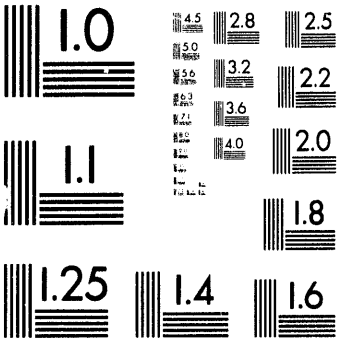
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EVIDENCE FROM MODULATED ECH FOR CONVECTIVE-LIKE TRANSPORT

by

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EVIDENCE FROM MODULATED ECH FOR CONVECTIVE-LIKE TRANSPORT*

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A theoretical understanding of cross-field transport in magnetized plasmas remains elusive despite enormous effort. In part, this is due to the absence of a diagnostic which measures directly a transport coefficient, such as energy diffusivity. One obvious technique to observe transport is to impose a perturbation and observe the dynamic response of the plasma. Modulated electron cyclotron heating (ECH) is ideal for probing energy transport since it can be well-localized in space and time, and the energy is deposited into the thermal electrons on a time scale much shorter than the characteristic transport time scales. Coherent addition of periodic perturbations by Fourier analysisallows detection of perturbations down to the 1 eV level. Use of Fourier techniques also lessens the computational difficulties by reducing the equations back to ordinary differential equations, assuming that the transport coefficients are not functions of time. This simplification of the model also leads to a better conceptual understanding of the capabilities of the technique through the development of analytic solutions in limiting cases.

EQUILIBRIUM AND DYNAMICAL EQUATIONS

The starting point for the transport analysis presented here is a conventional treatment of the plasma as a collection of fluids — electron, main ions, and impurity ions. The observables are moments of the single particle velocity space distribution for each species. These observables obey conservation equations derived by taking moments of the Boltzmann equation. The focus of this paper is the electron energy equation, but an analogous treatment of any of the moment equations is possible. The electron energy conservation equation is:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}\,nT\right) + \nabla\cdot\left(\frac{5}{2}\,\Gamma T + q\right) = Q \quad , \tag{1}$$

where n and T are the electron density and temperature respectively. The equation is quite simple conceptually; Q is a local source (or sink, if negative) which must be balanced either by the local change in energy (first term) or the transport of energy (second term). The quantity $\frac{5}{2}\,\Gamma T + q$ is the energy flux, where $\frac{5}{2}\,\Gamma T$ represents the energy carried by particle

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flux and q is the heat flux. The transport coefficients are defined by choosing a form for the heat flux. A quite general form will be used:

$$q = -n\chi\,(r,\,T,\,\nabla T)\,\nabla T + n\,U\,(r,\,T)\,T + \xi\,(r) \quad , \tag{2}$$

where χ is the diffusivity, U is a convective velocity, and ξ is a contribution to the heat flux which has no temperature dependence. Energy "convection" by some means other than particle transport sounds contradictory, but standard cross terms such as energy fluxes driven by density gradients appear in this form. The nomenclature is adopted because of the mathematical similarity with true convection without any underlying physical implication about the transport mechanism.

To obtain an equation which describes the dynamical behavior of an energy perturbation, the simplifying assumption that no density or particle flux perturbation occurs is made. This assumption is well-justified for modulated ECH and is consistent with the DIII–D data. For a perturbation $\tilde{T}$ much smaller than the equilibrium temperature T_0, a dynamical equation for $\tilde{T}$ is found by linearizing Eq. (1):

$$\frac{\partial \tilde{T}}{\partial t} = D\nabla^2\tilde{T} - V\nabla\tilde{T} - \frac{\tilde{T}}{\tau} + S \quad . \tag{3}$$

