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Comments on Finite Larmor Radius Models for Ion Cyclotron Range of Frequencies Heating in Tokamaks

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The accuracy of standard finite Larmor radius (FLR) models for wave propagation in the ion cyclotron range of frequencies (ICRF) is compared against full hot plasma models. For multiple ion species plasmas, the FLR model is shown to predict the presence of a spurious second harmonic ion-ion type resonance between the second harmonic cyclotron layers of two ion species. It is shown explicitly here that the spurious resonance is an artifact of the FLR models and that no absorption occurs in the plasma as a result of this "resonance."

Theoretical and numerical models of ion cyclotron range of frequencies (ICRF) heating in tokamaks are frequently predicated upon finite Larmor radius (FLR) expansions of the Vlasov-Maxwell system of equations for the wave fields and plasma response. In models based on a dispersion relation description, the full hot plasma dielectric tensor[1] is expanded to second order in the parameter $k_\perp \rho_i$. Here, $k_\perp$ is the wave vector component perpendicular to the equilibrium magnetic field, **B**, ρ_i is the ion Larmor radius and the approximation is made that $k_\perp \rho_i << 1$. The resulting dispersion relation is either fourth or sixth order in $k_\perp$, depending on whether or not the parallel component of the wave field, $E_{//}$, is assumed to vanish or remain finite, respectively. In models based on direct solution of the wave equations, the Vlasov-Maxwell system may be solved perturbatively[2] in orders of $k_\perp \rho_i$. For one dimensional plasma models in which the direction of the inhomogeneity, x, is perpendicular to **B** , the resulting wave equation is either fourth order or sixth order in d/dx, again depending on the treatment of $E_{//}$. Correspondence between the dispersion relation models and the differential models is established by letting $d/dx \rightarrow i\, k_\perp$ in the homogeneous limit.

It is well known that the FLR models do not correctly describe certain plasmas of experimental interest in which $k_\perp \rho_i \geq 1$ for some ion species in the plasma. For example, in fundamental minority ion heating schemes,[3] the minority ions are accelerated by the applied wave fields to energies of 1 MeV or higher. For these ions, the small $k_\perp \rho_i$ limit is invalid. A second example in which the FLR expansion is invalid is in the study of ICRF heating in deuterium - tritium (D-T) plasmas in which the fusion generated 3.5 MeV alpha particles may resonantly interact with the applied ICRF waves.

While it is widely recognized that the FLR models encounter difficulties when the small $k_\perp \rho_i$ limit is violated, it is not as well known that difficulties may also occur in plasmas with multiple ion species in which the condition $k_\perp \rho_i << 1$ is satisfied in spatial regions of interest. The difficulties can be traced to the vanishing of the highest order terms in either d/dx or $k_\perp$. One such difficulty with the standard fourth order dispersion relation model was recently studied by Chow, Fuchs and Bers.[4] They noted for a two ion species plasma in which the majority ion is not resonant with the ICRF but the minority ion is resonant, that a spurious resonance and an associated spurious mode with very large $k_\perp$ appear near the location where the coefficient of the fourth order term in the dispersion relation (with $E_{//} = 0$) can vanish. By analogy, one may demonstrate that the same difficulty occurs for the fourth order differential equation model, though in this case it is the vanishing of the coefficient of the highest order derivative which causes the problem.

In general, models based on truncated FLR expansions are flawed whenever the coefficient of the highest order derivative or of the term of highest order in $k_\perp$ vanishes, leading to the presence of an artificial resonance.[2,5] Inclusion of higher order terms in the expansions resolves the resonance into a mode conversion region where waves associated with the lower order terms can couple with waves arising from the higher order terms in $k_\perp \rho_i$. This problem was first studied in detail by Perkins[5] for the ion-ion hybrid resonance which occurs in D-T plasmas as well as in deuterium majority - hydrogen minority plasmas. In this report, a manifestation of this breakdown that occurs in two ion species plasmas in which only second (or higher) harmonic heating is present is discussed. It is shown that a spurious resonance occurs at a spatial location between the second (or higher) harmonic layers for each ion species. Though the "resonance" is reminiscent of the ion-ion hybrid resonance which occurs between the fundamental cyclotron layers of two ion species,[6] in this case it can be shown explicitly that no absorption occurs as a result of the "resonance."

