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Basic Principles Approach for Studying Nonlinear Alfvén Wave-Alpha Particle Dynamics

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Abstract

An analytical model and a numerical procedure are presented which give a kinetic nonlinear description of the Alfvén-wave instabilities driven by the source of energetic particles in a plasma. The steady-state and bursting nonlinear scenarios predicted by the analytical theory are verified in the test numerical simulation of the bump-on-tail instability. A mathematical similarity between the bump-on-tail problem for plasma waves and the Alfvén wave problem gives a guideline for the interpretation of the bursts in the wave energy and fast particle losses observed in the tokamak experiments with neutral beam injection.

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I Introduction

Progress in confinement of plasmas in tokamaks has reached the stage where researchers can now meaningfully address the plasma physics issues that will arise when there is appreciable partial pressure present in energetic particles produced by the fusion reaction. These particles, unlike most thermal tokamak plasmas, have speeds that resonate with Alfvén waves, at frequencies that are below the diamagnetic drift frequency of the hot particles, which is typically 100 times larger than the diamagnetic frequency of the background plasma. As a consequence, the resonant particle Alfvén wave interaction taps the "universal instability drive" of the hot particles, which in turn may lead to enhanced radial diffusion. The relevancy of the above concern is illustrated by the bursts of rapid particle loss that has been observed in experiments in tokamaks that have been especially designed to simulate the conditions of an ignition system.[1,2]

One goal of the theoretical studies of this problem at the Institute for Fusion Studies, is to understand the nonlinear consequences of the alpha particle-Alfvén wave interaction at a fundamental level. We begin with the presumption that most of the nonlinear physics can be understood as a consequence of weak turbulence theory, where the wave dynamics is basically determined by linear theory, together with the nonlinear consequences of the interaction of the waves with resonant particles.

Several other general properties of a system are needed to define the problem. First of all, the instability driving component is established through the input of a very weak steady source (in the ignition problem this source comes from the fusion products produced by the background plasma; in present day experiments the source arises from the injection of energetic neutral beams). Secondly, when the particles created by the source relax by the classical transport processes inherent in a given system, a steady-state distribution is formed with a shape that can excite microinstabilities. Thirdly, the waves of the background plasma, in the absence of the source, have a finite damping rate. The fourth consideration is that in general the waves have a discrete spectrum, so that at small enough amplitudes of the background wave, a particle resonates with only one wave, which cannot produce global diffusion of the particles.

These considerations define a general problem, which the alpha particle-Alfvén wave interaction satisfies. Hence, considerable basic understanding can be attained by studying more simplified problems, such as the bump-on-tail instability of plasma waves, in which the "bump" is fed by an energetic source, and where simple models are used to describe classical relaxation.

Now suppose the steady-state system is predicted to be linearly unstable based on classical transport theory. Critical questions that can now be analyzed in a systematic way are: (1) whether the excited waves are steady or whether they appear in pulsating bursts, (2) whether the instability produces global diffusion or just local tailoring of the distribution function to achieve stability without greatly changing the predictions of classical transport theory.

The case where global diffusion does occur needs a great deal of further study. However, even for this case, we have achieved significant insight which will be described below.

In this paper we primarily address a simple paradigm model of a bump-on-tail instability. However, along with the description of the paradigm, we will also indicate what the analogous features of the alpha particle-Alfvén wave problem.

II Generic Problem

As a paradigm, we consider the bump-on-tail problem for the excitation of plasma waves. The "bump" is formed from the steady injection of energetic particles. Steady state is achieved by allowing for particle annihilation, which physically can arise from charge-exchange with low energy neutrals. For simplicity we consider only a one-dimensional system with phase space coordinates (x, v).

In the absence of waves, the equation for the evolution of the distribution function f is given by

$$\frac{\partial f}{\partial t} = S(v) - \nu_a(v) f \tag{1}$$

where $S(v)$ is the particle source, and ν_a is the rate of annihilation. The steady-state distribution for Eq. (1) is

$$f_0(v) = \frac{S(v)}{\nu_a(v)} \; . \tag{2}$$

We assume $f_0(v)$ to have shape shown in Fig. 1. In this case the region where $\partial f_0/\partial v > 0$ is the "free energy reservoir" region that can excite plasma waves. We further assume, that the plasma waves can only be excited at discrete phase velocities (in Fig. 1 these are at v_1, v_2, v_3). In addition, in the absence of the beam, these discrete waves are damped from transport processes that are inherent in the background plasma.

