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Tests of Non-Local Interferences in Kaon Physics at Asymmetric ϕ-Factories.*

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April 16, 1993

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ABSTRACT

Tests of non-local interference effects in the two-kaon system are proposed. The first kind of tests consists of measuring the amount of destructive interference between $K_S \rightarrow K_L$ regeneration processes of two distant kaons. The second kind deals with constructive interference. These tests could be performed at an asymmetric ϕ-factory. Estimates are given of the number of events predicted by orthodox quantum mechanics and kaon regeneration theory in various suitable experimental conditions. The impact on local theories if the predictions of quantum mechanics hold is discussed.

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Contents

1 Introduction.

Whenever quantum theory makes surprising predictions, it deserves to be checked experimentally. The object of the tests suggested here is the interference effect between two $K_S \rightarrow K_L$ regeneration processes of two different kaons at two different locations, centimeters apart. That kind of interference cannot be described as an interference between waves propagating in space. In the literature it is referred to as a non-local phenomenon proper to quantum theory, [1]. It is related to the EPR-paradox [2], as are many properties of the $K^0\overline{K^0}$ system, [3].

Such non-local effects are also present in systems of two photons, of course if the predictions of quantum mechanics are correct, [4]. Because non-local effects are surprising in the context of special relativity, it was considered worthwhile to verify that they actually exist in nature. This is why predictions that depend on these effects in optics have been checked experimentally, [5]. The same argument can be made to justify tests of non-local effects on two-kaon systems because, in spite of the success of quantum mechanics in optics, dominant phenomena in optics and in kaon physics are different. In the past, kaon physics has shown violations of principles much harder or impossible to detect in other domains of physics, [6].

In the literature, tests of the non-locality of quantum mechanics using the system of two neutral kaons created by the decay of ϕ-mesons have been proposed long ago, [7]. They are based on measurements of the kaon strangeness. They have been criticized as to their scope and significance, [8]. More recently, another test, based on kaon regeneration at an asymmetric ϕ-factory[1], has been proposed, [9]. It escapes most of the criticism of Ref. [8]. This test checks quantum mechanics in a case where a strong non-local *constructive* interference effect is expected between two distant kaon regeneration processes. Following up on Ref. [9], this paper describes tests involving non-local *destructive* interference, which are easier to perform experimentally. Then it describes more tests of *constructive* interferences at ϕ-factories with different machine-parameters than Ref. [9]. In addition, the significance of these different tests is analyzed and discussed in terms of which local theory is ruled out if orthodox quantum theory is verified.

2 Destructive Interference Tests.

2.1 General Description.

As in Ref. [9], these tests require measurements in four different experimental setups at an asymmetric ϕ-factory. In the first setup, sketched in Fig. 1a, there is no material in the way of either of the two neutral kaons produced by the ϕ-decay. In each event, one of the neutral kaons is emitted in a slightly upward direction, above the e^+e^--beam line: let us call it the "up" particle. The other kaon is emitted in a downward direction, below the beam line: it will be called the "down" particle. Because of conservation of charge conjugation in ϕ-decay, half of the $\phi \rightarrow K^0\overline{K^0}$ events have a K_L as the "up" particle and a K_S as the "down" particle; the other half are the other way around. The first half will be called : type $K_L K_S$ events; and the second half : type $K_S K_L$. In both cases, the first symbol in the event-type refers

[1] i.e. an asymmetric e^+e^--collider of a total energy in the center of mass equal to the mass of the ϕ-meson, 1020 MeV,

to the "up" kaon and the second symbol to the "down" kaon. In the setup of Fig. 1a, there are never any $K_L K_L$ or any $K_S K_S$ events, i.e. any events where the "up" and the "down" kaons are in the same state.

In the second setup, a piece of material is interposed on the path of the "up" and nothing on the "down" particle, as sketched on Fig. 1b. Some "up" K_S get regenerated into K_L in that material while the associated "down" kaon remains a K_L in vacuum. Therefore some two kaon systems initially of type $K_S K_L$ will be transformed into systems of type $K_L K_L$ and produce $K_L K_L$ events. If the piece of material is interposed on the "down" path instead (Fig. 1c), there are also $K_L K_L$ events because of $K_S \rightarrow K_L$ regeneration of some "down" kaons associated with an "up" kaon remaining a K_L. Regardless of which path the piece of material is on, events of type $K_L K_L$ will be seen. Similarly events of type $K_S K_S$ will be observed because of $K_L \rightarrow K_S$ regeneration. Because of these regeneration effects, such a piece of material interposed on a kaon path will be called a "regenerator".

In the tests of this paper, we consider only coherent regeneration, i.e. regeneration where the kaon emerges with its momentum unperturbed and where no nucleon or nucleus recoils in the regenerator. If there are two regenerators, one on the "up" and one on the "down" path (Fig. 1d), there will be a regeneration process in each of them. Orthodox quantum theory predicts that the coherent regeneration process of the "up" kaon and the coherent regeneration process of the "down" kaon are also coherent with one another. If both regenerators are identical and placed at the same distance from the interaction point, if one counts only non-interacting and coherently regenerated kaons (recognizable by their angle of emergence from the material), quantum mechanics predicts that the interference term is