Only the radial derivatives of the temperature are important, but the ∇ notation is retained to account for the effects of geometry without obscuring the effects of the various dependences of the transport. The right-hand of Eq. (3) has an "effective" diffusivity D, convective velocity V, and damping time τ. S is the normalized modulated source. These effective transport coefficients can be related back to the equilibrium transport coefficients defined in Eq. (2):

$$D = \frac{2}{3}\left(\chi + \frac{\partial\chi}{\partial\nabla T}\,\nabla T_o\right) \quad , \tag{4}$$

$$V \equiv \frac{2}{3}\left(U - \frac{\partial\chi}{\partial T}\,\nabla T_o + \frac{\partial U}{\partial T}\,T_o\right) - \frac{2}{3}\left(\chi + \frac{\partial\chi}{\partial\nabla T}\,\nabla T_o\right)\frac{\nabla n}{n}$$
$$- \frac{2}{3}\,\nabla\left(\chi + \frac{\partial\chi}{\partial\nabla T}\,\nabla T_o\right) + \frac{\Gamma}{n} \quad ,$$

$$\frac{1}{\tau} \equiv \frac{Q_{\rm OH}}{nT_o} + \frac{Q_{\rm ei}}{3nT_o}\left(\frac{T_o - 3T_i}{T_o - T_i}\right) + \frac{5}{3}\,\frac{\nabla\cdot\Gamma}{n} - \frac{2}{3}\,\frac{\Gamma\nabla n}{n^2}$$
$$+ \frac{2}{3}\,\frac{\nabla n}{n}\left(U - \frac{\partial\chi}{\partial T}\nabla T_o + \frac{\partial U}{\partial T}T_o\right) + \frac{2}{3}\nabla\cdot\left(U - \frac{\partial\chi}{\partial T}\nabla T_o + \frac{\partial U}{\partial T}T_o\right) \quad ,$$

$$S \equiv \frac{2}{3}\,\frac{\tilde{Q}_{\rm ECH}}{n} \quad .$$

It is clear now why these quantities which appear in the linearized equation are called "effective" transport coefficients. They play the role of diffusivity, convective velocity, or damping mathematically, but their physical origin may be completely different. For example, even if V and ξ are identically 0, a convective behavior appears in the dynamics of a heat pulse due to a spatial gradient in either density or diffusivity.

ANALYTIC SOLUTIONS

A great deal of physical insight into the effects of these effective transport coefficients on the dynamics of the heat pulse can be gained by solving the equation in slab geometry in simple limiting cases. The simplest possible case is for $V = 1/\tau = 0$ and D constant in space and time. Assuming a periodic perturbation in time which is spatially localized, the equation governing the evolution of the heat pulse in the source-free region is simply $i\omega\tilde{T} = D\ \nabla^2\tilde{T}$. The solutions are $\tilde{T} \propto e^{kx}$ where

$$k = \pm(1+i)\sqrt{\frac{\omega}{2D}} \quad . \tag{5}$$

The sign indicates the direction in which the pulse is moving. These solutions can be written in the form $\tilde{T} = A(x)e^{i\phi(x)}$ where $A(x)$ is the variation of the amplitude with position and $\phi(x)$ is the variation of the phase with position which are the experimentally measured quantities after a Fourier transform in time. Notice that $d\phi/dx = Im(k)$ and $A'/A = Re(k)$. The effective diffusivity D can be determined from either measurement of the phase gradient or the amplitude scale length:

$$D = \frac{\omega}{2}\Big/\left(\frac{d\phi}{dx}\right)^2 = \frac{\omega}{2}\ L_{\hat{T}}^2 \quad . \tag{6}$$

Analytic solutions can be obtained for the case of constant V and $1/\tau$. The solutions are still of the form $\tilde{T} \propto e^{kx}$, where now

$$k = \frac{V}{2D} \pm \Omega\ (\cos\Theta + i\sin\Theta) \quad , \tag{7}$$

$$\Omega \equiv \left(\frac{\omega}{D}\right)^{1/2}\left[\left(\frac{V^2}{4\omega D} + \frac{1}{\omega\tau}\right)^2 + 1\right]^{1/4} \quad ,$$

$$\Theta \equiv \frac{1}{2}\tan^{-1}\left[1\Big/\left(\frac{V^2}{4\omega D} + \frac{1}{\omega\tau}\right)\right] \quad .$$