[For the alpha particle-Alfvén wave problem the source is the alpha particles, produced by fusion. The relaxation process is predominantly due to the drag on alpha particles from background plasma electrons at a rate ν_α, and pitch-angle scattering from background ions at a rate ν_p. These processes do not cause significant spatial diffusion. As a result, a steady-state "slowing down" distributio forms with a space gradient arising from spatial variation of the source. An analogous plot to Fig. 1, would be $f_0(r)$ as a function of minor radius r (at fixed energy and magnetic moment). The space gradient taps the universal instability drive when $\omega < \omega_* = \dfrac{n}{R\,\omega_{c\alpha}\,m_\alpha}\,\dfrac{1}{n_\alpha}\,\dfrac{d}{dt}\,\langle n_\alpha\,E_\alpha\rangle$ with n the toroidal mode number, n_α the alpha particle density, R the tokamak major radius, E_α the mean alpha particle energy, m_α the particle mass and $\omega_{c\alpha}$ the cyclotron frequency in the poloidal magnetic field].

The shape of the distribution function in Fig. 1 indicates that there is free energy available from the region where $\partial f_0/\partial v > 0$, to self-excite waves. However, with weak instabilities, this free energy can only be tapped if the waves of the system can resonate with particles in the region. For the plasma wave problem the resonance condition is

$$\omega - kv = 0 \ . \tag{3}$$

This resonance produces a contribution, γ_L, to the growth rate that has the following structure,

$$\gamma_L \propto \frac{\partial f}{\partial v} \ . \tag{4}$$

However, for instability to arise, we need γ_L to exceed γ_d, the intrinsic damping rate of the waves in the absence of the energetic particle component. [The analogous parameters for the alpha particle problem is that the resonance condition is given by $\omega - n_\varphi \omega_\varphi - m_\theta \omega_\theta = 0$, where ω is the mode frequency, ω_φ and ω_θ are the frequencies related to the "toroidal" and "poloidal" motions of the particle, and n_φ and m_θ are integers. When the Alfvén frequency is less than the alpha particle diamagnetic frequency, the growth rate γ_L is proportional to $\partial f_0/\partial r$]. When there is instability, we need to consider how to describe the nonlinear evolution of unstable modes. When we have a sufficient number of modes, we can use quasilinear theory. However, this theory is inapplicable in the case of isolated modes. The transition between discrete mode theory and quasilinear theory can be determined from the following consideration. If we have a single mode, the characteristic nonlinear parameter is the trapping frequency, ω_b of resonant particles which is given by

$$\omega_b = \left(\frac{ekE_k}{m}\right)^{1/2} \tag{5}$$

with E_k the electric field amplitude of mode k, with k the wavenumber, m the particle mass [the corresponding trapping frequency for the Alfvén problem is somewhat complicated; it is explicitly given in Ref. 3 and it has the structure that $\omega_b \propto B_*^{1/2}$ with B_* the perturbed amplitude of the magnetic field]. When there is instability, the amplitude grows until

$$\omega_b \sim \gamma_L \ . \tag{6}$$

At this point a single wave will saturate in an undriven system and the single wave has tapped all the free energy it is capable of. The distribution locally flattens around the resonance velocity, $v \equiv v_{ph} = \omega/k$ in an interval $k\Delta v \sim \omega_b \sim \gamma_L$ as shown in Fig. 2 [in the Alfvén wave problem the distribution flattens in space at fixed energy and magnetic moment by an amount given by $\delta r \partial/\partial r(\omega - n_\varphi \omega_\varphi - m_\theta \omega_\theta) = \omega_b$ in the approximation $\omega_\varphi = v_\| / R$ and $\omega_\theta = v_\| / qR$ which is used in Ref. 3, the flattened region is defined by $\delta r = -\omega_b / \frac{\partial}{\partial r}(m_\theta v_\| / qR)$.

One can show that kinetic energy lost by the distribution function has been converted to wave energy and that this balance is consistent with the condition $\omega_b \sim \gamma_L$.

This picture is accurate if this natural saturation width, $\omega_b \sim \gamma_L$ is less than the spacing $k(v_i - v_{i+1})$ between adjacent phase velocities. On the other hand, when a given particle can resonate with many different waves, the interaction is properly described by quasilinear theory if $\gamma_L \gg k(v_i - v_{i+1})$. Hence, there are two major divisions in the nonlinear description. One is where the nonlinear evolution of the waves is determined by discrete mode theory, and the other is where quasilinear theory plays a crucial role.

