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Statistical Gamma Transitions in ^{174}Hf

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Abstract: Statistical spectrum extracted from the ^{172}Yb(α,2n)^{174}Hf reaction was fit with Monte Carlo simulations using a modified GDR E1 strength function and several formulations of the level density.

I. Introduction

After formation in a fusion-evaporation reaction, the compound nucleus decays by a combination of three types of gamma-transitions:[1] (i) statistical transitions, which remove excitation energy over the yrast line, but not much spin; (ii) quasi-continuum, collective, stretched, dipole and quadrupole transitions, which remove angular momentum, but not much energy; and (iii) discrete transitions, deexciting known, near-yrast levels. The quasi-continuum, quadrupole transitions go down collective bands that are approximately parallel to the yrast line, and form a bump on the continuum spectrum at $E_\gamma \approx 1$ MeV in rare-earth nuclei formed at high spin, as in (heavy-ion,xn) reactions. The statistical spectrum depends on the average gamma-transition strength and the level density, and we hope to extract data on these quantities. An accurate determination of the statistical spectrum is also important to studies of the E2 quasi-continuum spectrum, for the accurate removal of the statistical background.

II. Experiment

In order to isolate the statistical spectrum, it is necessary to populate the target nucleus with low angular momentum, minimizing the yield of quasi-continuum quadrupoles. The ^{172}Yb(α,2n)^{174}Hf reaction was used with a beam of 28 MeV α particles from the Notre Dame Tandem Van de Graaff accelerator. Reaction γ rays were recorded using the University of Pittsburgh Gamma Array, which consists of 6 Compton suppressed Ge detectors and 14 BGO scintillators. The Ge detectors were in-plane: two at 30°, two at 150°, one at 90° and one at 105° relative to the beam direction. Thirty million γ-γ coincidence events were recorded up to $E_\gamma = 5$ MeV. In addition, response functions of the Ge detectors were measured using 14 radioactive gamma sources with energies ranging from 0.122 to 2.754 MeV. The total projected gamma-coincidence spectrum (Fig. 1), shows a smoothly decreasing exponential, with no broad structure at 1 MeV, which is expected in a fusion reaction with heavy ions.

III. Extraction of Statistical Spectrum

In order to extract the continuum spectrum from the data, the following procedure was used. For each detector, spectra were gated on the $4^+\rightarrow2^+$, $6^+\rightarrow4^+$, and the $8^+\rightarrow6^+$ lines in order to select only ^{174}Hf transitions. The lines from the (n,n'γ) reactions were then fitted and subtracted out. The background from Compton-scattered γ rays was removed by the unfolding procedure[2] of Radford, et al. After their Compton spectra were removed, the discrete lines were fitted and subtracted from the unfolded spectrum. The remaining spectra for the six detectors were corrected for the relative efficiencies of the individual detectors as a function of E_γ. The intensity as a function of angle was assumed to follow a Lengendre polynomial, with the coefficient of $P_4(\cos\theta)$, $A_4 = 0$. The A_0 and A_2 coefficients can then

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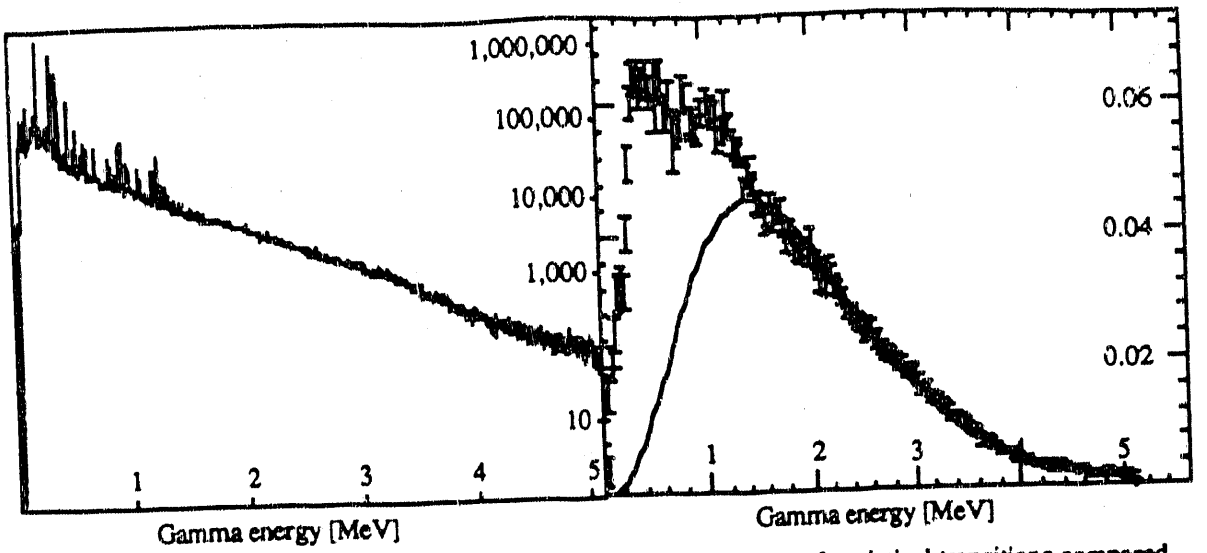

Fig.1. Total projected spectrum.

