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Title: **CHIRAL LIMIT OF QCD**

Author(s): Rajan Gupta
Theoretical Division
Los Alamos National Laboratory
University of California
Los Alamos, NM 87545
E-mail : rajan@pion.lanl.gov

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Chiral limit of QCD

Rajan Gupta [a]

[a]T-8 Group, MS B285, Los Alamos National Laboratory, Los Alamos, New Mexico 87545 U. S. A.

This talk contains an analysis of quenched chiral perturbation theory and its consequences. The chiral behavior of a number of quantities such as the pion mass m_π^2, the Bernard-Golterman ratios R and χ, the masses of nucleons, and the kaon B-parameter are examined to see if the singular terms induced by the additional Goldstone boson,η', are visible in present data. The overall conclusion (different from that presented at the lattice meeting) of this analysis is that even though there are some caveats attached to the indications of the extra terms induced by η' loops, the standard expressions break down when extrapolating the quenched data with $m_q < m_s/2$ to physical light quarks. I then show that due to the single and double poles in the quenched η', the axial charge of the proton cannot be calculated using the Adler-Bell-Jackiw anomaly condition. I conclude with a review of the status of the calculation of light quark masses from lattice QCD.

1. INTRODUCTION

The main question this review attempts to answer is "should the ostrich care about the alarmists view of quenched QCD"? The alarmists are two groups, Sharpe and collaborators [3] [16] [18] and Bernard and Golterman [1] [2]. They have calculated, using quenched chiral perturbation theory, a number of quantities to 1-loop and point out that in the quenched approximation η' loops give rise to unphysical terms in the chiral expansion, and in many cases the chiral limit is singular. Also, the coefficients in the chiral expansion (including those of the normal chiral logs) are different in the full and quenched theories. The ostrich are the rest of us who wish to continue using the chiral expansions derived for the real world for extrapolating quenched data to the chiral limit. The answer, as I show in this talk, is, unfortunately, YES they should care.

The artifacts due to η' loops can potentially invalidate all the extrapolations to the chiral limit. The hope is that since these are loop corrections and potentially large only in the limit $m_q \to 0$, therefore, there might exists a window in m_q where the leading order chiral expansion is valid and sufficient, albeit with coefficients different from those in full QCD. Extrapolations of the quenched data from this range to the physical light m_u may prove to be sensible, and the difference between the full and quenched coefficients taken as a measure of the goodness of the quenched approximation. With this goal in mind I analyze the existing quenched data in the range $m_s/4 - m_s$ and show that terms induced by the η' are already visible and statistically significant.

In Section 9 I review the status of calculations of $\overline{m}$ and m_s. The quenched Wilson fermion data for $\overline{m}$ is almost a factor of two larger, even at $\beta = 6.4$, than that for quenched staggered or $n_f = 2$ staggered or Wilson fermion data. The estimates of m_s depend on whether K or K^* or ϕ is used to set the strange scale. These systematic differences are much larger than statistical errors and need to be brought under control.

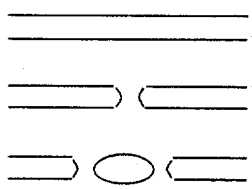
2. QUENCHED CHIRAL PERTURBATION THEORY

Morel [5] gave a Lagrangian description of the quenched theory by introducing ghost quark fields with Bose statistics. This Lagrangian approach has been further developed by Bernard-Golterman into a calculational scheme. To the order we will be concerned with $\mathcal{L}_{BG}$ is

$$\begin{aligned}\mathcal{L}_{BG} &= \frac{f^2}{8}\mathrm{str}\Big[(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) + 2\mu(M\Sigma + M\Sigma^\dagger)\Big] \\ &+ \alpha_0\partial_\mu\Phi_0\partial_\mu\Phi_0 - m_0^2\Phi_0^2 \qquad (1)\end{aligned}$$

where $f = f_\pi = 132\ MeV$ is the pion decay constant, $\Sigma = \exp(2i\Pi/f)$, M is the hermitian

Figure 1. The pseudoscalar propagator, (b) the hairpin vertex, and (c) the one bubble contribution to the η' propagator in full QCD which after summation of all diagrams has the form shown.