The dimensionless quantity $\omega\tau$ is an indicator of the importance of damping; if $\omega\tau \to \infty$, then damping is not important. The dimensionless quantity $V^2/4\omega D$ gives the relative importance of convection. If $V^2/4\omega D \to 0$, then convection is not important. It is instructive to evaluate the purely diffusive model estimators for D:

$$D_{ph} \equiv \frac{\omega}{2}\Big/\left(\frac{d\phi}{dx}\right)^2 = D\left[1 + \left(1 + \frac{1}{\alpha^2}\right)^{1/2}\right]\alpha \quad , \tag{8}$$

$$D_{amp} \equiv \frac{\omega}{2}\ L_{\hat{T}}^2 = D\ \left\{\frac{V^2}{2\omega D} \pm \frac{2V^2}{\omega D}\ \left[(1+\alpha^2)^{1/2} + \alpha\right]^{1/2} + (1+\alpha^2)^{1/2} + \alpha\right\}^{-1} \quad ,$$

$$\alpha \equiv \frac{V^2}{4\omega D} + \frac{1}{\omega\tau} \quad .$$

If $V^2/4\omega D \rightarrow 0$, D_{ph} and D_{amp} behave as shown in Fig. 1. In the case of damping alone, these estimates of D are independent of the direction of the heat pulse motion. In the case $\omega\tau \rightarrow \infty$, this symmetry is broken as shown in Fig. 2. The + sign in D_{amp} in Eq. (8) is for a pulse moving in the direction opposite to the convective velocity. Therefore, the ideal experiment is an off-axis perturbation where both the amplitude and phase information are available for pulses moving both directions.

The intuition gained by this analytical analysis can be shown to carry over to realistic geometries as long as the modulation source is localized and well away from the boundaries and to cases where the coefficients are functions of space. While these simple estimators of D [Eq. (6)] are not in general valuable to evaluate D, they are clearly useful to determine from the Fourier transformed data the character of the equation, i.e., whether convective-like or damping-like terms are important. This analysis clearly shows that there are many reasons why a diffusivity determined from purely diffusive perturbation analysis should be significantly different from a power balance diffusivity. It is exactly this difference which makes the perturbative techniques a powerful tool for probing transport when properly applied.

EXPERIMENTAL RESULTS

The results of the previous section will be applied to two sets of ECH modulation data from the DIII–D tokamak — a "standard" NBI-heated L–mode discharge and an off-axis ECH discharge where the electron flux is approximately zero at radii smaller than the resonance radius. The DIII–D ECH system has been described in detail elsewhere.[1] All that is important here is that the beam pattern provides a well-localized source ($\Delta\rho < 0.1$) and the single-pass absorption is greater than 80%. The heat pulses are detected by two soft x–ray (SXR) arrays; one images a full horizontal view of the plasma with 32 diodes and the other images a nearly vertical view of the plasma from the outer to the inner edge with 16 diodes. An inversion technique is applied to both the equilibrium and the co- and quadrature-spectra of the Fourier harmonics to provide a local perturbation amplitude and phase. These quantities are then

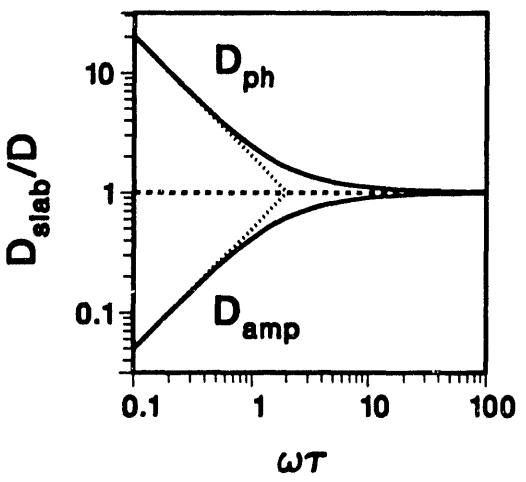

Fig. 1. The effect of damping on D_{ph} and D_{amp} as a function of $\omega\tau$. The dotted lines are the asymptotic solutions as $\omega\tau \rightarrow 0$.

