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EFFICIENCY OF FAST WAVE CURRENT DRIVE FOR A WEAKLY RELATIVISTIC PLASMA

by

S.C. CHIU, C.F.F. KARNEY,* and Y.R. LIN-LIU

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*Princeton Plasma Physics Laboratory

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S.C. CHIU and Y.R. LIN-LIU
General Atomics, San Diego, CA 92186-9784

C.F.F. KARNEY
Princeton Plasma Physics Laboratory, Princeton, NJ 08543

ABSTRACT

Current drive by fast waves (FWCD) is an important candidate for steady-state operation of tokamaks. Major experiments using this scheme are being carried out on DIII–D. There has been considerable study of the theoretical efficiency of FWCD.[1–7] In Refs. 4 and 5, the nonrelativistic efficiency of FWCD at arbitrary frequencies was studied. For DIII–D parameters, the results can be considerably different from the Landau and Alfvén limits. At the high temperatures of reactors and DIII–D upgrade, relativistic effects become important. In this paper, the relativistic FWCD efficiency for arbitrary frequencies is studied. Assuming that the plasma is weakly relativistic, *i.e.*, T_e/mc^2 is small, an analytic expression for FWCD is obtained for high resonant energies ($u_{ph}/u_{T_e} \gg 1$). Comparisons with the results from a numerical code ADJ[7] and the nonrelativistic results[5] shall be made and analytical fits in the whole range of velocities shall be presented.

I. INTRODUCTION

Current drive by fast waves (FWCD) is an important candidate for steady-state tokamak experiments. For this reason, there are major experiments being carried out or planned in DIII–D, Torre-Supra, JET, and other tokamaks. There was also considerable theoretical work.[1] Fisch and Karney[2] and Ehst and Karney[3] have obtained empirical formulae for the Landau and Alfvén limits. The authors[4,5] recently obtained a unified nonrelativistic formula applicable to both limits and in intermediate regimes. This is especially useful for intermediate frequencies such as in DIII–D. As the electron temperature increases to above 10 keV, relativistic effects are expected to be increasingly important. One of the authors[6,7] made fully relativistic studies of the current drive efficiency in the Landau and Alfvén limits. This paper extends the work of Ref. 5 for arbitrary frequencies to include relativistic effects and extends the work of Ref. 6 to intermediate regimes. Analytic expressions for the efficiency are obtained that agree well with the numerical adjoint code ADJ. We find that in the Alfvén limit the nonrelativistic efficiency formula is quite accurate even at energies when relativistic effects are separately important in absorption and current density.

II. WEAKLY RELATIVISTIC EFFICIENCY OF FWCD

For all practical purposes, the assumption that $\lambda^{-1} = T_e/mc^2 \ll 1$ is valid, *i.e.*, the plasma is weakly relativistic. We shall assume this in the following; nevertheless, the resonant electron energy can still be fully relativistic.

We express momentum per unit mass W in units of $V_{T_e} = (T_e/mc)^{1/2}$, also the efficiency $J_\parallel/P_D$ in units of $2e/m\nu_0 V_{T_e}$ where ν_0 is the electron collision frequency. Then

$$\eta = J_\parallel/P_D = G(W_p)/D_A(W_p) \quad , \tag{1}$$

where W_p is the resonant momentum per unit mass. For a Maxwellian distribution, the absorption function $D_A(W_p)$ can be expressed in terms of elementary functions. For large λ (the weakly relativistic approximation),

$$D_A(W_p) \simeq \left[\frac{2}{\gamma_p^3} + \frac{\alpha_R - 1}{\gamma_p}\left(1 + \frac{1}{\gamma_p}\right) + \frac{\alpha_2}{\gamma_p}\right] - \frac{1}{\lambda\gamma_p^2}\left[\frac{2(\alpha_R - 1)}{\gamma_p} + \alpha_2\right]$$
$$+ \frac{1}{\lambda^2\gamma_p^3}\left[\frac{6}{\gamma_p^2} + \frac{6(\alpha_R - 1)}{\gamma_p} + 2\alpha_R\right] + O(\lambda^{-3}) \quad , \tag{2}$$

where

$$\alpha_R + i\alpha_I = -\frac{c^2\Omega_e}{V_{T_e}^2\omega}\frac{D(\epsilon_{33}^R - i\epsilon_{33}^I)}{|\epsilon_{33}^2|(S - n_\parallel^2)} \quad , \tag{3}$$