$$\frac{1}{p^2 + m^2}$$

$$\frac{1}{p^2 + m^2} \, m_0^2 \, \frac{1}{p^2 + m^2}$$

$$\frac{1}{p^2 + m^2 + m_0^2}$$

quark mass matrix, μ sets the scale of the mass term, and str is the supertrace over quarks and ghost quarks. The last two terms involve the field $\Phi_0 = (\eta' - \tilde{\eta}')/\sqrt{2}$, where $\tilde{\eta}'$ is the ghost field companion to the η'. These terms are treated as interactions and give rise to "hairpin" vertices (see Fig. 1) in the η' propagator. This introduces two new parameters, m_0^2 and a momentum dependent coupling $\alpha_0 p^2$, in the quenched analysis. In the full theory this vertex and the tower generated by the insertion of bubble diagrams sums to give η' its large mass, $m_0^2/(1-\alpha_0)$, while in the quenched theory the η' remains a Goldstone boson and its propagator has a single and double pole.

The strength of the vertex, m_0^2, has been calculated on the lattice by the Tsukuba Collaboration [4] by taking the ratio of the disconnected to connected diagrams. It has also been determined using its relation to the topological susceptibility

$$m_0^2 = 2n_f \chi_t / f_\pi^2 = m_{\eta'}^2 + m_\eta^2 - 2m_K^2 \qquad (2)$$

measured on pure gauge configurations. These methods give $750 < m_0 < 1150\ MeV$. The parameter that occurs repeatedly in the chiral expansion of quenched quantities is $\delta \equiv m_0^2/24\pi^2 f_\pi^2$. Using $f_\pi = 132\ MeV$ and $m_0 = 900\ MeV$, the mean of the above estimates, gives $\delta \approx 0.2$, however, its value could be different, in particular smaller depending on the statistical and systematic errors in lattice data. One of the goals, therefore, is to extract its value from the lattice data for the chiral behavior of as many quenched observables as possible.

Let me first give an intuitive picture of why the η' propagator gives extra contributions. The enhanced logs due to the η' are infrared divergent, so it suffices to consider the $p^2 = 0$ limit in the η' propagator. The single pole term is akin to the pion in the full theory, $1/m_\pi^2$, while the double pole term (due to the hairpin vertex diagram) is $\frac{1}{m_\pi^2} m_0^2 \frac{1}{m_\pi^2}$. Thus any time there is a normal correction term like $m_\pi^2 \mathrm{Ln} m_\pi^2$ from pion loops there will also be a singular term of the form $\frac{m_0^2}{m_\pi^2} m_\pi^2 \mathrm{Ln} m_\pi^2 = m_0^2 \mathrm{Ln} m_\pi^2 \sim \delta \mathrm{Ln} m_\pi^2$. This is exactly what one finds in the chiral expansion for m_π^2. Similarly, in the case of $m_{nucleon}$ the regular chiral correction is $\propto m_\pi^3$, and the η' gives an extra term $\propto m_0^2 m_\pi$. My goal is to expose these extra terms in the present lattice data for different observables, and extract δ from them.

Further details on the formulation of the quenched chiral lagrangian and on the calculation of 1-loop corrections are given in Refs. [1] [3] [16]. The results of the 1-loop corrections in the full and quenched theories show that

- the expansion coefficients are different,
- there are enhanced chiral logs,
- there are no kaon loops with strange sea quarks,
- values for parameters like f, μ,.. are different

in the quenched expressions. I will assume that this difference is implicit in all subsequent discussion even when the same symbols are used for the two theories. Before addressing the question, what are the consequences of these differences for the various physical quantities and are they significant in the present data, I would like to mention the difference in the strategies, after 1-loop corrections have been calculated, of the two groups of alarmists. I find that knowing their respective emphasis helps in reading their papers.

Sharpe and collaborators focus on determining those quantities that can be extracted reliably from quenched simulations. To do this they use real world (or commonly accepted) values to determine the chiral parameters and require that the chiral corrections are small in both the full and quenched expressions, as well as in their difference. Observables satisfying these conditions are the "good" candidates. Bernard and Golterman concentrate on testing quenched χPT by forming ratios of quantities which are (a) free of $O(p^4)$ terms in $\mathcal{L}_{cpt}$ and (b) independent of the ultraviolet cutoff used to regularize loop integration. The quenched chiral expansion of such ratios then have terms proportional to the extra parameter δ. Since these terms can be singular in the chiral limit, it is necessary to assume that there exists a window in quark mass where the 1-loop results are reliable. Then δ can be determined from fits to the quenched expression, provided the fits to the quenched and full theory are significantly different.