Consider ICRF heating in a two ion species plasma in which the frequency of the applied ICRF waves is chosen so that the second harmonic ion cyclotron resonance of each ion species may affect the wave propagation and/or damping. The one dimensional fourth order kinetic wave equation may be written in the following form:[7]

$$\frac{1}{\gamma_1} L \frac{\partial^2}{\partial x^2} - 2\left(\frac{\gamma_1+\gamma_2}{\gamma_1}\right) L u - \frac{\partial^2 u}{\partial x^2} - k_\perp^2 u = 0 \qquad (1)$$

where $k_\perp^2 = (\gamma_2^2 - \gamma_1^2)/\gamma_1$, $\gamma_1 = k_z^2 - \omega^2 K_{xx0}/c^2$, $\gamma_2 = -i\omega^2 K_{xy0}/c^2$, and

$$L = (\omega^2 K_{xx1}/c^2)\partial/\partial x + (\omega^2 K_{xx2}/c^2)\partial^2/\partial x^2 \,. \qquad (2)$$

Here, K_{xx0} and K_{xy0} are the diagonal and off-diagonal elements of the cold plasma dielectric tensor and K_{xx1} and K_{xx2} are the FLR corrections to second order in $(k_\perp \rho_i)^2$. In particular, note that the coefficient of the highest order derivative in d/dx, namely, K_{xx2}, can be written approximately as:

$$K_{xx2} \cong \frac{i n_{1i} v_{Ti1}}{4 m_{i1} k_z} \frac{Z(\zeta_{-2,i1})}{(\omega - \omega_{ci1})^2} + \frac{i n_{2i} v_{Ti2}}{4 m_{i2} k_z} \frac{Z(\zeta_{-2,i2})}{(\omega - \omega_{ci2})^2} \qquad (3)$$

where small, nonresonant electron and ion terms have been neglected, as described in Refs. 2 and 7. The argument of the plasma dispersion function,[8] Z, can be written as $\zeta_{-2,\,i1} = (\omega - 2\,\omega_{ci1}(x))/k_z v_{ti1}$, and similarly for the second ion, i2. The remaining coefficients, n_i, v_{ti}, m_i and ω_{ci} are the density, thermal speed, mass and cyclotron frequency of species 1 and 2 as noted and are also functions of x. Setting $K_{xx2} = 0$ then defines the spatial location where the truncated FLR model becomes invalid. Because all of the coefficients in K_{xx2} are either positive or else have the same sign for both terms, the behavior is determined by the Z function contributions. The real part of Z is negative for $\zeta_{-2,i} > 0$ and positive for $\zeta_{-2,i} < 0$,[8] so K_{xx2} can vanish only between the second harmonic cyclotron layers of the two ion species. However, the imaginary part of K_{xx2} is always positive or zero, so the imaginary part of Kxx2 can vanish only in regions where the imaginary part ot the Z function vanishes, which is where $\zeta_{-2,i1} >> 1$ and $\zeta_{-2,i2} >> 1$. The exact location where the term K_{xx2} vanishes depends on the other physical quantities in Eq. (2), namely, the relative densities, masses, and temperatures of the two ion species. This is reminiscent of the case of the ion-ion hybrid resonance,[6] in which a true resonance occurs between the fundamental cyclotron layers of two ion species. However, here the "resonance" occurs in regions where the plasma absorption vanishes and involves roots which are present only in a truncated finite temperature description.

