = 1.89 \pm 0.04$, and $A_2/A_1(D_s \to \phi l\nu) = 0.70 \pm 0.09$, where the error estimate includes statistical errors and errors due to extrapolation to $q^2 = 0$ and to physical values of $(m_u + m_d)/2$ and m_s. The remaining sources of systematic errors are those due to $O(a)$ discretization errors and those due to quenching, which our results indicate may be small. We also comment on the validity of pole-dominance in these form-factors.

1. INTRODUCTION

Exclusive semi-leptonic decays of D and B mesons provide the cleanest measurements of the CKM quark mixing matrix. For example, the decay rate for $D \to Kl\nu$,

$$\frac{d\Gamma^{D\to Kl\nu}}{dq^2} = \frac{G_F^2}{24\pi^3} \left|V_c s\right|^2 p_K^3 \; f_+^2(q^2), \tag{1}$$

depends on kinematic factors, a single CKM matrix element V_{cs}, and the form factor $f_+(q^2)$. To extract CKM matrix elements from such processes requires non-perturbative calculations of the form-factors as they encapsulate all strong interaction effects. In this talk we report on results obtained from numerical simulations of lattice QCD.

2. LATTICE PARAMETERS

The results presented here have been obtained using the following lattice parameters. The $32^3 \times 64$ gauge lattices were generated at $\beta = 6.0$ using the combination 5 over-relaxed (OR) sweeps followed by 1 Metropolis sweep. Quark propagators are calculated on lattices separated by 2000 OR sweeps using the simple Wilson action. Periodic boundary conditions are used in all 4 directions, both during lattice update and propagator calculation. Quark propagators have been calculated using one version of Wuppertal smeared sources at $\kappa = 0.135$ (C), 0.153 (S), 0.155 (U_1), 0.1558 (U_2), and 0.1563 (U_3). These quark masses correspond to pseudoscalar mesons of mass 2800, 980, 700, 550 and 440 MeV respectively using $1/a = 2.25(10) GeV$ set by m_ρ. On each of the 63 configurations we make two independent measurements of the form-factors, which we average before doing the statistical analysis using the jackknife method.

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*Talk presented by R. Gupta at DPF94.

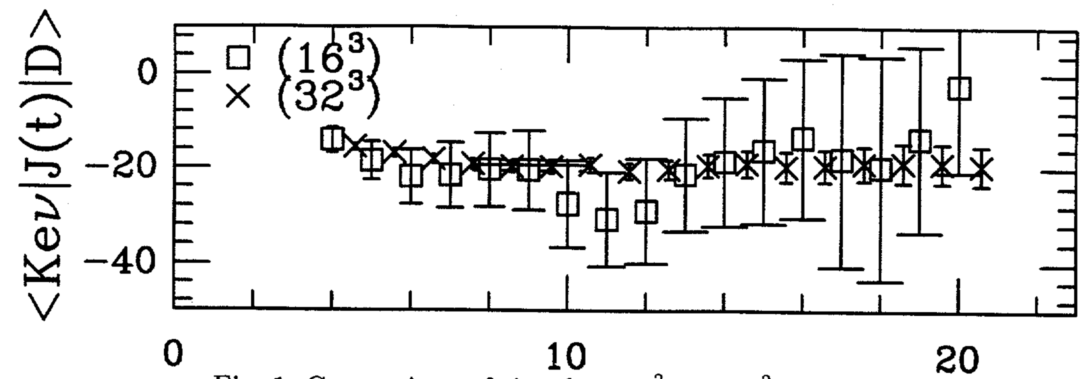

Fig. 1. Comparison of signal on 16^3 and 32^3 lattices.

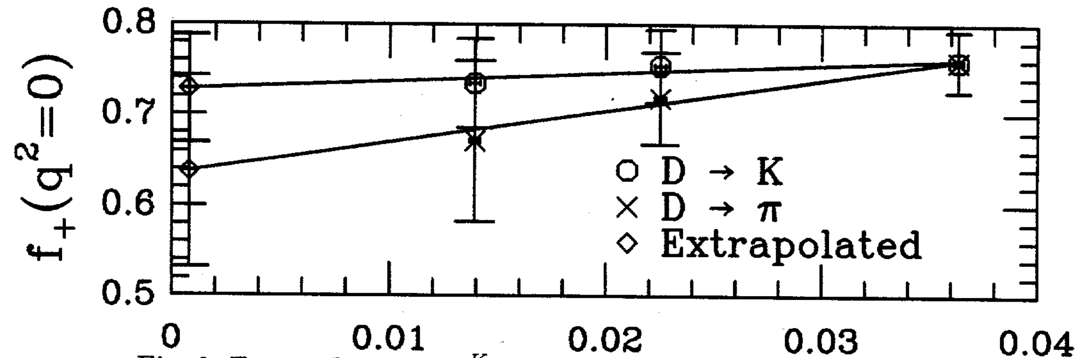

Fig. 2. Extrapolation of f_+^K and f_+^π in the light quark mass $\bar{m}a$.