III Nonlinear Discrete Mode Scenario

As we have already indicated, there is a natural saturation level for a discrete mode of an undriven system, which is determined by setting the trapping frequency equal to the linear growth rate. However, the mode amplitude of a driven system can be pumped to a level above the natural level. The pumping arises because new particles arrive in the resonance region because of the presence of the source and classical transport mechanisms (see Ref. 4). This allows the maintenance of a finite slope $\partial f_0 / \partial r$ in the resonance region. Roughly, the slope is reduced by a factor v_{eff}/ω_b, compared to the zero field amplitude case. [For our paradigm, $\nu_{\text{eff}} = \nu_a$, while for the alpha particle-Alfvén wave problem $\nu_{\text{eff}} = \nu_p (\omega/\omega_b)^2$.] Hence, the wave can grow as long as $\gamma_L \dfrac{\gamma_{\text{eff}}}{\omega_b} > \nu_d$. When equality is achieved, a steady-state wave is predicted whose amplitude is determined by the relation,

$$\gamma_L \frac{\nu_{\text{eff}}}{\omega_b} = \nu_d \ . \tag{7}$$

In order for the steady-state level predicted by Eq. (7) to be correct , it needs to exceed the natural trapping level given by Eq. (6), or equivalently,

$$\frac{\nu_{\text{eff}}}{\nu_d} > 1 \ . \tag{8}$$

It is shown in Ref. 5 that, if the field amplitude predicted by Eq. (7) is below that given by Eq. (6), or equivalently,

$$\frac{\nu_{\text{eff}}}{\nu_d} < 1 \tag{9}$$

that the lower amplitude wave solution is unstable, while the natural amplitude level, given by Eq. (6), cannot be a steady-state solution. The latter assertion follows from a basic energy conservation constraint. If a steady solution were possible, more power would be absorbed by the background plasma by dissipation, than is being injected into the system by the source. As a result there cannot be a steady-state response. Instead, a pulsating response is expected.

Thus the following scenario emerges when Eq. (9) is satisfied. When the distribution function acquires a slope close to the one given by classical theory, a wave grows to the level predicted by Eq. (6) and the distribution function flattens locally as indicated in Fig. 2. At that point the linear drive is saturated, but the persisting dissipation from the background plasma remains, which then damps the wave at a rate ν_d. After the wave is damped, the source can build up the slope of the distribution function, which occurs at a rate ν_{eff}. When the slope in resonance region becomes comparable to the one predicted by the transport theory without waves, the system is ready to produce another spontaneous wave pulse and the cycle repeats itself. During intermediate times of the buildup of the distribution function, its slope is finite, and precursor oscillations can conceivably arise. However, the saturation level of these precursor pulses are low and they cannot flatten the average slope that has been achieved by the distribution function in the interval about the frequency resonance condition given by Eq. (3) [or its generalization in the alpha particle-Alfvén wave problem], and thus the precursor structure does not change the general pulsation nature that has been described.

Even with several waves present, these saturation and pulsation mechanisms apply to each wave separately if the widths of the saturated waves do not overlap with neighboring modes. Hence, with several waves, the saturated waves would cause the distribution function to look like Fig. 3a. [It should be noted that the flattened regions at the different phase velocities do not necessarily arise at the same time.] However, when there is mode overlap of neighboring modes at the natural saturation level, stochastic motion of particles arises when adjacent modes are at comparable amplitudes. Then mode overlap allows individual particles to reach larger phase space regions than is possible when there is no orbit overlap. As a result, with orbit overlap from several modes, the distribution can flatten over a large region of velocity space. This global diffusion causes a drastic change in the saturated distribution as indicated in Fig. 3b. The solid curve indicates the flattened distribution when overlap is not quite fulfilled, while the dotted curve indicates the globally flattened distribution that arises when there is overlap. Observe, that much more particle kinetic energy has to be released to achieve the flattening than arises when overlap does not occur. This apparent loss of kinetic energy has been transformed into wave energy, which, according to quasilinear theory, determining the diffusion rates. We therefore infer that when overlap occurs, there is an explosion in phase space that rapidly flattens the overall distribution function. The large conversion to wave energy that is attained justifies the use of quasilinear theory to describe the explosive phase of the relaxation. [In the alpha particle problem, it is the particle density gradient at constant energy and magnetic moment that flattens as a result of stochastic motion due to the perturbed electric and magnetic fields. In this case energetic particles may diffuse to the boundaries and then be lost at energies comparable to the energy the particles are created at. Such a process then makes the achievement of ignition more difficult. This is the crucial degradation mechanism that we would like to understand and ultimately quantify.]