Fig.2. Spectrum of statistical transitions compared to single transition prediction (solid line)

be determined from the spectra at two angles. Figure 2 displays the isotropic portion of the continuum spectrum, normalized to the $2^+ \rightarrow 0^+$ total intensity.

To determine the average energy and angular momentum at which the statistical gamma cascade is entered and exited, it is first assumed that all cascades go through the $2^+ \rightarrow 0^+$ transition, so that the total number of cascades, N_c, is given by the $2^+ \rightarrow 0^+$ intensity, corrected for conversion. The average multiplicity, $\langle m_\gamma \rangle$, for each transition is determined by dividing its intensity by N_c. For discrete lines, the fitted intensity is used, and for the statisticals, the number of counts in the energy bin is used. The energy per cascade removed by a transition is given by $E_\gamma \langle m_\gamma \rangle$, where E_γ is the fitted peak energy for discrete transitions, and the average energy of a bin for the statisticals. The angular momentum removed is $\Delta I_\gamma \langle m_\gamma \rangle$. ΔI_γ is known for most of the discrete transitions, and is assumed to be 0.5 $\hbar$ for the statistical transitions. Table 1 gives the calculated total values.

IV. Calculation of the Statistical Spectrum

The spectral distribution, $\nu(E_\gamma)$, of primary γ rays is proportional to the average gamma-transition probability and the level density. If the statistical gammas were only electric dipole in nature[3], then

$$\nu(E_\gamma) \propto E_\gamma^3 f_{E1}(E_\gamma) \sum_{I_f} \rho(E_i - E_\gamma, I_f)/\rho(E_i, I_i) \qquad (1)$$

where f_{E1} is proportional to the average B(E1) value, ρ is the level density, i is the initial state, f is the final state, and I is the spin. In general, the spectral distribution would be composed of a sum of contributions of different multipolarities of both electric and magnetic transitions. For simplicity, in the present study it was assumed that the statistical cascade is composed of electric dipole transitions only, since these are the strongest in single particle

Table 1. Multiplicities of transitions, and energy and angular momentum removed.

	discrete	statistical	total
$\langle m_\gamma \rangle$	4.202 ± 0.475	4.262 ± 0.018	8.464± 0.493
E_γ [MeV]	1.680 ± 0.131	6.565 ± 0.121	8.245± 0.252
I_γ [$\hbar$]	6.705 ± 0.853	2.146± 0.009	8.851± 0.862

estimates. We take f_{E1} to be dominated by the giant E1 resonance, and that it exhausts the E1 sum rule[4,5], so that

$$f_{E1} = 50NZA^{-5/3}(1 + 0.8x)\, E_\gamma \Gamma_G \left\{(E_\gamma^2 - E_G^2)^2 + (\Gamma_G E_\gamma)^2\right\}^{-1} \qquad (2)$$

where x = 0.5 is the fraction[6] of exchange forces in the nuclear force. The energy and width of the GDR are estimated by $E_G \simeq 80\ A^{-1/3}$ MeV, which correspond to $E_G \simeq 14.6$ MeV for ^{174}Hf, and $\Gamma_G \simeq 5$ MeV, respectively. This expression matches the measured strength function from the photoabsorption spectrum, but overestimates the strength at low energies[7] ($E_\gamma < 5$ MeV) by about 30%.

Kadmenskii, Markushev, and Furman[8] (KMF) address this by including quasiparticle fragmentation in a microscopic calculation of the strength function. The result is to multiply f_{E1} at low E_γ by a factor

$$g = 0.7(E_\gamma^2 + 2U_f/3g(\varepsilon_F))/E_\gamma E_G \qquad (3)$$

where U_f is the energy of the final state of the transition and the density of single particle states at the Fermi energy is $g(\varepsilon_F)$. Since eq.3 fits the measured strength function[8], the KMF formulation of the E1 strength function is adopted for this work.