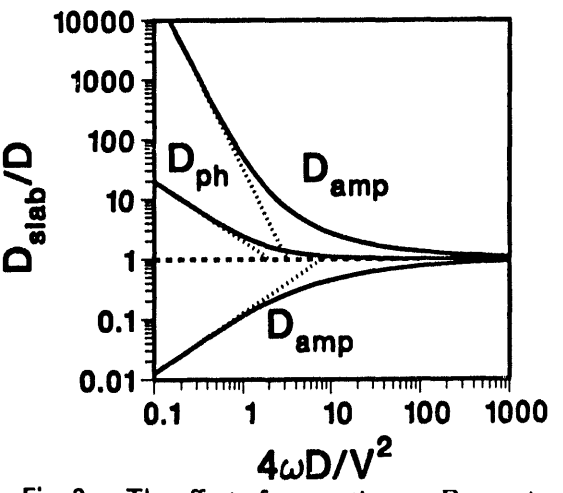

Fig. 2. The effect of convection on D_{ph} and D_{amp} as a function of $V^2/4\omega D$. The dotted lines are the asymptotic solutions as $V^2/4\omega D \rightarrow \infty$.

projected back onto magnetic flux surfaces. While the SXR intensity is dominated by changes in temperature, it is also a function of density. A local density measurement with sufficient time resolution to Fourier analyze is not available on DIII–D, but analysis of line-integrated density measurements detect no density perturbation.[2] Therefore, the modulation in the SXR intensity is assumed to be proportional to $\tilde{T}$.

The phase trajectories of the heat pulse at the fundamental frequency and sixth harmonic in a "standard" L–mode NBI-heated discharge are shown in Fig. 3. Notice that the phase gradient $d\phi/d\rho$ scales linearly with ω; therefore, D_{ph} scales like $1/\omega$. This is borne out by a series of similar discharges where the modulation frequency is varied from 25 to 80 Hz (see Fig. 4). This is exactly as predicted by Eq. (8). In general, the phase data alone cannot distinguish between V and $1/\tau$; however, the known sources of $1/\tau$ — the perturbed sources and particle convection terms — cannot account for the observed enhancement in D_{ph}. This constitutes indirect but credible evidence of an effective convection in the linearized equation.

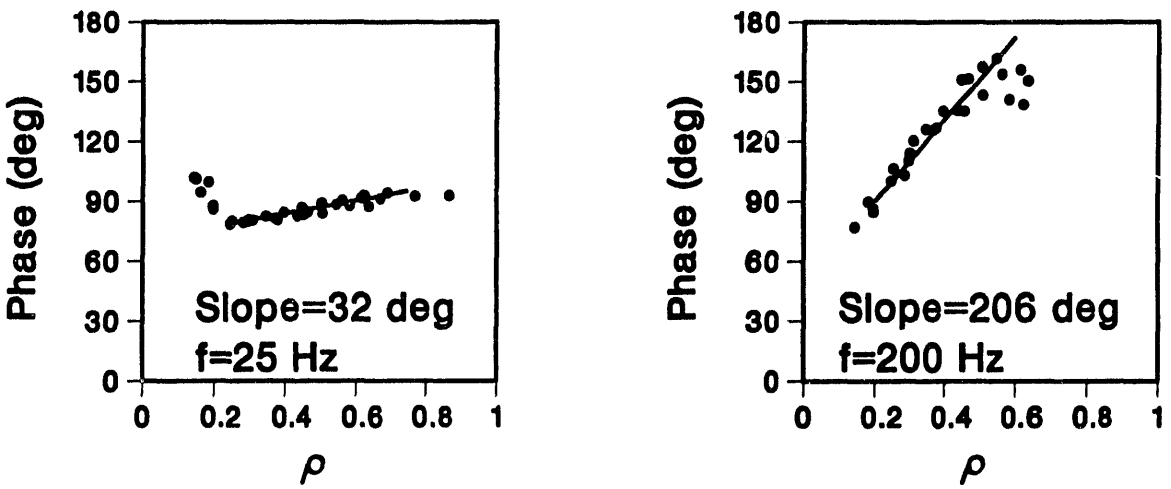

Fig. 3. Heat pulse phase referenced to the ECH signal versus ρ for the fundamental and sixth harmonic. The phase gradient is approximately linear in f.