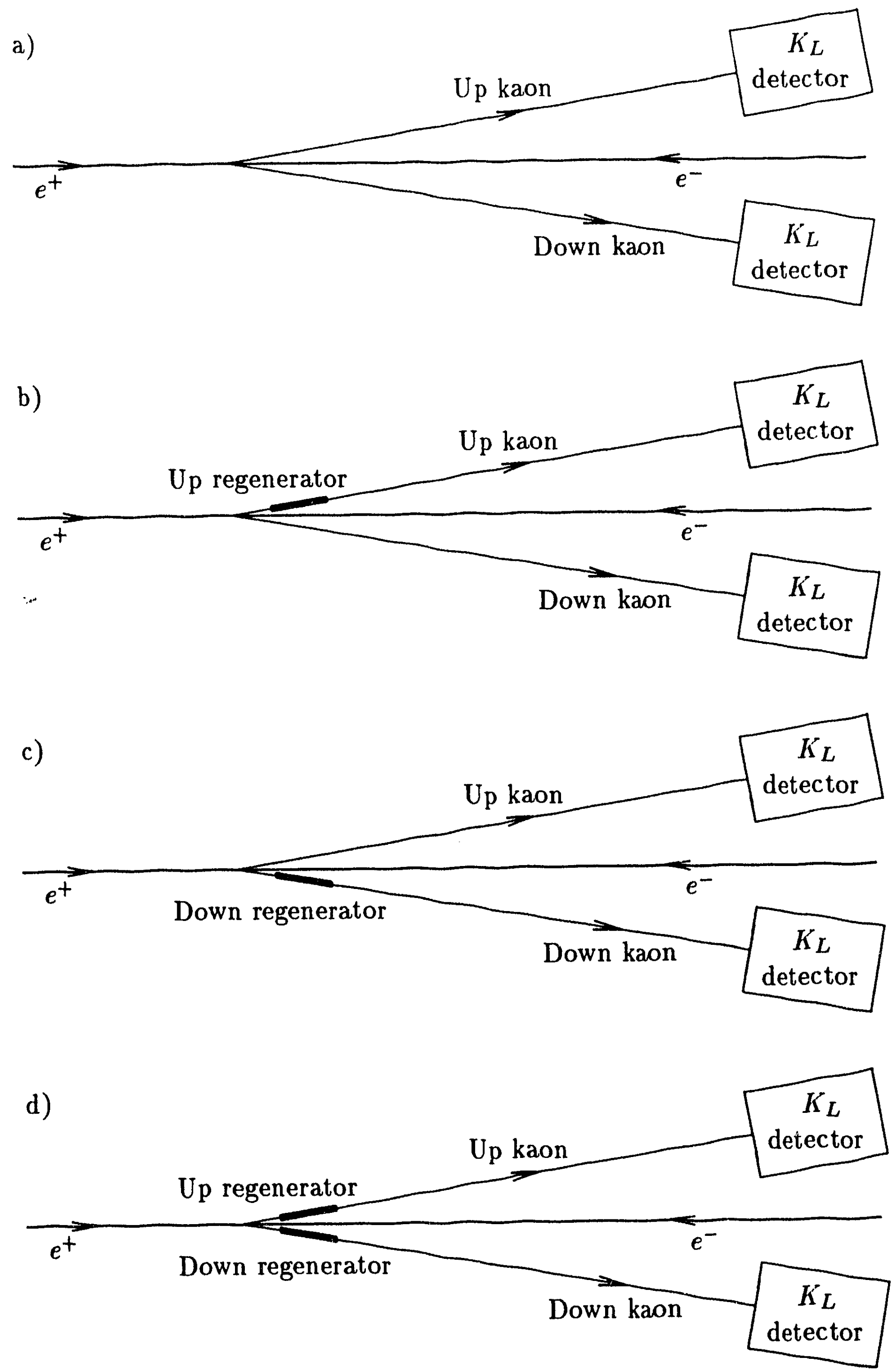

Figure 1: *Setups for the destructive interference test.*

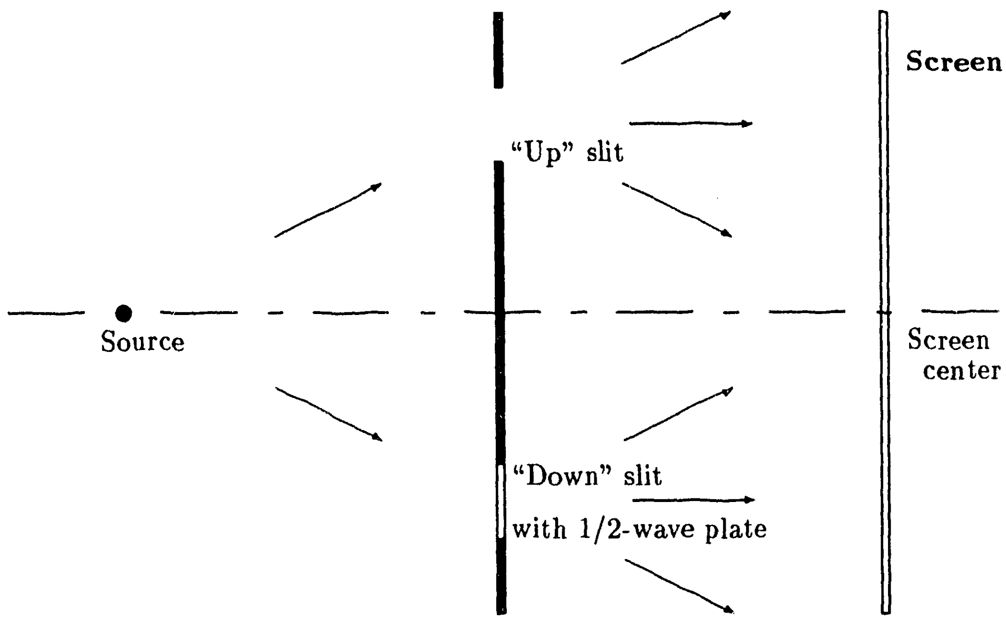

Figure 2: *Two-slit experiment with half-wave plate in one of the slits.*

negative and the effect is 100% destructive. There are no K_LK_L and no K_SK_S events in the setup of Fig. 1d, just as in the setup of Fig. 1a. With the two pieces of material, an interference phenomenon is taking place between two processes occurring at different locations and acting on two particles that will never meet again. The two processes obviously do not act independently but there is no apparent wave or particle to carry an influence of one of the processes onto the other or to coordinate the future behavior of the two kaons. In that sense this interference effect is a non-local effect.

To illustrate the non-local character of that phenomenon, let us compare this kaon experiment to the version of the two-slit photon experiment that is shown on Fig. 2. Because of the 1/2-wave plate located in the lower slit, the interference between the beams passing through the upper- and through the lower slits is 100% destructive at the center of the screen. There the photon intensity is zero with the two slits open, just like the number of $K_L K_L$ events in the two-kaon experiment with two regenerators. However, in the two-slit experiment, the effect can be described by waves propagating from the slits and merging to generate an interference pattern on the screen. Light is restored at the center of the screen if any obstruction is placed on the path of either one of the two waves, as if something participating in that interference effect was travelling along each of the paths. In the case of the kaon experiment, it is different. One could completely isolate the two kaons from one another by an enclosure of absorptive material, starting any time after they separated at the e^+e^-–colliding point, even before they reach the regenerators. The interference effect would be unchanged. There is some conspiracy between the two regeneration processes when two regenerators are in place, but there is no known vehicle to explain how a regenerator can influence the state of the kaon traversing the other regenerator.

2.2 Computations in Orthodox Quantum Mechanics.

The charge conjugation of the ϕ-meson is $C = -$, [10]. Because charge conjugation is conserved in ϕ-decay, the state vector of the two-kaon system at the e^+e^-–interaction point is

$$\psi = \frac{1}{\sqrt{2}} \left(|K_L> \ |K_S> \ - \ |K_S> \ |K_L> \right) . \qquad (1)$$

Let us call N the number of $\phi \rightarrow K^0\overline{K^0}$ produced. Without regenerator on either path, there are

$$n^0_{LS} = \frac{1}{2} N \qquad \text{events of type } K_L K_S , \qquad (2)$$

$$n^0_{SL} = \frac{1}{2} N \qquad \text{events of type } K_S K_L , \qquad (3)$$

and no event of another type, i.e. :

$$n^0_{LL} = 0 \qquad \text{events of type } K_L K_L , \qquad (4)$$

$$n^0_{SS} = 0 \qquad \text{events of type } K_S K_S . \qquad (5)$$

Let us call U the evolution operator on one kaon traversing the regenerator. If there is just one regenerator on the "up" particle (Fig. 1b), the state vector of the two-kaon system after traversal is

$$\psi = \frac{1}{\sqrt{2}} \left(\left(U|K_L >\right) |K_S > - \left(U|K_S >\right) |K_L > \right) . \qquad (6)$$

Let us call p_{thru} the probability that a single K_L traverses the material without interaction and p_{regen} the probability that a K_S is regenerated into a K_L coherently, i.e. without a nucleus breaking up or recoiling in the regenerator. The number n^{up}_{LL} of $K_L K_L$ events and the number n^{up}_{LS} of $K_L K_S$ events with the regenerator on the "up" path are

$$n^{\mathrm{up}}_{LL} = \frac{1}{2} N \left| < K_L| \, U|K_S > \right|^2 = \frac{1}{2} N \, p_{\mathrm{regen}} , \qquad (7)$$

$$n^{\mathrm{up}}_{LS} = \frac{1}{2} N \left| < K_L| \, U|K_L > \right|^2 = \frac{1}{2} N \, p_{\mathrm{thru}} . \qquad (8)$$