IV Explosive Pulsation Scenario

The previous arguments indicate that when γ_L exceeds a critical level, which we call γ_{Lc}, there will be an explosive collapse of the distribution function. Now let us consider the evolution of the system when γ_{0L} appreciably exceeds γ_{Lc} where γ_{0L} is the linear growth predicted from classical transport theory. In this case there are three distributions of interest as shown in Fig. 4. The dashed one is the one predicted by classical transport theory. The solid one is the quasilinear plateau that results after the phase space explosion. The dotted curve is the slope of the distribution function when $\gamma_L = \gamma_{Lc}$. After an explosive collapse the distribution function is in the form of the quasilinear plateau shown by the solid curve. The steady source that is present would then allow the distribution function to increase and the slope will rise on the scale of the global relaxation rate ν_{gbl}, which is in balance with the particle input rate $S(r)$ ($\nu_{\text{gbl}} \sim \nu_a$, for our annihilation model, while for the alpha particle problem $\nu_{\text{gbl}} \sim \nu_d$). First γ_L will become large enough to exceed ν_d, which will allow discrete mode instabilities. These modes will saturate at a level determined by Eq. (6) and the distribution function will locally flatten to quench the instability drive. However, the overall average slope of the distribution does change significantly and f and $\partial f/\partial v$ continue to increase in time until the dotted curve is approached. At this time, an onset of instability can trigger an excitation of several modes which cause the rapid transformation of the kinetic free energy of the distribution function into wave energy. This energy conversion stops when the distribution collapses to the flattened plateau-like distribution function. This transformation occurs on the time scale γ_{Lc}^{-1}. Afterwards, the waves damp at the rate ν_d and the system is ready to repeat itself.

We see that in the case of the explosive scenario, the system stores appreciably less energy in energetic particles, than is predicted by classical transport theory. In the alpha particle problem, the alpha particles will be appreciably broadened in space than predicted by classical transport theory and a significant number of energetic alpha particles may even be lost to the walls. One of the principal goals of our investigation is to develop the tools to quantify the description of this radial diffusion and loss process.

V Simulations

To verify our ideas we are in the process of developing numerical procedures and simulation codes for the dynamics of kinetic instabilities driven by fast particles in presence of a steady-state particle source and classical transport processes.

We find that the Alfvén wave-alpha particle problem is mathematically identical to the bump-on-tail problem for plasma waves, except for the structure of particle source and sink. It is possible, however, to include model source and sink terms into the bump-on-tail problem in a physically relevant way.

In our numerical simulations, we use a mapping technique to follow resonant particles on time steps long compared to the wave period, though short compared to a growth time. This technique will be described elsewhere.

Here we present preliminary results for the bump-on-tail problem in a single mode case assuming that the source and annihilation terms are described by Eq. (1). We demonstrate two different nonlinear regimes discussed in Sec. III. The first one is where the background damping rate is sufficiently low so that a steady-state wave can be maintained according to the prediction of Eq. (8). In this simulation the system starts with no energetic particles but with the source turned on. As particles are injected and stored, the resonant particles excite a plasma wave, whose wave energy is shown as a function of time in Fig. 5a. We see that the wave energy approaches a steady state level. The distribution function is shown in Fig. 5b and one should note the nearly flattened distribution function at the resonant velocity, which here is taken at $v = 0$.

The calculated mode amplitude in the steady-state regime agrees well with the analytical estimate given by Eq. (7). This is illustrated by Fig. 6 that shows the dependence of the particle bounce frequency at saturation on the background damping rate.

The difference between the theoretical curve and the simulation results can be attributed to the fact that, at a low damping rate, the plateau around the resonant velocity $v = 0$ (see Fig. 7) extends beyond the interval where the unperturbed distribution function is of constant slope. The second case is for a larger damping rate, which causes the wave energy to pulsate in time, as shown in Fig. 8a. When we examine the shape of the distribution for this case, we see that distribution function near $v = 0$, has an appreciable slope just prior to the explosive onset (see Fig. 8b), and the local distribution is flattened when the wave energy achieves its maximum level (see Fig. 8c).

Studies are continuing for the investigation of the many mode case with the intention of demonstrating the explosive scenario described above. If this work is successful, the numerical tools we are developing for the bump-on-tail plasma wave problem should be applicable to the interesting alpha particle problem.