3. m_π^2 VERSUS m_q

Gasser and Leutwyler [6] [11] show that in full QCD

$$m_\pi^2 = 2\mu m_q(1+\frac{1}{2}L(m_\pi)-\frac{1}{6}L(m_\eta)+O(m_q)) \quad (3)$$

where $L(m) = m^2\text{Ln}(m^2/\lambda^2)/8\pi^2 f^2$. Bernard and Golterman [1], and Sharpe [3] show that these logs are absent in the quenched approximation. Instead (for $\alpha_0 = 0$) they get

$$(m_\pi^2)_Q = 2\mu m_q(1-\delta\text{Ln}(\frac{m_\pi^2}{\lambda^2})+\ldots). \quad (4)$$

where λ is some typical scale of χSB. This expression has been refined by Sharpe, who summed up the leading logs for the degenerate case $m_u = m_s$. We use his result [10]

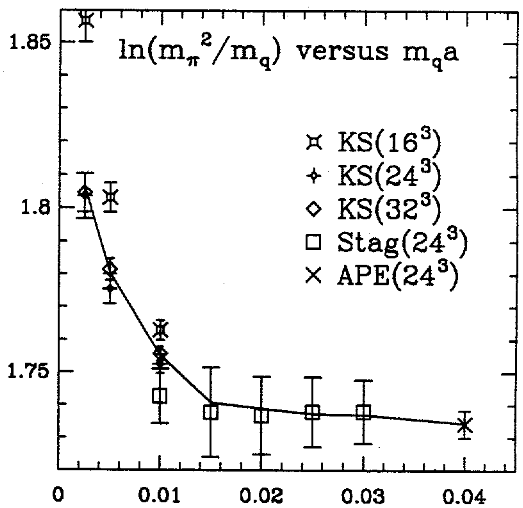
$$\text{Ln}\frac{(m_\pi^2)_Q}{m_q} = c_0 - \frac{\delta}{1+\delta}\text{Ln}m_q + c_1 m_q + c_2 m_q^2 \quad (5)$$

to extract δ from a compendium of staggered fermion data at $\beta = 6.0$ [7][8][9]. Expressing all quantities in lattice units, the best fit gives

$$\text{Ln}\frac{(m_\pi^2)_Q}{m_q} = 1.54-0.044\text{Ln}m_q+1.2m_q-2.8m_q^2. \quad (6)$$

This implies that $\delta \approx 0.053$, *i.e.* much smaller than the full QCD value 0.2; however, the breaking of flavor symmetry in staggered fermions has an interesting consequence for this analysis. The η' operator is a singlet under staggered flavor, and different from the Goldstone pion which has flavor γ_5. Thus one should use the corresponding non-Goldstone pion mass in terms that come from the η'. I use the $\tilde{\pi}$ (which has flavor $\gamma_4\gamma_5$) mass as it is better measured and consistent with

Figure 2. Fit to staggered $\frac{m_\pi^2}{m_q}$ data at $\beta = 6.0$. The estimate for strange quark mass is $m_s a \approx 0.025$



the flavor singlet case. The data and fit using $\tilde{\pi}$ mass in the log term is shown in Fig. 2 and gives

$$\text{Ln}\frac{(m_\pi^2)_Q}{m_q} = 1.35-0.13\text{Ln}m_{\tilde{\pi}}^2+1.5m_\pi^2-2.4m_\pi^4. \quad (7)$$

In this form the coefficient of the $\text{Ln}m_q$ term is δ. Thus $\delta \approx 0.13$, a value consistent with the estimate 0.2 based on the calculation of m_0^2. Also note that since the mass of flavor singlet state, $\tilde{\pi}$, does not vanish as $m_q \to 0$, therefore, there is no singularity at finite a due to the enhanced logs.

The above analysis show that if one wants to extract the value of constant term A_π, the quenched data would give a significantly different result depending on the kind of fit used. If one assumes that the 5 data points by the Staggered collaboration [7] represent a window in which χPT is valid and chiral corrections are negligible, *i.e.* the relation $m_\pi^2 = A_\pi m_q$ is valid (as expected at small enough m_q in full QCD) then one gets $m_\pi^2 = 5.87m_q$ [7], whereas Eq.7 gives $A_\pi \sim 3.9$, a significantly different value.