S, D, ϵ_{33} are plasma dielectric functions, W_p is the resonant momentum, and $\alpha_2 = (\alpha_R - 1)^2 + \alpha_I^2$. One can derive analytic expressions for $G(W_p)$ in the high $[G_H(W_p)]$ and low $[G_L(W_p)]$ energy limits. In the high energy limit, $G_H(W_p)$ is fairly complicated, although it is in terms of elementary functions and can be efficiently calculated. We only write down the leading order term in a W_p^{-1} series:

$$G_H(W_p) \simeq W_p^2 a_1(\gamma_p)\left[\frac{2}{\gamma_p^3} + \frac{2(\alpha_R - 1)}{\gamma_p^2} + \frac{\alpha_2}{\gamma_p}\right] \quad , \tag{4}$$

where γ_p is the relativistic-γ of resonant electrons,

$$a_1(\gamma_p) \approx \frac{4}{\gamma_p^2(Z + 1 + 4\kappa_1\gamma_p)} \quad , \tag{5}$$

where $\kappa_1 = K_1(\lambda)/K_2(\lambda)$, K_i are modified Bessel functions. On the other hand, the low energy limit is given by

$$G_L(W_p) = \frac{1}{W_p}\left[\frac{1}{4}I_4(\infty) + (\alpha_R - 1)I_2(\infty) + \alpha_2 I_0(\infty)\right] \quad , \tag{6}$$

where

$$I_n(\infty) = \int dW \frac{2\pi V_{T_e}^3 f_M}{n_0}\frac{\chi_s}{\gamma^2}W^n \quad , \tag{7}$$

χ_s being the Spitzer function, f_M the Maxwellian. $I_n(\infty)$ are functions of Z and λ and can be tabulated or approximated by simple functions. We can obtain a smoothly joint function of $G(W_p)$ as follows:

$$G(W_p) = e^{\lambda(\gamma_p - 1)}G_L(W_p) + \left[1 - e^{-\lambda(\gamma_p - 1)}\right]G_H(W_p)$$
$$+ \frac{W_p e^{-\lambda(\gamma_p - 1)}}{\gamma_p(Z + 1 + \kappa_1\gamma_p)}\left\{\frac{A + BZ}{Z + 1} + \frac{C\alpha_R}{Z + 4}W_p^2\right\} \quad , \tag{8}$$

where $A = 11.5\,\alpha_2 + 23\,\alpha_R - 1.5$, $B = 7.5\,\alpha_2 + 15\,\alpha_R + 4.5$, and $C = 8$. There are weak temperature dependences in A, B, C which can be neglected for parameters of interest. The exponential factors are suggested by the background Maxwellian.

The analytic expression $\eta_A = J_\parallel/P_D$ is compared with the numerical evaluation by the adjoint code ADJ, and η_{NR} for the non-relativistic limit of η. In the relativistic calculations, the bulk temperature is taken to be 25 keV. Figure 1 is a comparison of η_A with results from ADJ for various values of Z. The agreement is quite good. Figure 2 compares the relativistic calculations for the Alfvén ($\alpha_R = \alpha_I = 0$) and Landau ($\alpha_R \gg 1$) limits at 25 keV with non-relativistic calculations. The striking characteristic is that the non-relativistic η_{NR} appears quite accurate in the Alfvén limit even at relativistic energies ($\gamma_p \lesssim \sqrt{2}$), while in the Landau limit the divergence between η_{NR} and η (relativistic) occurs at much lower energies ($\sim$ 100 keV). The result at the Alfvén limit is surprising because relativity is supposed to have an effect at $\gamma = \sqrt{2}$. In a closer look at the absorption (Fig. 3), we find that the relativistic effect is actually quite prominent at lower energies. Apparently, when $\alpha_R = \alpha_I = 0$, the relativistic decrease in $j_\parallel$ and in P_D are similar so that the ratio is rather insensitive to λ, the bulk temperature. The accuracy of Eq. (5) becomes worse as γ_p becomes larger than about $\sqrt{2}$. This can be improved with a more accurate function. Results will be presented in a more detailed paper.