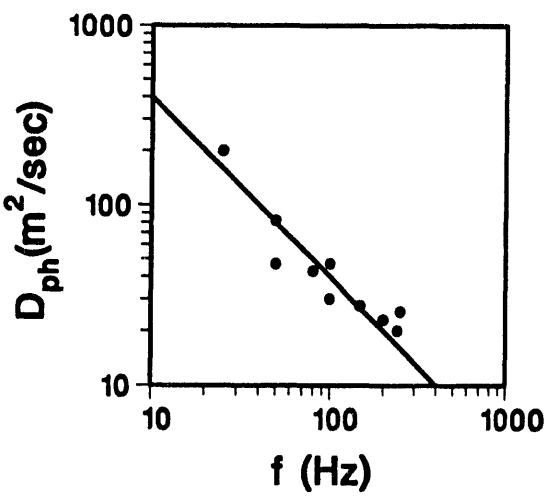

Fig. 4. D_{ph} versus modulation frequency for three similar discharges with f = 25, 50, and 80 Hz.

Direct evidence of an effective convection is found in the off-axis ECH discharge. In higher power, higher density discharges of this type, power balance analysis indicates a net inward energy flux in the electron fluid at radii smaller than the resonance radius.[3] The amplitude and phase at the fundamental frequency as a function of ρ are shown in Fig. 5. Also shown are the inverted local amplitude and phase. The amplitude scale lengths are clearly different for the inward- and outward-going pulses and indicate an inward effective convection. This is in agreement with the power balance which also requires an inward energy flux. A term of the form $U \propto 1/L_n$ such as is found in neoclassical theory and many drift wave models would satisfy the data. Further analysis along these lines is in progress.

SUMMARY

A general linearization of the electron energy balance equation shows that the transport coefficients in the linearized equation are not equivalent to those in the equilibrium equation, but are related in a well-defined manner. A simple analytic analysis of the linearized equation shows that estimates of the diffusivity not accounting properly for the effective convection and damping can be in error by orders of magnitude. This is seen experimentally in both a "standard" NBI-heated L–mode discharge and an off-axis ECH discharge. Both cases indicate the presence of a significant effective convection in the linearized equation. In the case of the off-axis ECH, this corroborates the power balance analysis, which requires an inward energy flow to support the observed gradients.

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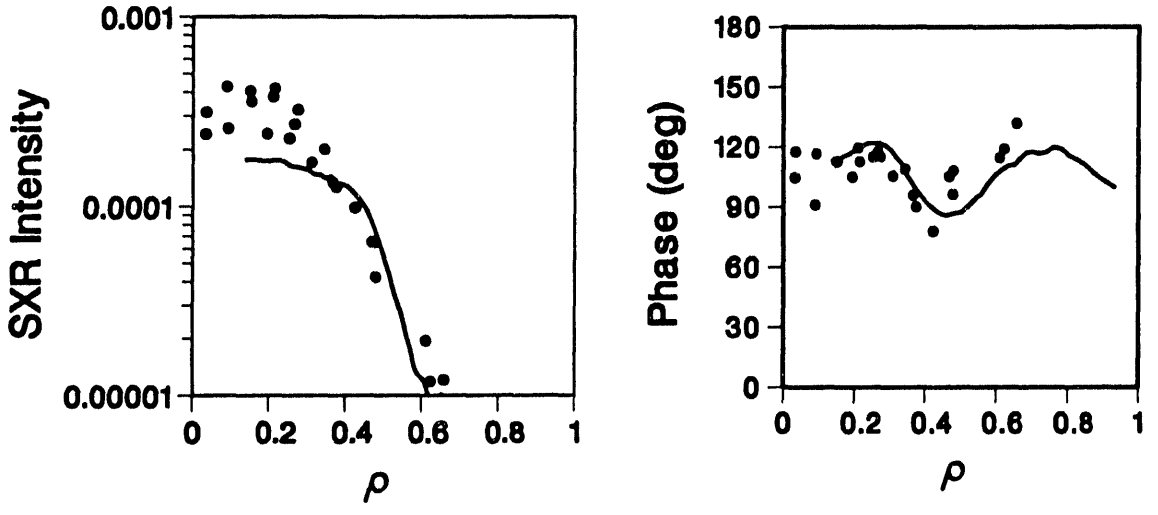

Fig. 5. The line-averaged (symbols) and inverted (lines) SXR amplitude and phase for the fundamental frequency during off-axis ECH. The phase is symmetric about the heating location, but the amplitude scale length is asymmetric indicating convection.

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