VI Conclusions

We have outlined a "first principle" approach to the problem of alpha particle interaction with unstable Alfvén modes. There are important similarities between this problem and the classical textbook problem of the bump-on-tail instability which suggest that a general approach to describing an important class of nonlinear self-consistent kinetic problems can be developed. In particular we have shown how several scenarios can occur in weak turbulence theory where mode saturation arises due to the interaction with resonant particles. In terms of theoretical technique, both problems can be treated within a general concept of nonlinear resonance in a Hamiltonian system. This approach allows fast numerical simulation of the system's behavior with the use of mapping techniques. We emphasize that the major saturation mechanism is the nonlinear flattening of the distribution function of the resonant particles whereas the nonlinear mode coupling is not expected to contribute significantly to saturation as long as the wave energy is sufficiently small. The reason that the wave energy can be kept at a relatively low level is that the source of the energetic particles is very weak so that the fraction of energetic particles is typically small. Two different nonlinear scenarios are predicted analytically and verified by our numerical simulations. Depending on the parameter range, the system either reaches a steady state saturation or exhibits quasiperiodic pulsations (bursts). The bursting scenario is a very plausible candidate for the interpretation of the bursts in the wave energy and fast particles losses observed the tokamak experiments with neutral beam injection.

Acknowledgments

This paper is dedicated to Norman Rostoker on the occasion of his official retirement from the University of California, though of course we expect his occasional seminal contributions to the physics of plasmas to continue.

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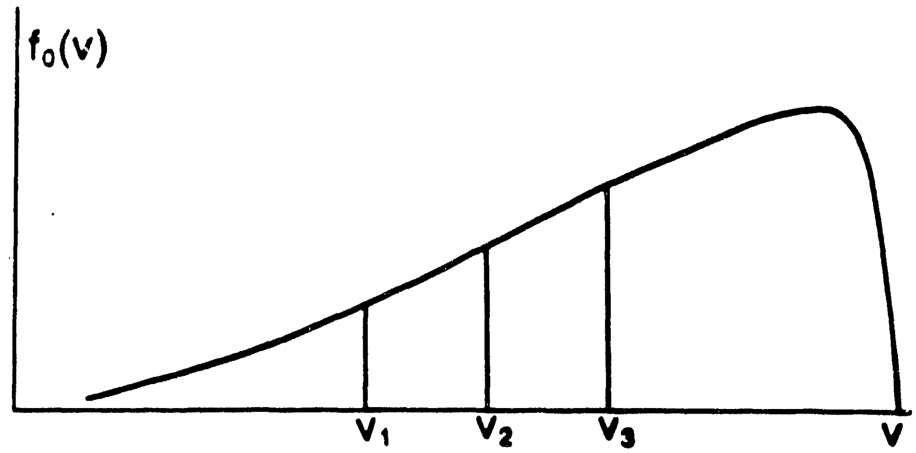

Fig. 1 Steady-state distribution function produced by particle source and annihilation. v_1, v_2 and v_3 refer to the phase velocities of the unstable modes.

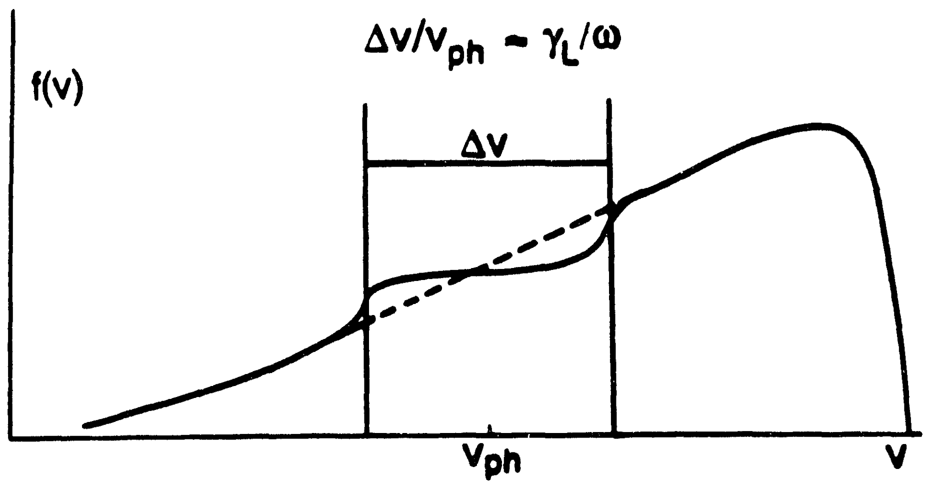

Fig. 2 Flattening of the particle distribution near the resonance due to interaction with an isolated mode. Dashed line shows the unperturbed distribution.