If that regenerator that was placed on the "up" particle is now placed on the "down" path instead, i.e. in the setup of Fig. 1c, the same evolution operator U applies to the "down" kaon. The two-kaon state vector becomes

$$\psi = \frac{1}{\sqrt{2}}\Big(\, |K_L> \; \big(U|K_S>\big) \; - \; |K_S> \; \big(U|K_L>\big) \Big) \; ; \qquad (9)$$

and the numbers n_{LL}^{down} of $K_L K_L$ events and n_{SL}^{down} of $K_S K_L$ events are

$$n_{LL}^{\text{down}} \;=\; \frac{1}{2}\, N \; \Big| < K_L| \; U|K_S > \Big|^2 = \frac{1}{2}\, N \; p_{\text{regen}} \; , \qquad (10)$$

$$n_{SL}^{\text{down}} \;=\; \frac{1}{2}\, N \; \Big| < K_L| \; U|K_L > \Big|^2 = \frac{1}{2}\, N \; p_{\text{thru}} \; . \qquad (11)$$

Now, if two identical regenerators are present, one on the "up" and one on the "down" paths (Fig. 1d), the state vector ends up

$$\psi = \frac{1}{\sqrt{2}}\Big(\big(U|K_L>\big) \; \big(U|K_S>\big) \; - \; \big(U|K_S>\big)\big(U|K_L>\big)\Big) \; ; \qquad (12)$$

and the number of $K_L K_L$ events is

$$n_{LL}^{\text{both}} \;=\; \frac{N}{2} \, \Big|< K_L| \; U|K_L > \; < K_L| \; U|K_S > - < K_L| \; U|K_S > < K_L| \; U|K_L > \Big|^2 \;=\; 0 \; , \qquad (13)$$

regardless of any hypothesis except the principles of quantum mechanics and conservation of charge conjugation.

The expression between the two $=$ signs of Eq. (13) contains the modulus of the difference of two terms, which happen to be equal when the two regenerators are identical. Each term corresponds to the contribution of the regeneration process in one regenerator and the transmission through the other. If the square of the modulus of the difference is developed, the square of each of the terms represents the contribution of the corresponding process and the double product represents the interference effect. Therefore the amount n_{LL}^{int} due to the interference effect is negative and its absolute value is equal to the number of $K_L K_L$ events obtained in a computation where the interference term would be omitted,

$$\begin{aligned} |n_{LL}^{\text{int}}| &= N \left| < K_L|U|K_L > \; < K_L|U|K_S > \right|^2 = N \; p_{\text{regen}} \; p_{\text{thru}} \\ &= 4\frac{n_{LL}^{\text{up}} n_{SL}^{\text{down}}}{N} = 4\frac{n_{LS}^{\text{up}} n_{LL}^{\text{down}}}{N} \; . \end{aligned} \tag{14}$$

Any deviation from the prediction zero of Eq. (13) represents a deviation from quantum mechanics. If such a deviation is ever found, one can express the amount of violation by the fraction by which the size of the interference effect is reduced, i.e. by the ratio

$$\epsilon^{\text{int}} = \frac{n_{LL}^{\text{both}}}{|n_{LL}^{\text{int}}|} \tag{15}$$

of the number of $K_L K_L$ found with two regenerators and the predicted amount of interference given by Eq. (14). [2] If no deviation is found, an upper limit for ϵ^{int} will be found. That upper limit is a quantitative estimate of the quality of the test.

The number of $K_S K_S$ events can be obtained by a similar calculation involving K_S absorption probability and $K_L \rightarrow K_S$ regeneration. However, in this paper, we focus on $K_L K_L$ events because, experimentally, it seems easier to identify K_L particles with negligible ambiguity than K_S. Indeed K_L can be recognized as neutral particles interacting strongly in a detector located downstream at a distance of several average K_S decay-lengths.

[2] In the terminology used in optics, $1 - \epsilon^{\text{int}}$ is called "visibility".

2.3 Quantitative Estimates.

In the setups with regenerators (Figs. 1b, 1c, and 1d), the numbers of K_LK_L, K_LK_S, and K_SK_L events can be predicted using measured cross sections, nuclear physics, and regeneration theory. The $e^+e^- \to \phi$ cross section is about 4 μb, [11], and the branching ratio of ϕ into neutral kaons approximately 34%, [10]. If the duration of the experiment is T and the average luminosity is $\mathcal{L}$,

$$N = 1.3 \times 10^{-30}\ \mathcal{L}T\ ; \tag{16}$$

$$\text{i.e.}\quad N = 3 \times 10^{7} \qquad \text{for } \mathcal{L} = 10^{31}\ \text{cm}^{-2}\text{s}^{-1} \tag{17}$$

$$\text{and } T = \text{one effective month.}$$

In the setup without any regenerator (Fig. 1a), according to Eqs. (2) and (3), half of these events are of type K_LK_S, and half are of type K_SK_L. There is no other type.

For the case of one regenerator either on the "up" path (Fig. 1b), or the "down" path (Fig. 1c), let us define ℓ as the thickness of the regenerator and λ as the K_L collision length in the material, [12]. Neglecting the decay probability of the K_L, one can predict the probability p_{thru} for the K_L not to interact in the "up" regenerator as

$$p_{\text{thru}} = \Big| < K_L|\ U|K_L > \Big|^2 = e^{-\ell/\lambda}\ . \tag{18}$$

Given the atom density ν in the regenerator, the difference Δf of forward scattering amplitude of K^0 and $\overline{K^0}$ by these atoms, the K_S mean lifetime τ_S, the kaon mass m, the K_L–K_S mass difference Δm, and the time t taken by the kaon in its own rest frame to traverse the regenerator, one can compute the probability p_{regen} of $K_S \to K_L$ regeneration [13],

$$\rho = \frac{\pi\nu}{\frac{1}{2\tau_S} - i\Delta m} \frac{\Delta f}{m} \left(1 - e^{(-\frac{1}{2\tau_S}+i\Delta m)t}\right) , \qquad (19)$$

$$p_{\text{regen}} = \left| < K_L|\, U|K_S > \right|^2 = |\rho|^2 \, p_{\text{thr..}} \, . \qquad (20)$$

Using the data of regeneration experiments, [14], Eqs. (7), and (8), one can predict the numbers n_{LL}^{up}, n_{LS}^{up}, n_{LL}^{down}, and n_{SL}^{down} of $K_L K_L$, $K_L K_S$, and $K_S K_L$ events in various setups. For experimental conditions indicated in Table 1, i.e. various e^+–energies and regenerator thicknesses, these numbers of events have been computed and are also shown in Table 1. From these numbers and Eq. (14), one derives the values of $|n_{LL}^{\text{int}}|$ indicated on the last row.

The quality of a test depends on how much $K_L K_L$ background there is in the setup with two regenerators. A major source of background may be due to K_S decaying into two neutral pions and producing four misidentified γ-rays. If the uncertainty on the contribution of this background can be made as small as 10^{-5} of the number of $K_S \to \pi^0\pi^0$ decays, [3] then the uncertainty on n_{LL}^{both} will be about 100, and the quality of the test, i.e. the upper limit for ϵ^{int} of Eq. (15), will be of the order of the percent. [4]

[3] implying a reasonable average of only 5% misidentified γ's,

[4] This corresponds to a "visibility" between 98.5 and 99.5%.