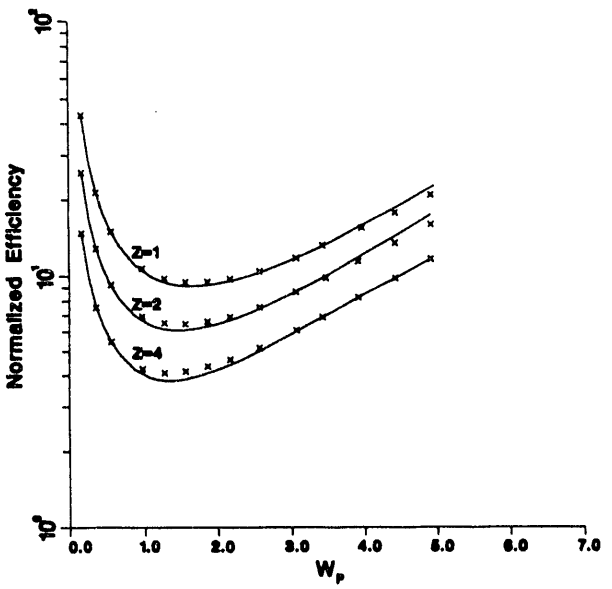

FIG. 1. Normalized J/P versus resonant momentum for $1/\lambda = T_e/mc^2 = 0.05$ for $Z = 1, 2, 4$. Solid lines are from ADJ code; crosses are from the analytic expression.

III. CONCLUSIONS

A general weakly relativistic theory of current drive is presented for parallel acceleration. The analytic expression for efficiency is found to be in good agreement with numerical results from the ADJ code. An interesting result is that the cold efficiency formula is quite accurate up to 0.5 MeV for the Alfvén limit but less so in the Landau limit. From absorption, it is found that even in the Alfvén limit, relativistic effects set in at lower energies, but this is compensated by a similar effect on $j_\parallel$ so that the ratio is not much changed.

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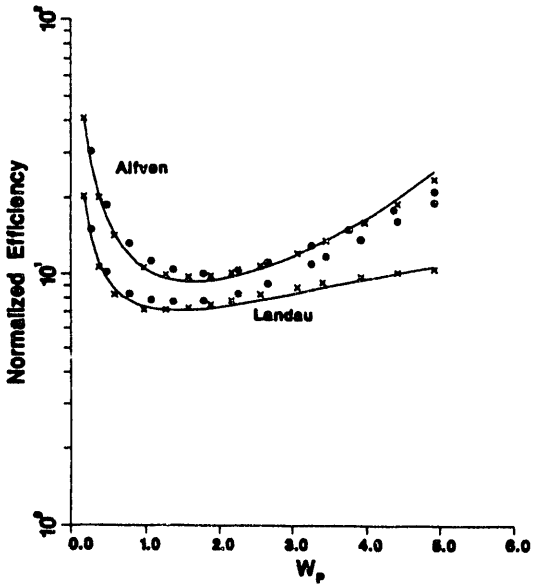

FIG. 2. Comparison between relativistic and non-relativistic J/P for $Z = 1$ at the Landau and Alfvén limits. Solid lines are relativistic ADJ results at $Z = 1$ and $\lambda^{-1} = 0.05$; crosses are from corresponding analytic expression; solid circles are non-relativistic results.

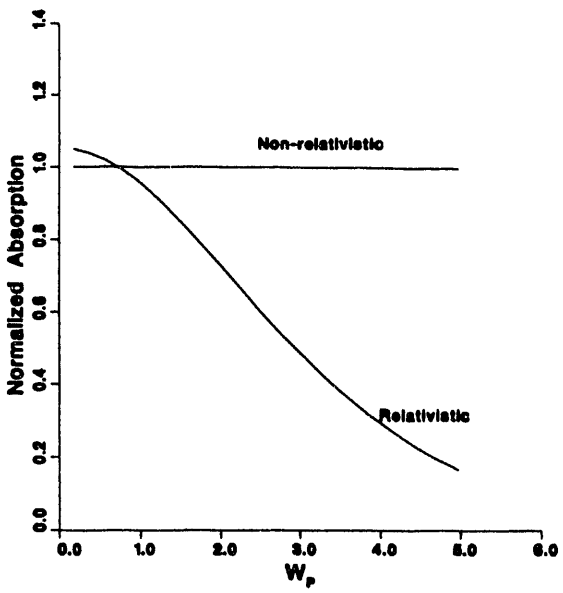

FIG. 3. Normalized absorption D_A versus W_p at the Alfvén limit.

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