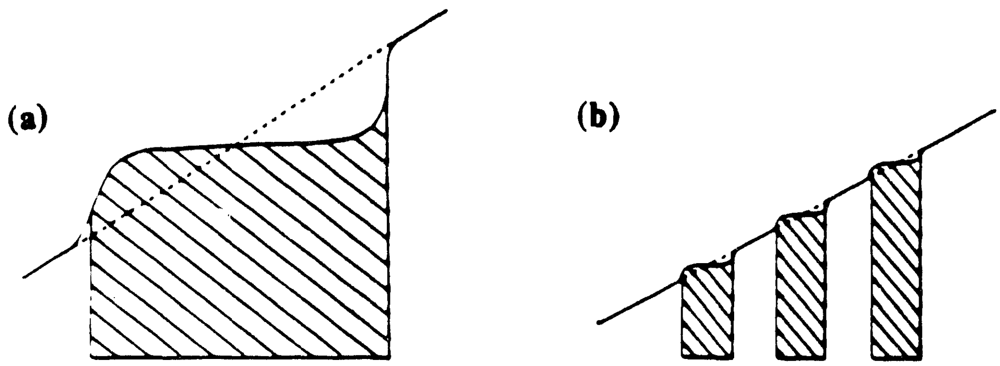

Fig. 3 Effect of resonance overlapping, In (a) modes do not overlap, and the relaxed distribution just has local flattening, with the general shape of the inverted equilibrium distribution preserved. When there is mode overlapping as in (b), the distribution flattens completely over the entire spectrum, with a much larger conversion of free energy to wave energy.

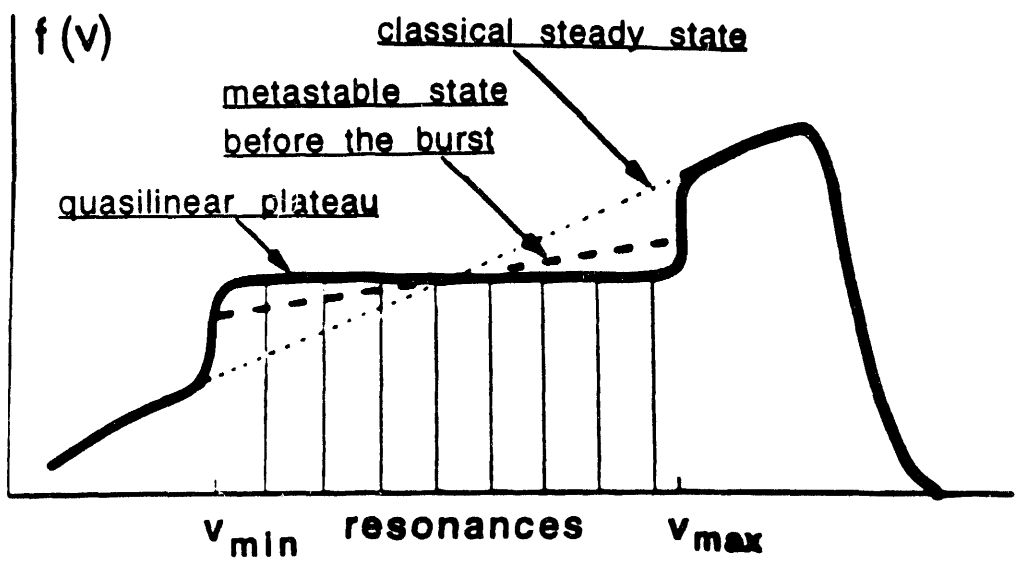

Fig. 4 Quasilinear regime of pulsations with many mode.