Factory e^+ energy	(GeV)	2	3	9
e^- energy	(MeV)	130	90	30
Integrated luminosity/setup	(pb^{-1})	25	25	25
# of $K^0\overline{K^0}$ events : N	(millions)	30	30	30
Regenerator material		copper	copper	graphite
Regenerator length : ℓ	(cm)	5	5	15
<u>One regenerator</u>				
$K_L K_L$ events :	$n_{LL}^{\rm up}=n_{LL}^{\rm down}$	17000	7200	6000
$K_L K_S$ or $K_S K_L$ events :	$n_{LS}^{\rm up}=n_{SL}^{\rm down}$	9 10^6	9 10^6	9 10^6
<u>Both regenerators</u>				
Amount of interference :	$\lvert n_{LL}^{\rm int}\rvert$	20400	8600	7200

Table 1: *Orthodox predictions for the destructive interference test, at three possible ϕ-factories.*

3 Alternate Theories.

3.1 One-Particle Quantum Theory.

To provide a focus to these tests of non-local interferences predicted by orthodox quantum mechanics, let us construct an alternate theory that cannot be disproved on the base of the results of experiments performed to date, but that is local at least as far as kaon-physics is concerned. The easier theory one can construct with these properties is one that predicts that single kaons obey quantum mechanics, but that systems of two kaons behave like sets of independently interacting kaons, each one described by a wave function of its own. Checks of quantum mechanics in kaon physics have been performed, directly or indirectly, when performing regeneration experiments, but these tests involve single kaons, [15], not systems of two kaons such as those produced at a ϕ-factory. In the literature, the assumptions of this one-particle quantum theory are often referred to as the Furry hypothesis, [16].

In this one-particle quantum theory, the "up" kaon is described by a wave function

$$\psi^{\mathrm{up}} = u_L \, |K_L> + \, u_S \, |K_S> \, , \tag{21}$$

and the "down" kaon, independently, by

$$\psi^{\mathrm{down}} = d_L \, |K_L> + \, d_S \, |K_S> \ . \tag{22}$$

The sample of kaon systems produced by ϕ-decay is made of systems with various wave-functions ψ^{up} and ψ^{down}, i.e. with various values for the coefficients u_L, u_S, d_L, and d_S. There is a statistical distribution of u_L, u_S, d_L, and d_S, which is yet unspecified. Let us denote the expectation value of a quantity by a bar over it. In the setup without regenerator,

$$\frac{n^0_{LL}}{N} = \overline{|u_L d_L|^2} , \tag{23}$$

$$\frac{n^0_{LS}}{N} = \overline{|u_L d_S|^2} , \tag{24}$$

$$\frac{n^0_{SL}}{N} = \overline{|u_S d_L|^2} , \tag{25}$$

$$\frac{n^0_{SS}}{N} = \overline{|u_S d_S|^2} . \tag{26}$$

In the two setups with a regenerator on the "up" path, the wave function of Eq. (21) is transformed into

$$\psi^{\mathrm{up}} = u_L \ U|K_L > + \ u_S \ U|K_S > , \tag{27}$$

and the probability amplitude for observing a K_L :

$$< K_L|\psi^{\mathrm{up}} = t \ u_L + r \ u_S , \tag{28}$$

where
$$t = < K_L| \ U|K_L > , \tag{29}$$

and
$$r = < K_L| \ U|K_S > ; \tag{30}$$

$$p_{\mathrm{thru}} = |t|^2 , \tag{31}$$

$$p_{\mathrm{regen}} = |r|^2 . \tag{32}$$

Similarly, in setups with a regenerator on the "down" path, the probability amplitude for observing a "down" kaon is :

$$< K_L|\psi^{\mathrm{down}} = t \ d_L + r \ d_S . \tag{33}$$

When the regenerator is only on the "up" path, the fractions of $K_L K_L$ and of $K_L K_S$ events are

$$\frac{n^{\mathrm{up}}_{LL}}{N} = \overline{|t \ u_L + r \ u_S|^2 \ |d_L|^2} = \overline{|t \ u_L d_L + r \ u_S d_L|^2} , \tag{34}$$

$$\frac{n_{LS}^{\text{up}}}{N} = \overline{|t\, u_L + r\, u_S|^2\, |d_S|^2} = \overline{|t\, u_L d_S + r\, u_S d_S|^2}\,. \tag{35}$$

When the regenerator is only on the "down" path, the fractions of $K_L K_L$ and $K_S K_L$ events are

$$\frac{n_{LL}^{\text{down}}}{N} = \overline{|u_L|^2\, |t\, d_L + r\, d_S|^2}\,, \tag{36}$$

$$\frac{n_{SL}^{\text{down}}}{N} = \overline{|u_S|^2\, |t\, d_L + r\, d_S|^2}\,. \tag{37}$$

With both regenerators, the fraction of $K_L K_L$ events is

$$\begin{aligned} \frac{n_{LL}^{\text{both}}}{N} &= \overline{|t\, u_L + r\, u_S|^2\, |t\, d_L + r\, d_S|^2} \\ &= \overline{\left|rt(u_L d_S + u_S d_L) + t^2\, u_L d_L + r^2\, u_S d_S\right|^2}\,. \end{aligned} \tag{38}$$

In Appendix, assuming Eqs. (34), (35), and (38), i.e. the premises of one-particle quantum theory, demonstrations are made of Ineqs. (65), (66), and (74), involving the numbers of events n_{LL}^{up}, n_{LS}^{up}, and n_{LL}^{both}. These inequalities can be used to test if the results of the experiment proposed here are consistent with that theory.

There are experimental studies of ϕ-decays into neutral kaons, [10]. The $K_L K_L$ and the $K_S K_S$ modes are known to be suppressed with respect to the $K_L K_S$ and the $K_S K_L$ modes. These studies were made without regenerator, as in the setup of Fig. 1a. Therefore we can also expect the number of $K_L K_L$ and of $K_S K_S$ events to be small in the experiment in the setup without regenerator. Suppose it is the case and suppose the numbers of events obtained from the experimental data are introduced in Ineqs. (65), (66), and (74). The constraints imposed by one-particle quantum theory reduce to

$$|t| \gtrsim \sqrt{\frac{n_{LS}^{\text{up}}}{n_{LS}^{0}}}\,, \tag{39}$$

$$|r| \gtrsim \sqrt{\frac{n_{LL}^{\mathrm{up}}}{n_{SL}^{0}}}\ , \tag{40}$$

thus
$$n_{LL}^{\mathrm{both}} \gtrsim \frac{(n_{LL}^{\mathrm{up}} n_{LS}^{\mathrm{up}})(n_{LS}^{0} + n_{SL}^{0})}{n_{LS}^{0} n_{SL}^{0}}\ . \tag{41}$$

Ineq. (41) is violated by the predictions of orthodox quantum mechanics. Indeed, if the predictions of Eqs. (2), (3), (7), and (8) hold, then Ineq. (74) predicts

$$n_{LL}^{\mathrm{both}} \gtrsim |n_{LL}^{\mathrm{int}}|\ , \tag{42}$$

where $|n_{LL}^{\mathrm{int}}|$ is the value indicated on the last row of Table 1. Orthodox quantum theory predicts $n_{LL}^{\mathrm{both}} = 0$. It follows that the proposed experiment can discriminate between one-particle- and orthodox quantum theory.