In Fig. 1a, the roots of the fourth order dispersion relation for a D-T plasma with equal ion densities are displayed for $k_{//} = 3$ m^{-1}. Parameters typical of Tokamak Fusion Test Reactor (TFTR)[9] supershot experiments were chosen: central electron density, n_{e0}, equal to 7.0x13 cm^{-3}, central electron temperature, T_{e0}, equal to 10 keV, central ion temperature, T_{i0}, equal to 20 keV, equilibrium toroidal magnetic field at the center of the plasma, $B_{T0} \sim 4.3$ T, plasma major radius, R_0, equal to 2.62 m, plasma minor radius, a, equal to 0.95 m, and oscillator frequency, f, equal to 43 MHz. To simplify the figures presented here, flat density and temperature profiles were chosen. For these parameters, the FLR expansion becomes invalid when $k_\perp^2 \sim 10^4$ m^{-2}. A second harmonic ion-ion hybrid-like resonance can be seen at $x = R - R_0 \sim 0.42$ m. Fig. 1b displays the corresponding full hot plasma dispersion relation.[1] The resonance-like behavior at x=0.42 m is absent in the full hot plasma model. Comparing Figs. 1a and 1b, the main effect of the resonance which appears in the truncated FLR model is a significant distortion of the behavior of the ion Bernstein wave (IBW) root which lies between the D second harmonic layer on the outside the plasma at $x \sim 1.36$ m and the T second harmonic layer at $x \sim 0$. Furthermore, the slope of the IBW wave in the vicinity of the T second harmonic layer is less than that obtained with the full model, even for $k_\perp$'s below the limit where $k_\perp \rho_i \sim 1$. For second harmonic T heating scenarios, the distortions of the IBW wave in the vicinity of the second harmonic T layer are most important. While the differences between the FLR and full models near the second harmonic T layer are modest for 50-50 D-T plasmas, the differences become more pronounced as the relative tritium concentration is decreased. In Fig. 2, the differences are displayed for a plasma consisting of 90% D and 10% T. The slope as well as the zero crossing of the IBW root near the second harmonic T layer is significantly different in the FLR model. This behavior, which is qualitatively similar to that discussed by Chow, Fuchs and Bers,[4] may lead to inaccurate modeling of the single pass scattering coefficients by the FLR model.

Though the behavior of the IBW root is modeled incorrectly with the FLR expansions, the fast wave (FW) root is largely unaffected by the truncation. In Fig. 3, the behavior of the fast wave root in the vicinity of the second harmonic T layer as determined from the FLR model is contrasted with that obtained from the full hot plasma dispersion relation for the parameters used in Fig. 2. While the wave propagation (real part) of the fast wave root is reproduced nearly exactly by the FLR model, the wave damping (imaginary part) predicted by the FLR model differs from that of the exact model by less than 5% for this case. Hence, the FLR model is reliable for plasmas in which the absorption is dominated by the fast wave and in which the FLR condition on the ion energies is satisfied. It is precisely for these plasmas that the reduced order approximation[10] for calculating wave propagation and damping in tokamak geometry was formulated. Since the reduced order approximation is built upon the hot plasma fast wave root in an FLR description of the plasma-wave interactions, an accurate representation of the fast wave root by the FLR model is required to insure accuracy of the derived power deposition profiles. However, for plasmas in which significant mode conversion to the ion Bernstein wave is anticipated, quantitative predictions based on the FLR models are suspect whenever the behavior of the full hot plasma IBW root is not accurately reproduced by the FLR models.

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Fig. 1 The standard fourth order dispersion relation, shown in (a), is compared to the full hot plasma dispersion relation, shown in (b), for TFTR supershot plasmas: B_0 = 4.3 T, n_{e0} = 7.0 x 13 cm^{-3}, T_{e0} = 10 keV, T_{i0} = 20 keV, $\eta_T = \eta_D$ = 0.5, R_0 = 2.62 m, a = 0.95 m, $k_{//}$ = 3 m^{-1}, and f = 43 MHz. A spurious second harmonic ion-ion type resonance, introduced by the FLR expansions, is evident in (a) near x ~ 0.4 m.

Fig. 2 The IBW roots derived from the standard fourth order dispersion relation and from the full hot plasma dispersion relation are shown in (a) and (b), respectively, for a plasma consisting of 90% D and 10% T. The remaining parameters are the same as those used in Fig. 1. The spurious resonance now occurs near x ~ 0.1 m, in the vicinity of the second harmonic T layer near x ~ 0.

Fig. 3 A magnified plot of the real and imaginary parts of the fast wave root is given using both the full hot plasma dielectric tensor and the FLR approximated model for the plasma defined in Fig. 2. Note that the results from the two models nearly overlap.