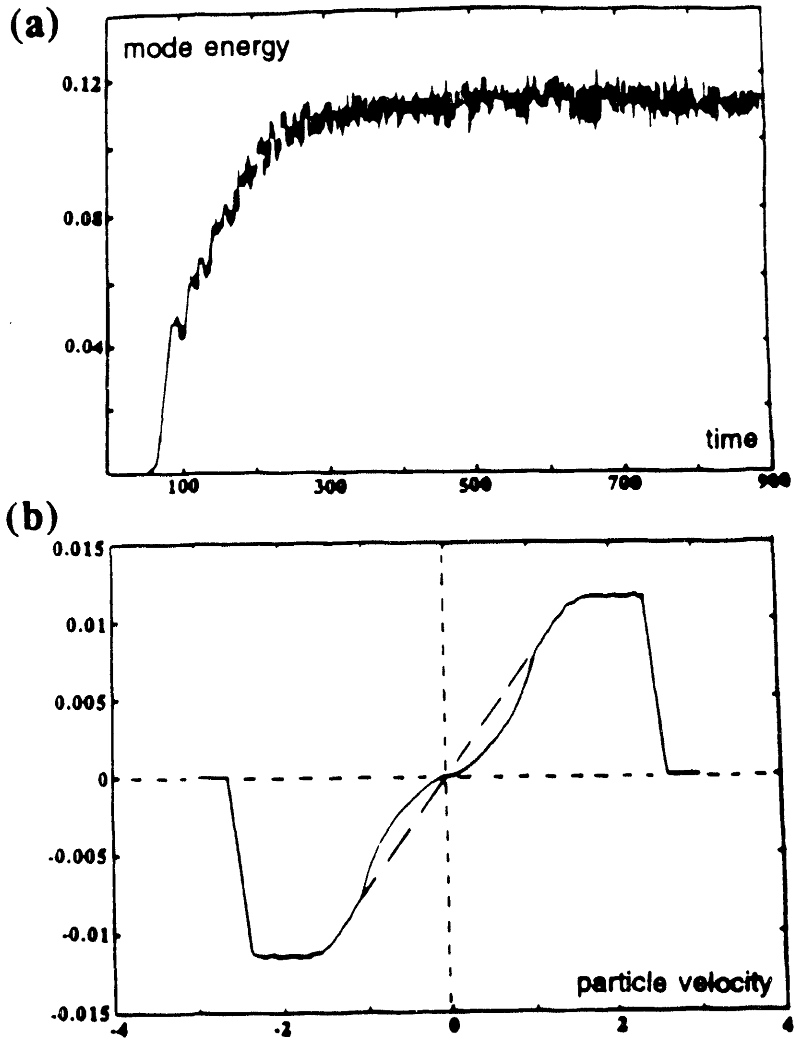

Fig. 5 Steady-state nonlinear saturation of an isolated mode (numerical result). (a) Time dependence of the mode energy. (b) Flattened particle distribution near the resonance: solid line — distribution at mode saturation, dashed line — unperturbed distribution.

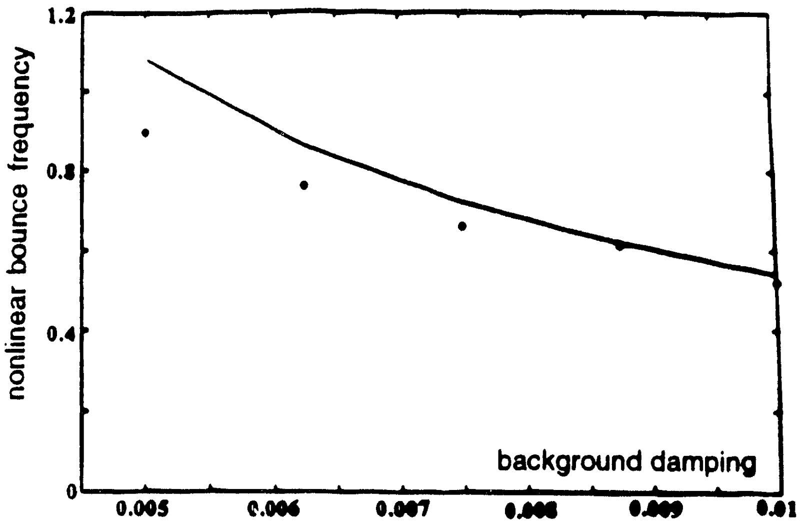

Fig. 6 Scaling law for saturation. — - analytical theory ($\omega_b = 1.9\nu_a\gamma_L/\nu_d$), ◆ -simulations.

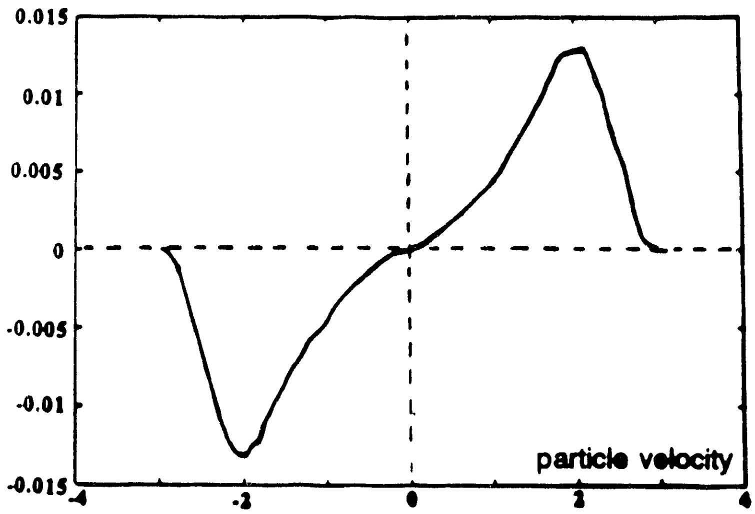

Fig. 7 Particle distribution at mode saturation with small background damping ($\nu_d = 0.005$, Fig. 6).