3.2 Uncorrelated Hidden Variables.

If the predictions of quantum theory turn out to be correct in one of the tests of Sect. 2, other local theories would be eliminated than just the one-particle quantum theory. Consider a theory that pictures regenerators as systems of atoms fluctuating all the time and having time dependent regeneration properties. The theory would associate different values of some hidden variables Λ^{up} and Λ^{down} to the different configurations of the "up" and of the "down" regenerators, respectively; predictions for the outcome of an experiment would imply averaging over all the values taken by these hidden variables Λ^{up} and Λ^{down}.

The constraints imposed by such a theory to its predictions can be derived as it was done for one-particle quantum theory in the Appendix of this paper. Instead of constant probability amplitudes t and r, one deals with amplitudes of transmission and regeneration in the "up" regenerator that are functions $t(\Lambda^{\rm up})$ and $r(\Lambda^{\rm up})$; $t(\Lambda^{\rm down})$ and $r(\Lambda^{\rm down})$ in the "down" regenerator. These amplitudes fluctuate with $\Lambda^{\rm up}$ and $\Lambda^{\rm down}$. For the predictions, averages have to be taken over the values of the functions $t(\Lambda^{\rm up})$, $r(\Lambda^{\rm up})$, $t(\Lambda^{\rm down})$, and $r(\Lambda^{\rm down})$ as well as over u_L, u_S, d_L, and d_S.

For the sake of simplicity, in this paper, the demonstration will be restricted to thec ies of this type but where, at the time of the ϕ-decay, there are no K_LK_L and no K_SK_S, only K_LK_S and K_SK_L systems. Eqs. (4) and (5) are satisfied in the setup without regenerators, but not necessarily Eqs. (2) and (3). Let us define the random quantities

$$p^{\rm up}_{\rm thru} = \mid t(\Lambda^{\rm up}) \mid^2 , \tag{43}$$

$$p^{\rm up}_{\rm regen} = \mid r(\Lambda^{\rm up}) \mid^2 , \tag{44}$$

$$p^{\rm down}_{\rm thru} = \mid t(\Lambda^{\rm down}) \mid^2 , \tag{45}$$

$$p^{\rm down}_{\rm regen} = \mid t(\Lambda^{\rm down}) \mid^2 . \tag{46}$$

They are the transmission and the regeneration probabilities of an "up" kaon in the "up" regenerator when its variables have the value $\Lambda^{\rm up}$; and the transmission and regeneration probabilities in the "down" regenerator when its variables are $\Lambda^{\rm down}$, respectively. With only the "up" regenerator in place, K_LK_L events have a "down" K_L, therefore the "up" kaon was born K_S and must have been regenerated into K_L when traversing the "up" regenerator. Similarly, K_LK_S events are due to systems born K_LK_S with the "up" K_L surviving the traversal. Averaging over the variables $\Lambda^{\rm up}$:

$$n^{\rm up}_{LL} = n^0_{SL} \ \overline{p^{\rm up}_{\rm regen}} , \tag{47}$$

$$n^{\rm up}_{LS} = n^0_{LS} \ \overline{p^{\rm up}_{\rm thru}} . \tag{48}$$

Similarly, with only the "down" regenerator in place :

$$n_{LL}^{\text{down}} = n_{LS}^{0} \; \overline{p_{\text{regen}}^{\text{down}}} \,, \tag{49}$$

$$n_{SL}^{\text{down}} = n_{SL}^{0} \; \overline{p_{\text{thru}}^{\text{down}}} \,. \tag{50}$$

In the setup with both regenerators, the number n_{LL}^{both} of $K_L K_L$ events is the sum of the number of "up" K_S regenerated into K_L with the corresponding "down" kaon surviving as a K_L; plus the number of "down" K_S regenerated into K_L with the "up" kaon surviving as a K_L :

$$n_{LL}^{\text{both}} = n_{SL}^{0} \; \overline{p_{\text{regen}}^{\text{up}} \, p_{\text{thru}}^{\text{down}}} \; + \; n_{LS}^{0} \; \overline{p_{\text{thru}}^{\text{up}} \, p_{\text{regen}}^{\text{down}}} \,. \tag{51}$$

In this context, locality can be interpreted as the Λ^{up} variables that influence transmission and regeneration in the "up" regenerator not being statistically correlated to the Λ^{down} variables that influence the same phenomena in the "down" regenerator. Then, in a local theory,

$$\overline{p_{\text{regen}}^{\text{up}} \, p_{\text{thru}}^{\text{down}}} = \overline{p_{\text{regen}}^{\text{up}}} \; \overline{p_{\text{thru}}^{\text{down}}} \,, \tag{52}$$

$$\overline{p_{\text{thru}}^{\text{up}} \, p_{\text{regen}}^{\text{down}}} = \overline{p_{\text{thru}}^{\text{up}}} \; \overline{p_{\text{regen}}^{\text{down}}} \,, \tag{53}$$

Then Eqs. (48), (47), (50), (49), (52), and (53) imply :

$$n_{LL}^{\text{both}} = \frac{n_{LL}^{\text{up}} \, n_{SL}^{\text{down}}}{n_{SL}^{0}} + \frac{n_{LS}^{\text{up}} \, n_{LL}^{\text{down}}}{n_{LS}^{0}} \,. \tag{54}$$

This constraint is abided by the kind of hidden variables theories we have constructed here. It is violated by orthodox quantum mechanics, for which n_{LL}^{up}, n_{SL}^{down}, n_{LS}^{up}, and n_{LL}^{down} are not zero, as shown in Table 1, but n_{LL}^{both} is, as stated in Eq. (13). Therefore an experiment of Sect. 2 would necessarily eliminate either orthodox quantum mechanics or the class of local theories considered here.

For Eq. (54) to hold, it is enough that Eqs. (52) and (53) hold. This is obviously true if not only the regenerators but also the kaons were described with hidden variables Λ^{up} and Λ^{down} (in addition to their wave function), as long as the hidden variables of the "up" kaon are not correlated to the ones of the "down" kaon. Furthermore, for reason of continuity, Eq. (54) would apply approximately if Eqs. (52) and (53) were held only approximately. For the same reason of continuity, Eq. (54) would apply approximately if, contrarily to the restrictive assumption of this section, the number of $K_L K_L$ and of $K_S K_S$ events in setup without regenerator are not zero but almost zero. There is a large class of theories for which Eq. (54) holds approximately. On the contrary, for orthodox quantum mechanics, Eq. (54) is violated by a huge amount, which is of the order of the number of regenerated events of all kinds observed in the experiment. Therefore if the predictions of orthodox quantum mechanics turn out to be valid in an experiment of the kind described in Sect. 2, that whole class of local theories where Eq. (54) holds even approximately would become untenable.

3.3 The EPR-Argument.

There are hidden-variable theories that do not exhibit any contradiction with the orthodox predictions for the experiments of Sect. 2. Consider again the hidden variables Λ^{up} and Λ^{down} of Sect. 3.2. One can construct theories where the variables Λ^{up} and Λ^{down} are not statistically independent. If Λ^{up} and Λ^{down} are correlated in such a way that :

$$p^{\mathrm{down}}_{\mathrm{thru}} = 0 \qquad \text{for any } \Lambda^{\mathrm{up}} \quad \text{for which } p^{\mathrm{up}}_{\mathrm{regen}} \neq 0 \ , \tag{55}$$

$$p^{\mathrm{up}}_{\mathrm{thru}} = 0 \qquad \text{for any } \Lambda^{\mathrm{down}} \text{ for which } p^{\mathrm{down}}_{\mathrm{regen}} \neq 0 \ , \tag{56}$$

then the prediction $n_{LL}^{\rm both} = 0$ of Eq. (13) is reproduced by Eq. (51), without $n_{LL}^{\rm up}$, $n_{LS}^{\rm up}$, $n_{LL}^{\rm down}$, and $n_{SL}^{\rm down}$ of Eqs. (47), (48), (49), and (50) being zero. Such theories obviously can be made compatible with the predictions of Table 1 and of Eq. (13). Like the non-local interference term in orthodox quantum mechanics, these correlations between $\Lambda^{\rm up}$ and $\Lambda^{\rm down}$ make the number of $K_L K_L$ events dependent on the presence of both regenerators at the same time.