Finite size effects in m_π increase the value of $(m_\pi^2)_Q/m_q$, so one might attribute the 4% deviation at $m_q = 0.0025$ in Fig. 2 to this artifact. Fortunately, Kim and Sinclair [8] have obtained high statistics data for $m_q = 0.0025,\ 0.005,\ 0.01$

on lattices of size $L = 16$, 24, and 32 as shown in Fig. 2. There is clear indication of finite size effects on $L = 16$ lattices, but the near agreement between $L = 24$ and 32 data confirms that $L = 32$ is essentially infinite volume, *i.e.* all the points used in the fit are, within their statistical errors, free of finite size effects. To conclude, the data show that the lowest order chiral expansion has broken down and the effects of η' logs are manifest for $m_q < m_s/2$. A similar analysis with Wilson fermions is not yet useful because the lowest m_q used in simulations is $\sim 0.4 m_s$, *i.e.* the point where staggered fermions just start to show significant deviations.

4. BERNARD-GOLTERMAN RATIO R AND f_π

The chiral behavior of f_π in full QCD has been analyzed by Gasser and Leutwyler [11] to be

$$f_\pi = f[1 - L(m_\pi) - \frac{1}{2}L(m_K) + f(m_u + m_d + m_s)L_4 + m_u L_5] \quad (8)$$

where L_4 and L_5 are two of the $O(p^4)$ constants introduced by them. In the quenched theory (for $\alpha_0 = 0$) Bernard-Golterman and Sharpe get

$$f_\pi = f(1 + m_u L_5). \quad (9)$$

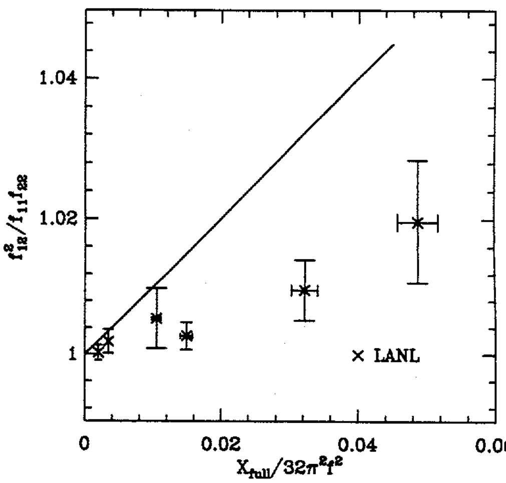
The absence of the pion and kaon chiral logs in the quenched expression is a $13-19\%$ effect (corresponding to the range $\lambda = 0.77 - 1\ GeV$ for the chiral symmetry breaking scale in $L(m)$) using full QCD parameters. In order to compare the full and quenched theories, Bernard-Golterman construct a ratio in a 4-flavor theory that is independent of the cutoff Λ and $O(p^4)$ terms,

$$R \equiv \frac{f_{12}^2}{f_{11'} f_{22'}} \quad (10)$$

where $m_1 = m_{1'}$ and $m_2 = m_{2'}$. The expression for R in the full and quenched theories is

$$R^F = 1 + \frac{1}{32\pi^2 f^2}\left[m_{11'}^2 \mathrm{Ln}\frac{m_{11'}^2}{m_{12}^2} + m_{22'}^2 \mathrm{Ln}\frac{m_{22'}^2}{m_{12}^2}\right]$$
$$R^Q = 1 + \delta\left[\frac{m_{12}^2}{(m_{11'}^2 - m_{22'}^2)}\mathrm{Ln}\frac{m_{11'}^2}{m_{22'}^2} - 1\right]. \quad (11)$$

Figure 3. The Bernard-Golterman ratio R versus the full QCD expression given in Eq.11.



where the quantity within [ ] (called X) increases with the mass difference $m_2 - m_1$.

The quenched Wilson data for R obtained by the LANL [12], UKQCD[13], and Bernard *et al.*[14] collaborations are shown in Figs. 3 and 4 versus the full and quenched expressions given in Eq.11. The slope of the fit to R^Q gives δ, while for R^F the expected slope is unity. The data favor the quenched expression and give $\delta = 0.10(3)$.