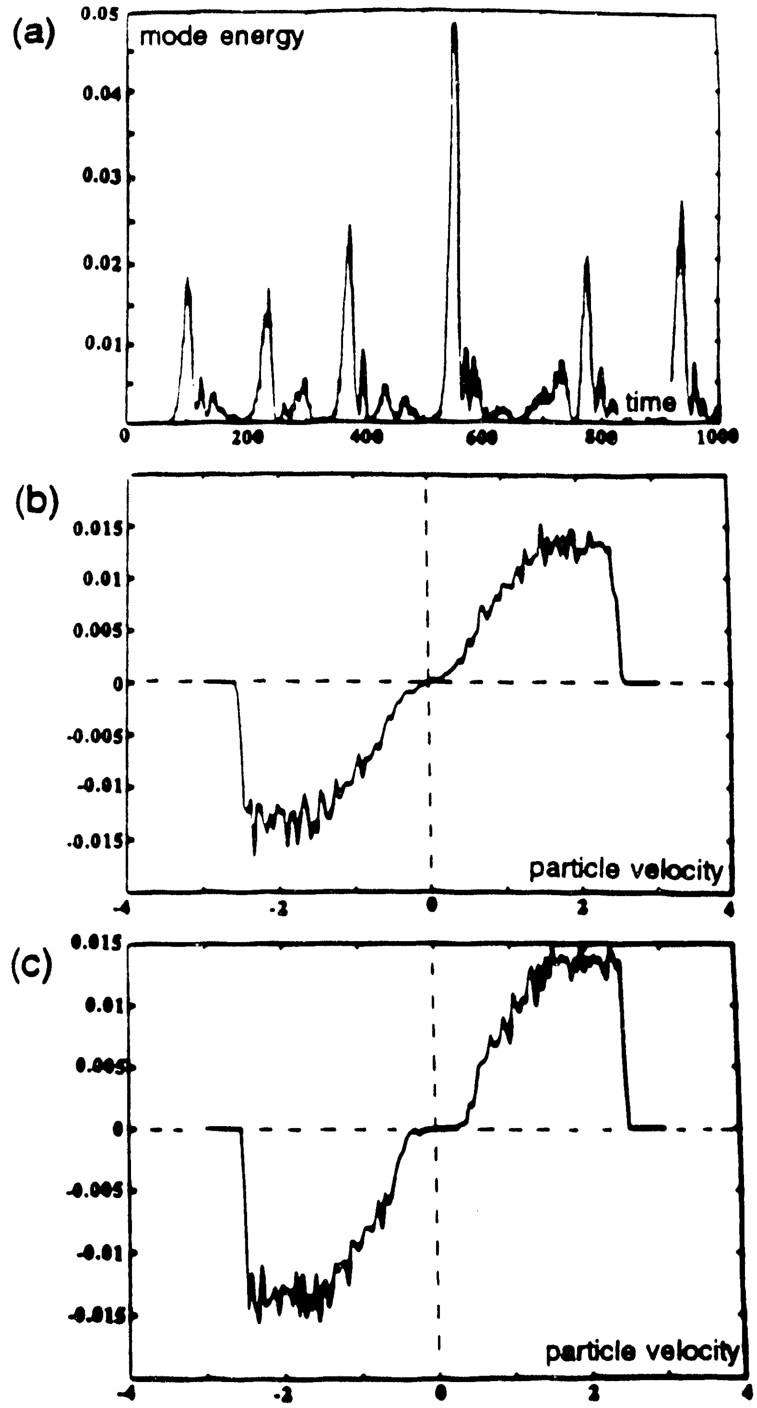

Fig. 8 Nonlinear bursts of an isolated mode ($\nu_d = 10\nu_a$). (a) Time dependence of the mode energy, (b) particle distribution prior to burst ($t = 225$), (c) particle distribution after a burst ($t = 250$).

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