One way to generate the strong correlations of Eqs. (55) and (56) between $\Lambda^{\rm up}$ and $\Lambda^{\rm down}$ is to assume a coordinated evolution of the hidden variables in the two regenerators. Whenever one regenerator is in a state that allows $K_S \to K_L$ regeneration, the other regenerator is in a state that forbids transmission of the conjugate kaon. This has to occur all the time, whatever thickness ℓ is chosen for the regenerators. A simpler solution is to consider the variables $\Lambda^{\rm up}$ and $\Lambda^{\rm down}$ as variables specific of the kaons and not of the regenerators at all. They could correspond to physical quantities generated at the time of the ϕ-decay and carried by each kaon from the ϕ-decay point to the regenerator located on its path. If ϕ-mesons decay only into pairs of kaons described by variables $\Lambda^{\rm up}$ and $\Lambda^{\rm down}$ satisfying Eqs. (55) and (56), the correlation would be generated by a local process, when the two kaons are still located at the same place. Then the transmission and regeneration probabilities in the "up" regenerator would be $p_{\rm thru}^{\rm up}$ and $p_{\rm regen}^{\rm up}$ depending only on $\Lambda^{\rm up}$, i.e. on a local quantity brought in by the incident "up" kaon; the physical processes in the "down" regenerator would have probabilities $p_{\rm regen}^{\rm down}$ and $p_{\rm thru}^{\rm down}$ depending only on the quantity $\Lambda^{\rm down}$ brought in by the "down" kaon. Therefore the theory would not involve any non-local effects.

Though such theory is local and reproduces the predictions of orthodox quantum mechanics, it describes kaons at birth as particles that are not simply K_L or K_S, but that need additional variables $\Lambda^{\rm up}$ or $\Lambda^{\rm down}$ to be described completely. In orthodox quantum mechanics K_L and K_S are pure states of a two-component quantum system. There is a statistical behavior common to all K_L and another behavior common to all K_S. A two-component wave function cannot accommodate for these additional variables $\Lambda^{\rm up}$ and $\Lambda^{\rm down}$. Therefore, according to this theory, quantum mechanics is incomplete in its description of what kaons are in reality.

In their famous paper, [2], Einstein Podolsky, and Rosen (EPR) made an argument that is essentially the same as this one to demonstrate that quantum theory is incomplete, under the assumption that only local phenomena exist in nature. EPR made their argument using a gedanken experiment only remotely related to this one. However there is enough analogy between the argument used by EPR and the one used in this section to call ours "the EPR-argument". The hypothesis of additional variables with the total correlation of Eq. (55) and (56) will be called the EPR-hypothesis.

If an experiment described in Sect. 2 is performed and gives the result predicted by quantum theory, i.e. Table 1 and Eqs. (13), one can conclude that there is coordination between regeneration and transmission processes taking place centimeters apart. These processes are either non-local or they depend on more variables than the ones used by quantum theory.

If the predictions of quantum theory hold in such experiment, it is possible yet to resolve the ambiguity left between the "non-local" interpretation and the "additional-variables" one. Another kind of experiment has to be made. That other kind is feasible in principle but more difficult than the one of Sect. 2. That is the subject of the next section.

4 Constructive Interference Tests.

4.1 Experimental Setups.

A contradiction between the predictions of orthodox quantum mechanics and a theory based on the EPR-hypothesis shows up in experiments similar to those of Sect. 2, but with some modifications shown on Fig. 3. Without regenerator (Fig. 3a), the setup is the same as before (Fig. 1a). With regenerators, the setup is changed in such a way that orthodox quantum theory predicts a constructive interference between the two regeneration processes. That interference effect is so strong that orthodox theory predicts more $K_L K_L$ events with both regenerators in place than the sum of the number of $K_L K_L$ events with only the "up" regenerator in place and the number of $K_L K_L$ events with only the "down" regenerator.

The "down" regenerator is moved downstream (Figs. 3c and 3d) from the position it had in the experiment of Sect. 2. In the setup with two regenerators (Fig. 3d), the process where an "up" kaon is regenerated in the "up" regenerator implies that the "down" kaon travels the distance between the e^+e^-–interaction point and the "down" regenerator as a K_L. In the process where the "down" regenerator is regenerated, it travels that distance as a K_S. Because of the K_L–K_S mass difference, orthodox quantum mechanics predicts a phase shift between the amplitudes due to the two processes. This phase shift transforms the destructive interference into a constructive effect.