Our procedure for extracting form-factors is very similar to that proposed by Lubicz *el al.*,[1] and a detailed paper is under preparation. The D meson is created at $\vec{p} = (0,0,0)$ and the momentum inserted by the current is carried by the final kaon. The five values of momenta analyzed are $\vec{p} = (0,0,0)$, $\vec{p} = (1,0,0)$, $\vec{p} = (1,1,0)$, $\vec{p} = (1,1,1)$, and $\vec{p} = (2,0,0)$ in units of $\pi/16a$. These correspond to roughly 0, 440, 625, 765, and 880 MeV respectively.

The use of large lattices to study form-factors leads to a dramatic improvement in reliability. In Fig. 1 we show a comparison of the signal in $\langle K \mid v_i \mid D \rangle$ with $\vec{p} = (\pi/8a, 0, 0)$ for our current data set (126 measurements) with a previous study using 35 16^3 lattices. The reduction in errors by a factor of ≈ 5 is consistent with the increase in statistics and lattice volume. In addition, the larger lattice allows measurements at three smaller values of non-zero momentum transfer, for which the signal is even better. These points bracket $q^2 = 0$ and allow a reliable extraction of $f(q^2 = 0)$, which we do in two ways. Our best fit uses a two parameter fit to the pole-dominance ansatz $f(q^2) = f(0)/(1 - q^2/\mathcal{M}^2)$. In the second method we fix the pole mass $\mathcal{M}$ to its lattice measured value. The relative merits of the two methods are discussed below.

We take $\kappa = 0.135$ as the physical charm quark. The ratio m_π^2/m_K^2 fixes the strange quark at $\kappa = 0.1550(2)$. The three light quarks $U_1 - U_3$ are used to extrapolate the $q^2 = 0$ data to the physical value of $\bar{m} \equiv (m_u + m_d)/2$ (fixed by the experimental ratio m_π^2/m_ρ^2) assuming that the form-factors depend linearly on the light quark mass. For example, the extrapolation of f_+ is shown in Fig. 2. Also, to calculate final ratios of form-factors, the ratios are taken at the very beginning of the jackknife process.

	$f_+(q^2=0)$			$f_0(q^2=0)$	
	(a)	(b)	EXPT.	(a)	(b)
$D \to Kl\nu$	0.73(6)	0.81(3)	0.77(4)	0.73(4)	0.72(2)
$D \to \pi l\nu$	0.64(11)	0.75(4)		0.64(6)	0.64(3)
$(D \to \pi l\nu)/(D \to Kl\nu)$	0.88(6)	0.93(2)	$1.29 \pm 0.21 \pm 0.11$	0.88(4)	0.88(2)

Table 1. Form factors, f_+ and f_0, extracted using (a) best fit and (b) lattice pole mass.

	V		A_1		A_2	
		exp.		exp.		exp.
$D \to K^*l\nu$	1.21(8)	1.16(16)	0.65(3)	0.61(5)	0.46(19)	0.45(9)
$D \to \rho l\nu$	1.04(12)		0.55(4)		0.16(24)	
$D_s \to \phi l\nu$	1.28(5)		0.66(1)		0.47(8)	

Table 2. Estimates for vector form factors from pole fits.

3. FINAL RESULTS

The final results for the decay $D \to Kl\nu$ are given in Table 1. Present errors preclude a serious test of the pole-dominance hypothesis even though the best fit value for $\mathcal{M}_{1^-}$ is about $10-20\%$ below the mass measured on the lattice and $20-30\%$ below the known experimental values. Since f_+ is known from experiments,[2] one can regard the lattice measurements as providing a measure of systematic errors due to quenching and lattice discretization that we cannot otherwise estimate. The data in Table 1 suggest that these are small, *i.e.* at the 10% level. Our data show that $(D \to \pi l\nu)/(D \to Kl\nu) = 0.88(6)$, a value at the lower end of the experimental estimate.[2]

In the case of the vector final states we find that the estimates for A_2 are not very stable for $\vec{p} = (1,1,1)$ and $(2,0,0)$, making a two free parameter fit unreliable. However, the point $\vec{p} = (1,1,0)$ lies very close to the desired limit $q^2 = 0$, and using this value as a consistency check, we find that the pole fits are reasonable. For V and A_1 the two kinds of fits give consistent estimates, therefore in Table 2 we give pole fit results as our best estimates for all three form-factors. The results for $D \to K^*l\nu$ are in surprisingly good agreement with the averaged experimental values.[2] The form factors for $D \to \rho l\nu$ are consistently smaller and we find little difference, qualitatively or quantitatively, between the two final states K^* and ϕ. The experimental errors in $D_s \to \phi l\nu$ are too large (see summary talk by Janis McKenna in these proceedings) to make a meaningful comparison.

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