The other major component in eq.1 is the level density. The effect of pairing on the level density is to reduce the number of states below the pairing gap to zero, but this effect decreases as a function of spin. This was simulated with a level density yrast line (LDYL), which was determined by fitting $E_{LDYL} = E_0 + (\hbar^2/2\mathcal{J})\, I(I+1)$ to the first levels lying above the ground state and beta-vibrational band for each spin, where $\mathcal{J}$ is an effective moment of inertia for the LDYL. The intrinsic excitation energy U is measured relative to the LDYL: $U = E_{ex} - E_{LDYL}$. The spin dependence of the level density is taken to be [9]

$$\rho(E_{ex}, I) = (2I+1)\left(\hbar^2/2\mathcal{J}\right)^{3/2} \rho(U,0) \qquad (4)$$

Three level density formulas have been used: the Constant Temperature formula (CT), the Fermi-gas model formula [9] (FG), and the Grossjean and Feldmeier Fermi-gas formula[10] (GFFG):

$$\textbf{CT}: \qquad \rho(U) = 1/T \exp(U/T) \qquad (5)$$

$$\textbf{FG}: \qquad \rho(U) = (\sqrt{a}/24)\, \frac{\exp(\sqrt{aU})}{(E+(3/2)T)^2} \qquad (6)$$

$$T = \sqrt{U/a + 9/16a} + (3/4)\, a$$

$$\textbf{GFFG}: \qquad \rho(U) = \delta(U) + \frac{\exp((U/T) + aT)}{\sqrt{48U}}\, \frac{1 - \exp(-aT)}{\sqrt{1 - (U/2T)\exp(-aT)}} \qquad (7)$$

$$U = aT^2/(1 - \exp(-aT))$$

The level density parameter $a = \pi^2 g(\varepsilon_F)/6$, and the nuclear temperature $T = \sqrt{U/a}$.

The CT formula arises from assuming a nuclear temperature, and using the Fermi-distribution for a simple calculation of the level density based on the density of single-particle states near the Fermi energy.

The FG formula arises from a detailed statistical mechanical calculation, in which it is assumed that the density of single particle states is a smooth function.

The GFFG formula is calculated following the same procedure as the FG formula, except that the ground state is taken into account with a delta function, thus fixing the divergence at low energy arising from the denominator in the standard Fermi-gas formula.

The statistical cascade, from an entry distribution in phase space, to within 1 MeV of the yrast line was modeled in a Monte Carlo simulation[11], using eq. 1 with the KMF strength function and the three level density expressions. The probability of decay by a given transition is given by $\nu(E_\gamma)$, and the cascades typically proceed through several transitions. The average entry point of the cascade was fixed at the value calculated from the experimental data. When using the CT level density, T was varied, and when using the two FG formulas, a was varied. The resulting spectra were normalized to the number of cascades.

In Fig.2, the experimental spectrum has been fit with the expression $A\,E_\gamma^3 \exp(-E_\gamma/T)$, where A is an amplitude, and T is analogous to temperature in eq. 5. This corresponds to a cascade consisting of a single γ ray, with a constant temperature level density, no change in spin, and a constant f_{E1} as a function of energy. This and related expressions[1,12,13] have typically been used to fit the continuum spectrum above 2.5 MeV, to then subtract the fit as the background to the quasi-continuum spectrum. In Fig.2, for an α-induced reaction, this fit is only good above 1.5 MeV, whereas for rare-earth nuclei formed at high spin, the quasi-continuum bump typically is centered around 1 MeV. Therefore, this expression is not adequate to determine the background of the quasi-continuum.

Figure 3 shows the results of the Monte Carlo simulations. These simulations consist of cascades of several statistical transitions, with realistic expressions for the level density