Unfortunately the number of "down" K_S available for regeneration in

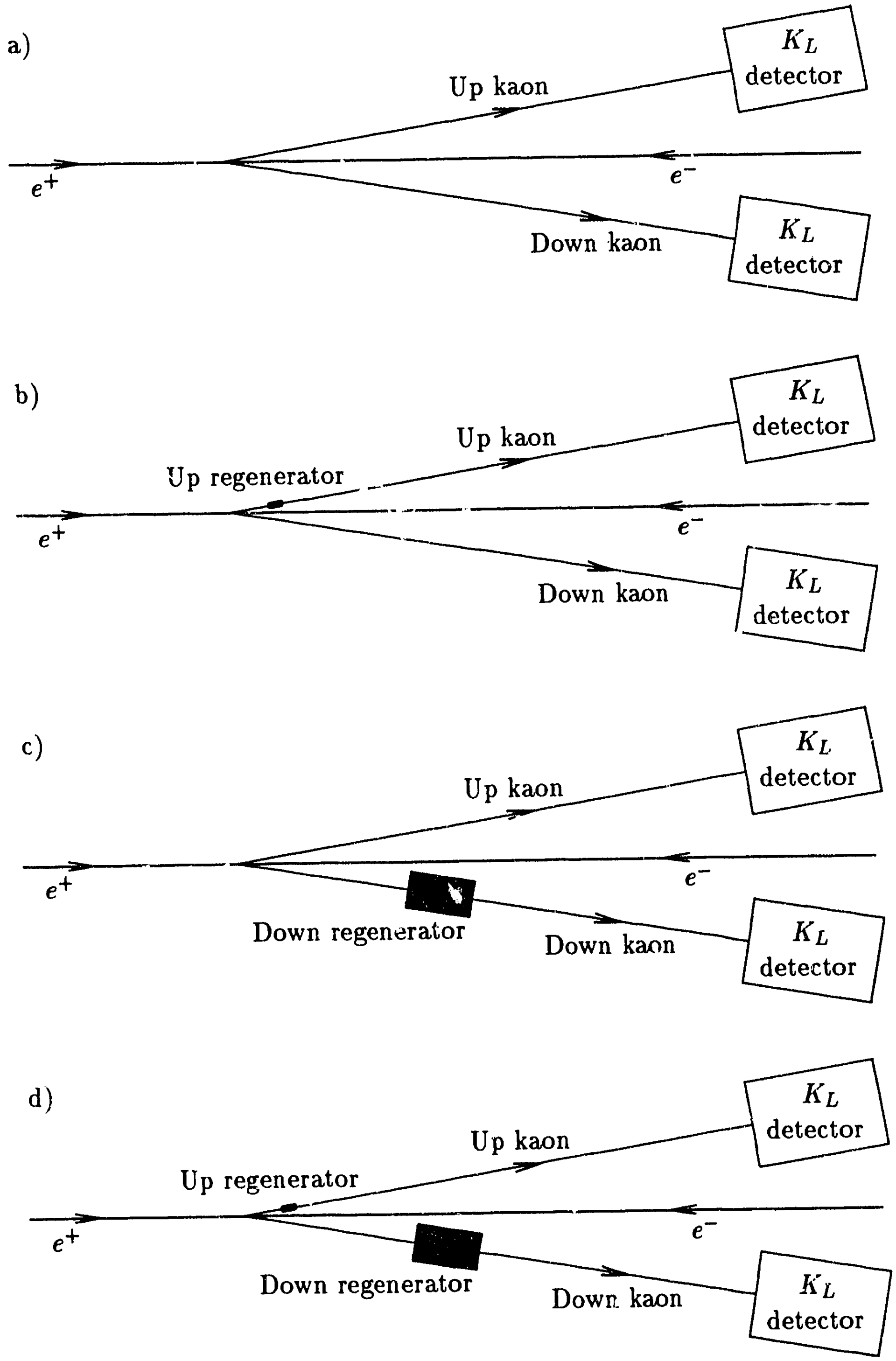

Figure 3: *Setups for the constructive interference test.*

Factory e^+ energy (GeV)	2.0	3.0	9.0
e^- energy (MeV)	130	90	30
Integrated luminosity/setup (pb^{-1})	150	400	600
# of $K^0\overline{K^0}$ events : N (millions)	200	500	1000
Regenerator material	copper	copper	graphite
"Up" regenerator thickness (mm)	4	4	10
"Down" regenerator thickness (cm)	5	5	15
"Down" regenerator distance (cm)	30	42	140
$K_L K_L$ events			
"Up" regenerator : n_{LL}^{up}	1840	1750	1600
"Down" regenerator : n_{LL}^{down}	1100	1290	910
Both regenerators : n_{LL}^{both}	3750	3820	3330

Table 2: *Orthodox predictions for the constructive interference test.*

this new experiment is reduced by a factor of about 100 due to K_S decay between the e^+e^-–interaction point and the "down regenerator. The length of the "down" regenerator has already been optimized more or less for maximum regeneration, taking into account absorption in the material. Therefore the number of regenerated kaons is 100 times less than in Sect. 2. To maximize the contradiction with the EPR-hypothesis, the two regeneration processes in the two regenerators have to contribute a comparable amount. Then the thickness of the "up" regenerator has to be reduced by a factor of 10 or so (Fig. 3b).

With such setups, the computation of the predictions of orthodox quantum mechanics is similar to the one of Sect. 2.2. Results for various experimental conditions are given in Table 2. The conditions in the first column of Table 2 are those of Ref. [9].

4.2 Bell's Inequality.

To show that the predictions of Table 2 are incompatible with the EPR-hypothesis of Sect. 3.3, let us go back to Eqs. (47), (48), (49), (50), and (51). These equations were derived in Sect. 3.2 under the assumptions that there are only K_LK_S and K_SK_L events in the setup without regenerator, that the statistical distribution of the quantity $p^{\rm up}_{\rm thru}$, which determines the probability for a kaon to traverse the "up" regenerator as an unperturbed K_L, and the distribution of the quantity $p^{\rm up}_{\rm regen}$, which determines the chance of a K_S to be regenerated there, are the same whether the "down" regenerator is in place or not; (and vice versa for the probabilities in the "down" regenerator). These assumptions are valid for the EPR-hypothesis. The only difference between the EPR-hypothesis and the theories considered in

Sect. 3.2 is that, for EPR, there is the correlation of Eqs. (55) and (56) between Λ^{up} and Λ^{down}. Yet a constraint can be derived from the fact that the quantities $p_{\text{thru}}^{\text{up}}$ and $p_{\text{thru}}^{\text{down}}$ are transmission probabilities. They are smaller than 1 for all values of Λ^{up} and Λ^{down}. Thus, for the numbers n_{LL}^{up} and n_{LL}^{down} of $K_L K_L$ events measured with one regenerator and the number n_{LL}^{both} measured with both, Eqs. (47), (49), and (51) imply :

$$n_{LL}^{\text{both}} < n_{LL}^{\text{up}} + n_{LL}^{\text{down}} . \tag{57}$$

A correction can be applied to take into account theories where the numbers n_{LL}^{0} and n_{SS}^{0} of $K_L K_L$ and $K_S K_S$ events are not zero in the setup without regenerator (Fig. 3a). Such theories would imply a generalized EPR-hypothesis. The correction amounts to allowing for the existence of some events where the ϕ-meson decays into two K_S particles and both K_S get regenerated. These events are fewer than the number n_{SS}^{0} of $K_S K_S$ events in the setup without regenerator. Therefore, after that correction, Ineq. (57) reads :

$$n_{LL}^{\text{both}} < n_{LL}^{\text{up}} + n_{LL}^{\text{down}} + n_{SS}^{0} . \tag{58}$$

Ineq. (58) is a form of Bell's inequality, [17]. It can be transformed into the form proposed by Clauser and Horne, [18]. Like the other forms of Bell inequalities, it is quite general. It applies to all local theories that give a label K_S or K_L to every kaon.

If a constructive interference test of the kind suggested here is performed and if it confirms the orthodox predictions, it would rule out not only the EPR-hypothesis of Sect. 3.3 and the one-particle quantum theory of Sect. 3.1, but a much larger class of local theories, as it is shown in Ref. [9]. Such test would be more powerful than the destructive interference test of Sect. 2. Unfortunately the rates of interesting events are smaller, for the same integrated luminosity. Thus more luminosity is needed and background problems are expected to be more severe.

In Ref. [9], it is pointed out that, if a constructive interference test of this kind is performed and if it turns out to confirm the predictions of quantum mechanics, there is still a class of local theories that would not get ruled out. These theories escape the constructive interference tests because of ambiguities in determining if a kaon is a K_L or a K_S on an event-to-event basis. The numbers of K_L and K_S can be determined very precisely only from a statistical analysis of the data. In the framework of quantum mechanics, K_L- and K_S-states are well defined (though not quite orthogonal) states. However, it is possible to construct theories where these states well defined in quantum mechanics are no longer relevant. All that a local theory has to reproduce is what can be observed directly, i.e. the distributions of times of decay of the kaons for each mode. These theories that escape the constructive interference tests use hidden variables to describe not only the regeneration processes but also times and modes of decay, without referring to K_L and K_S states. In Ref. [9], it is shown that such theories have very peculiar properties. Therefore a constructive interference test permits to discriminate between orthodox quantum mechanics and a very large class containing not so tortuous local theories.

In Ref. [8], other tests of locality were analyzed and it was also concluded that they could not violate Bell's inequalities. They would be less foolproof than the one suggested here.