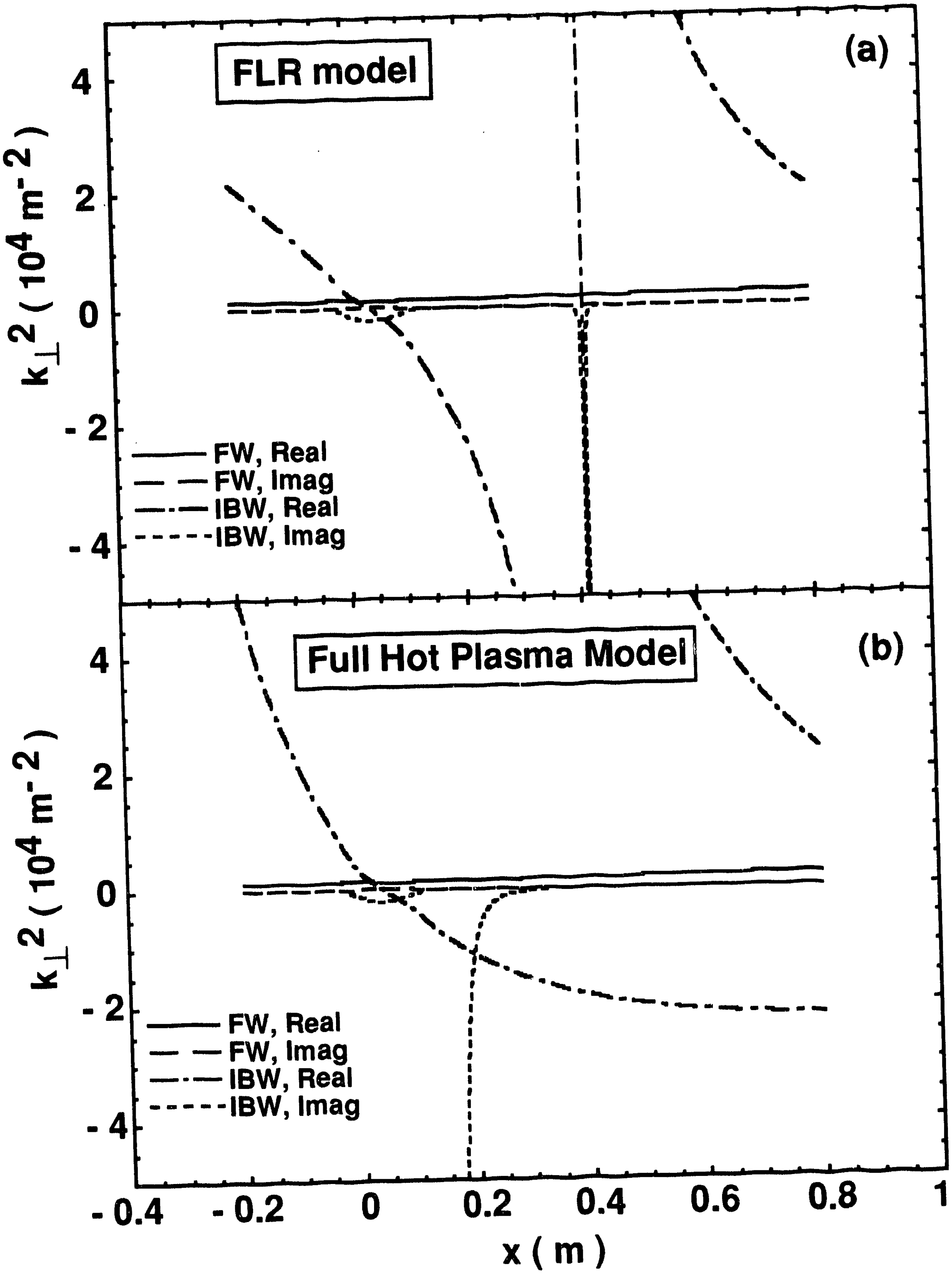
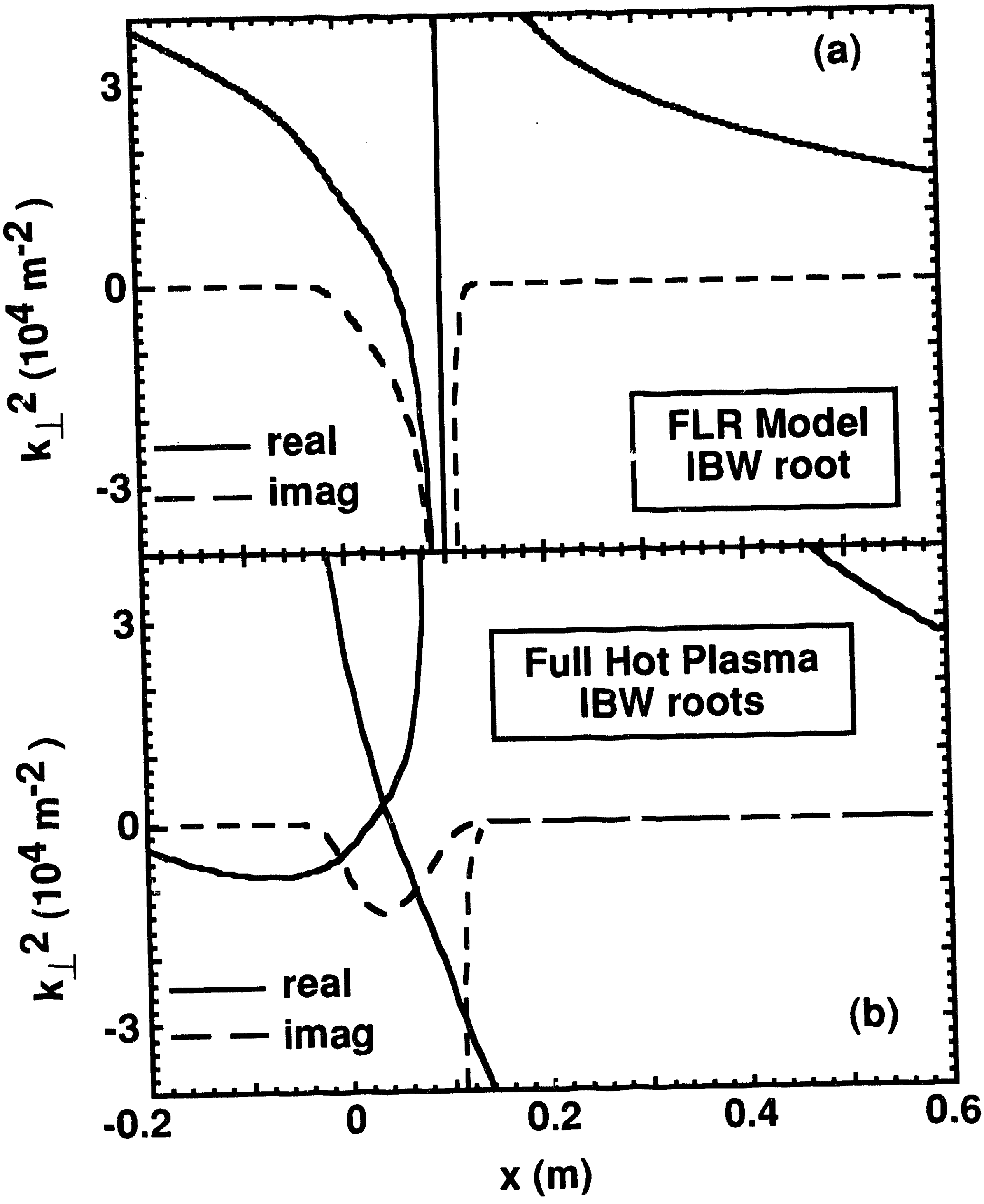
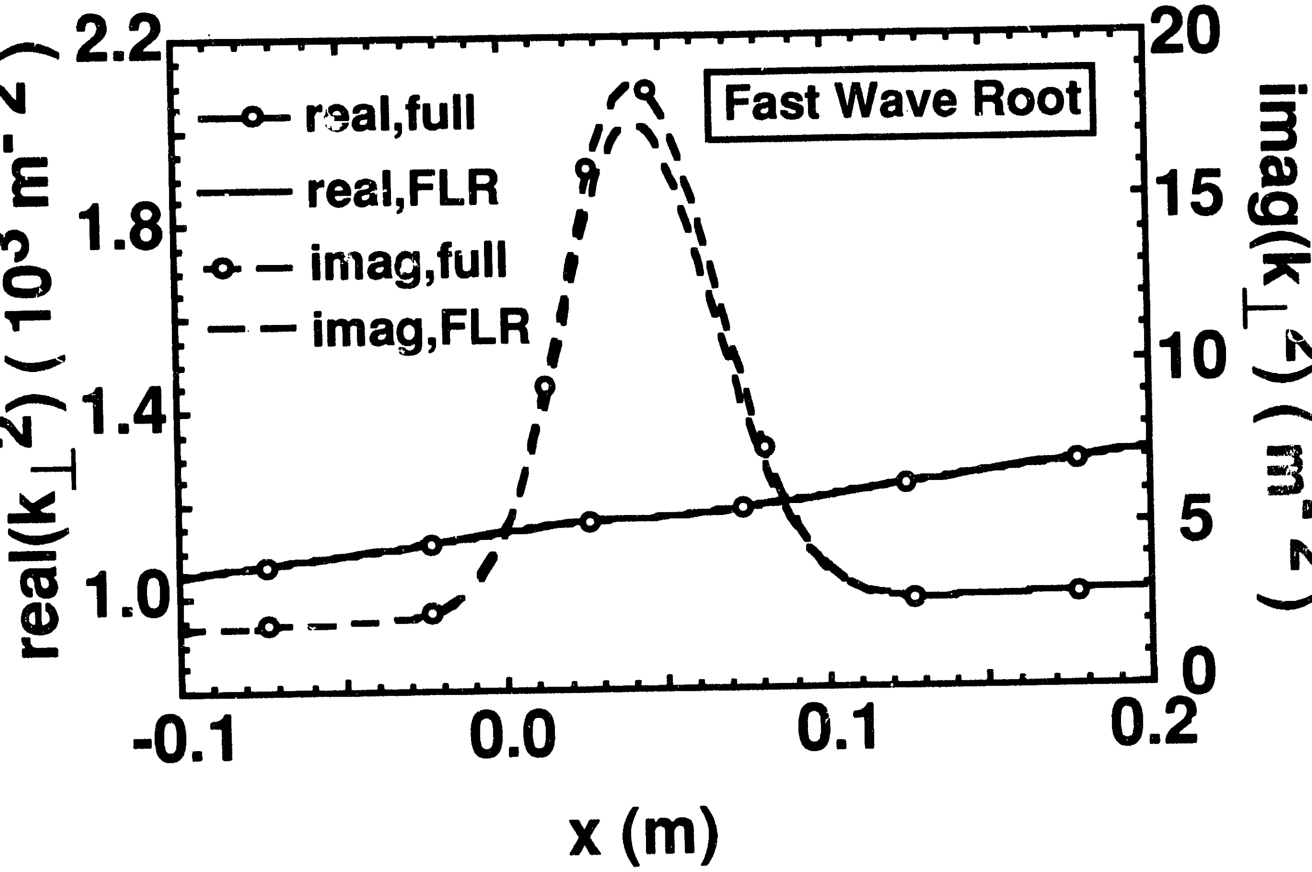

FLR model
(a)
Full Hot Plasma Model
(b)
k⊥2 (10⁴ m⁻²)
x (m)
FW, Real
FW, Imag
IBW, Real
IBW, Imag
4
2
0
- 2
- 4
- 0.4
- 0.2
0
0.2
0.4
0.6
0.8
1

Figure 1



(a)
FLR Model
IBW root
real
imag
3
0
-3
$k_\perp^2$ (10^4 m^{-2})
Full Hot Plasma
IBW roots
real
imag
(b)
-0.2
0
0.2
0.4
0.6
x (m)

Figure 2



real,full
real,FLR
imag,full
imag,FLR
Fast Wave Root
2.2
1.8
1.4
1.0
20
15
10
5
0
-0.1
0.0
0.1
0.2
real($k_\perp^2$) (10^3 m^{-2})
imag($k_\perp^2$) (m^{-2})
x (m)

Figure 3

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