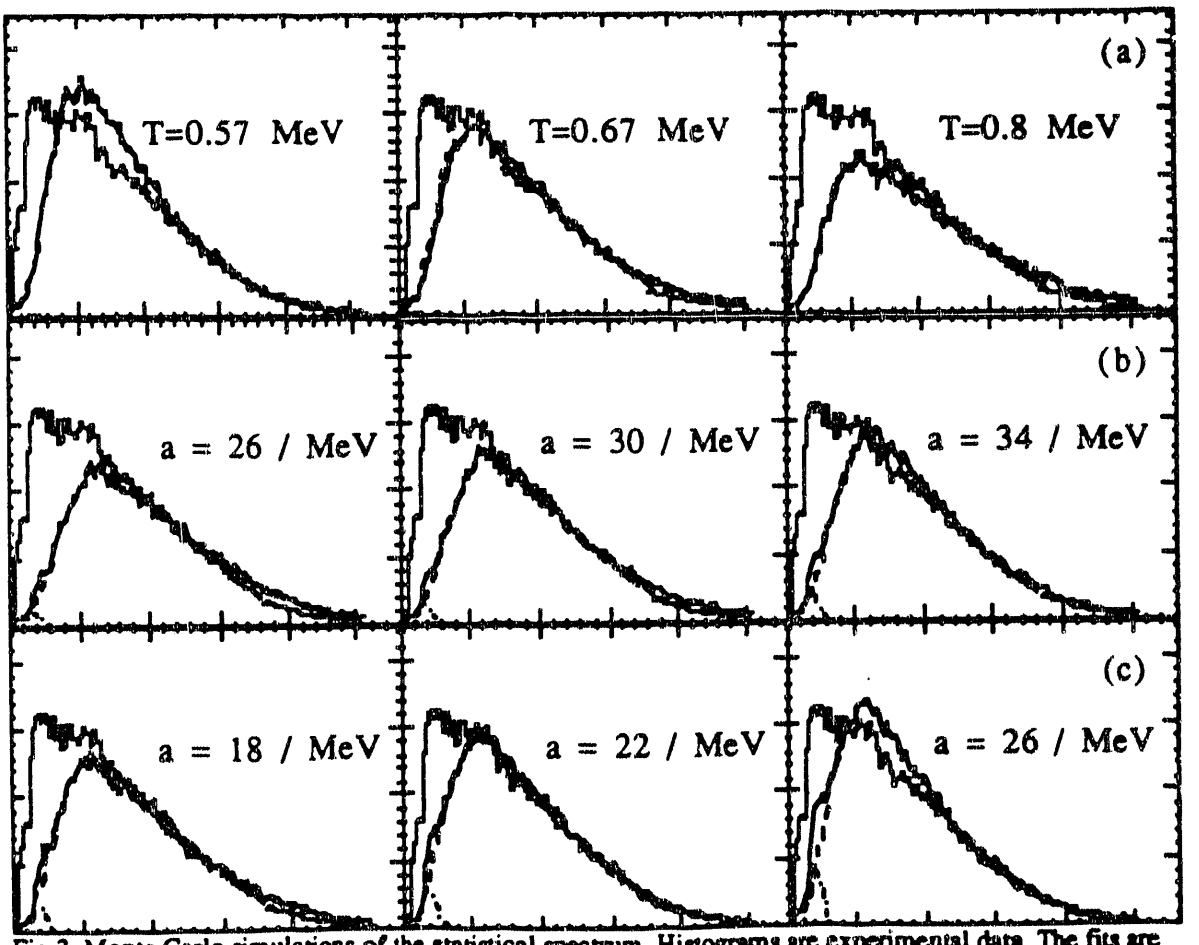

Fig.3. Monte Carlo simulations of the statistical spectrum. Histograms are experimental data. The fits are for different expressions for the level density: (a) CT formula, (b) FG formula, (c) GFFG formula

and the E1 strength function. All three expressions for the level density are able to produce a good fit of the data down to 1 MeV, with the GFFG formula giving the best fit, although comparable to the CT formula. In particular, we note that for the GFFG formula, the level density parameter, $\mathbf{a}$ = 22/MeV, is consistent with both a diffuse Fermi-gas formula estimate,[14] $\mathbf{a} = (A/14.61)\ (1 + 4\ A^{-1/3}) = 20.44$ / MeV, and $\mathbf{a} = A/8 = 21.75$ / MeV, determined from a fit of neutron resonance densities across a wide range of nuclear mass.[15] Similarly, if we take $\mathbf{a}$ = 21 / MeV and $U = \mathbf{a}T^2$ for the constant temperature formula, for an intrinsic excitation of 9 MeV, T = 0.65 MeV.

V. Summary and Conclusions

The spectrum of statistical gamma transitions was measured in the $^{172}Yb(\alpha,2n)^{174}Hf$ reaction. This spectrum was fit to a simple formula corresponding to a single transition in the statistical cascade, and to Monte Carlo simulations with multiple transitions in the cascade, using realistic expressions for the average gamma-transition probability, but considering only E1 transitions.

The data could not be fit with the single-transition model below E_γ = 1.5 MeV. With the Monte Carlo simulations it is possible to fit the data down to E_γ = 1 MeV. The GFFG formula gives the best fit, with level density parameter close to that expected, but the CT formula gives a comparable fit.

The yield of statisticals is underestimated by the realistic models at low energy. However, M1 and E2 transitions are expected to become more important as the cascade approaches the vicinity of the yrast line, and the statistical nature of the transitions eventually gives way to domination by nuclear structure effects.

In the study of quasi-continuum transitions, the use of the expression $A\ E_\gamma^3\ \exp(-E_\gamma/T)$ is not recommended, because it can only reproduce the observed statistical spectrum down to $E_\gamma \approx$ 1.5 MeV. An improved determination of the statistical background can be obtained by using a Monte Carlo simulation of a multiple transition cascade, using eq.1 to determine the transition probabilities, with a Constant Temperature level density and the KMF gamma-strength function. To determine the temperature, the data can be fit above $E_\gamma > 2.5$ MeV.

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