5 Summary and Conclusions.

Quantum theory has been tested extensively in optics and atomic physics, not only with single particles but also with systems of particles that are subject to non-local interference effects [5]. This paper suggests testing the existence of non-local interferences in two-kaon system. Systems of two-neutral kaons produced at an asymmetric ϕ-factory would be suitable to check that such non-local interferences do exist in neutral kaon physics, and that they exist to the full extent of the amount that is predicted. In a test of destructive interference, Sect. 2, the quality of a test is expressed by the ratio of the number of fake $K_L K_L$ events in setup with two regenerators in place to the number $|n_{LL}^{\rm int}|$ shown in Table 1.[5]

Theories involving only local phenomena in kaon physics can be constructed in such a way that they are not ruled out by experiments performed to date. The simplest one, called one-particle-quantum theory in this paper, gives different predictions than orthodox quantum mechanics for the tests described in Sect. 2. If a test of this kind were performed and if its results confirmed the orthodox predictions, this one-particle quantum theory would be ruled out. Other local hidden variable theories would also be eliminated, but not all of them. For instance, theories based on the EPR-hypothesis would not.

A larger class of local theories, including theories based on the EPR-hypothesis, gives different predictions than orthodox quantum theory for the constructive interference tests described in Sect. 4. If such a test were

[5]According to the terminology used in optics, the difference between 1 and this ratio is the "visibility."

performed in addition or instead of a test of Sect. 2, and if its results confirmed the predictions of orthodox quantum mechanics, it would rule out not only the one-particle quantum theory and the class of theories eliminated by a test of Sect. 2, but also that larger class of local theories which contains the EPR-hypothesis. Unfortunately, as can be seen from the numbers shown in Table 2, a constructive interference test is also more difficult experimentally. Beside, there would still remain a small class of local theories that would survive. They would be very peculiar theories involving hidden variables not only in the regeneration process but also in the decay times and decay modes of the kaons.

The significance of the tests proposed here comes from the large relative contribution of non-local interference effects in the predictions of quantum mechanics. That large contribution makes tests of *destructive* interference very sensitive to any unexpected reduction of its size. It also makes the predictions of orthodox quantum mechanics very different from the ones predicted by a large class of local theories, which would then be definitely ruled out if the results of at least one of these tests was shown to agree with the orthodox predictions.

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APPENDIX – Proof of Ineq. (41).

First, let us show two inequalities valid for any pair of random variables x and y. The relation

$$\overline{|x+y|^2} = \overline{|x|^2} + \overline{|y|^2} + 2Re\left\{\overline{x^* y}\right\} \ , \tag{59}$$

and Schwarz inequality imply :

$$|Re\{\overline{x^* y}\}| \ \leq \ \sqrt{\overline{|x|^2}}\sqrt{\overline{|y|^2}} \ , \tag{60}$$

$$\sqrt{\overline{|x+y|^2}} \ \leq \ \sqrt{\overline{|x|^2}} + \sqrt{\overline{|y|^2}} \ , \tag{61}$$

$$\sqrt{\overline{|x+y|^2}} \ \geq \ |\sqrt{\overline{|x|^2}} - \sqrt{\overline{|y|^2}}| \ . \tag{62}$$

Ineq. (61) can be applied to $x = t\ u_L d_L$ and $y = r\ u_S d_L$ figuring in Eq. (34) of Sect. 3.1. Using also Eqs. (23) and (25) :

$$\sqrt{n_{LL}^{\rm up}} \leq |t|\sqrt{n_{LL}^0} + |r|\sqrt{n_{SL}^0} \ . \tag{63}$$

Ineq. (61) can also be applied to $x = t\ u_L d_S$ and $y = r\ u_s d_S$ figuring in Eq. (35). Then Eqs. (24) and (26) give :

$$\sqrt{n_{LS}^{\rm up}} \leq |t|\sqrt{n_{LS}^0} + |r|\sqrt{n_{SS}^0} \ . \tag{64}$$

It is known that ϕ-mesons decay more into the $K_L K_S$ and $K_S K_L$ modes than into the $K_L K_L$and $K_S K_S$ ones, [10]. Therefore we can expect n_{LL}^0 and n_{SS}^0 to be smaller than n_{SL}^0 and n_{LS}^0 in setup without regenerators. Then, we can resolve Ineqs. (63) and (64) into

$$|t| \ \geq \ \frac{\sqrt{n_{LS}^{\rm up} n_{SL}^0} - \sqrt{n_{LL}^{\rm up} n_{SS}^0}}{\sqrt{n_{LS}^0 n_{SL}^0} - \sqrt{n_{LL}^0 n_{SS}^0}} \ , \tag{65}$$

$$|r| \geq \frac{\sqrt{n_{LL}^{\rm up} n_{LS}^0} - \sqrt{n_{LS}^{\rm up} n_{LL}^0}}{\sqrt{n_{LS}^0 n_{SL}^0} - \sqrt{n_{LL}^0 n_{SS}^0}} \ . \tag{66}$$

Furthermore Ineq. (62) can be applied to

$$x = rt(u_L d_S + u_S d_L) \tag{67}$$

and

$$y = t^2 u_L d_L + r^2 u_S d_S \ , \tag{68}$$

appearing in Eq. (38). Then

$$\sqrt{\frac{n_{LL}^{\rm both}}{N}} \geq |r||t|\sqrt{\overline{|u_L d_S + u_S d_L|^2}} - \sqrt{\overline{|t^2 u_L d_L + r^2 u_S d_S|^2}} \ . \tag{69}$$

Ineq. (61) can be also applied to the second term of the second hand of Ineq. (69), identifying

$$x = t^2 \ u_L d_L \ , \tag{70}$$

and

$$y = r^2 \ u_S d_S \ . \tag{71}$$

$$\sqrt{\overline{|t^2 u_L d_L + r^2 u_S d_S|^2}} \leq |t|^2\sqrt{\frac{n_{LL}^0}{N}} + |r|^2\sqrt{\frac{n_{SS}^0}{N}} \tag{72}$$

Finally, Schwarz inequality, as well as Eqs. (23), (24), (25), and (26) give :

$$\begin{aligned}\overline{|u_L d_S + u_S d_L|^2} &= \overline{|u_L|^2|d_S|^2} + \overline{|u_S|^2|d_L|^2} + 2Re\left\{\overline{u_L^* d_L u_S d_S}\right\} \\ &\geq \frac{n_{LS}^0}{N} + \frac{n_{SL}^0}{N} - 2\frac{\sqrt{n_{LL}^0 n_{SS}^0}}{N} \end{aligned} \tag{73}$$

Therefore

$$\sqrt{n_{LL}^{\rm both}} \geq |r||t|\sqrt{n_{LS}^0 + n_{SL}^0 - 2\sqrt{n_{LL}^0 n_{SS}^0}} - |t|^2\sqrt{n_{LL}^0} - |r|^2\sqrt{n_{SS}^0} \tag{74}$$

Assuming that n^0_{SS} and n^0_{LL} are not only smaller but much smaller than n^0_{LS} and n^0_{SL} (because that is the way ϕ-decays have been seen), the second hand of Ineq. (74) increases with t and r. Thus, the minimum of n^{both}_{LL} allowed by Ineq. (74) corresponds to the minimum values admissible for t and r. Such minimum values can be obtained by equations where the $\geq$ signs have been replaced by an $=$ sign in Ineqs. (65) and (66) and can be plugged into Ineq. (74). The resulting inequality is a consequence of our local hypothesis.

If the numbers n^0_{LL} and n^0_{SS} of $K_L K_L$ and $K_S K_S$ events are found to be negligible in the setup without regenerator, Ineq. (74) becomes Ineq. (41).

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