The caveat in this case is that the two points at largest X^Q are obtained with $m_2 = 2m_s$ and one could argue that 1-loop chiral perturbation theory is not reliable for these masses. Barring this, I believe that this quantity provides the cleanest determination of δ.

5. BERNARD-GOLTERMAN RATIO χ AND $\langle\bar\psi\psi\rangle$

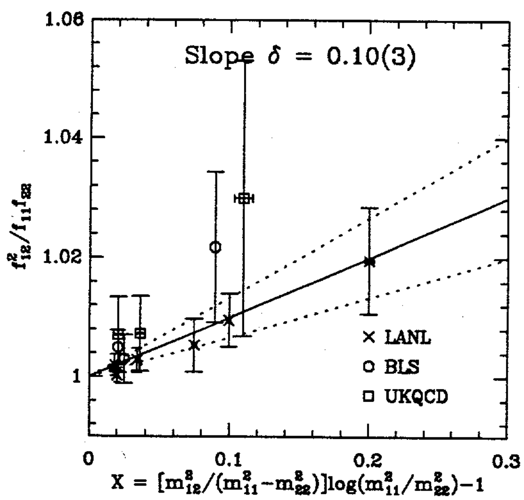
The second quantity constructed by Bernard-Golterman that is independent of Λ and $O(p^4)$ terms is

$$\chi = \frac{\langle\bar d d\rangle}{\langle\bar u u\rangle} - \left(\frac{M_{K^0}^2 - M_{K^+}^2}{M_{K^0}^2 - M_\pi^2}\right)\frac{\langle\bar s s\rangle}{\langle\bar u u\rangle} \quad (12)$$

for which χPT gives

$$\chi_{tree} = \frac{m_s - m_d}{m_s - m_u} \quad (13)$$
$$\chi_Q = \chi_{tree} + \delta[\mathrm{Ln}\frac{m_u}{m_d} - \frac{m_d - m_u}{m_s - m_u}\mathrm{Ln}\frac{m_u}{m_s}]$$

Figure 4. The Bernard-Golterman ratio R versus the quenched expression given in Eq.11.



$$\chi_F = \chi_{tree} + \frac{1}{8\pi^2 f^2}[M^2_{K^+}\mathrm{Ln}\frac{M^2_{K^+}}{M^2_\pi} - \frac{m_s - m_d}{m_s - m_u}M^2_{K^0}\mathrm{Ln}\frac{M^2_{K^0}}{M^2_\pi}]$$

To evalue these expressions requires data for the condensate at three values of m_q and pseudoscalar masses for the combinations $\pi = uu$, $K^0 = sd$, $K^+ = su$. At present only the staggered [7] and Wilson [12] fermion simulations at $\beta = 6.0$ by the LANL collaboration have all the necessary data. Their results for $\delta = (\chi - \chi_{tree})/Y$, where Y is the factor multiplying δ in the expression for χ_Q in Eq.14, are given in Table 1. The staggered data (the difference between Goldstone and non-Goldstone mass in terms that get contribution from η' loops has not been taken into account) has large errors and would give the wrong sign for δ. With Wilson fermions the condensate in the chiral limit can be calculated in two ways, using the GMOR relation or the Ward Identity as explained in Ref. [15]. At finite m_q there are lattice artifacts which we cannot control, nevertheless, the data give reasonable value for δ. This is probably fortuitous and I believe that much better data is needed in order to extract δ from the chiral condensate.

Table 1
The Bernard-Golterman ratio X

	Staggered	Wilson(GMOR)	Wilson(WI)
χ	0.549(30)	0.608(6)	0.614(5)
χ_{tree}	0.517(14)	0.620(2)	0.620(2)
χ_F	0.509(15)	0.616(2)	0.616(2)
δ		0.10(5)	0.05(4)

6. CHIRAL EXTRAPOLATION OF THE NUCLEON MASS

The behavior of baryon masses has been calculated in ChPT and has the general form [6]

$$M_B = \bar{M} + \sum c_i^{(2)} M_i^2 + \sum c_i^{(3)} M_i^3 + O(m_\pi^4 \mathrm{Ln} m_\pi) \quad (14)$$

where M_i are π, K, η meson masses. The term proportional to M_i^3 comes from pion loops and is $25\% - 50\%$ of M_B for the octet. For example, using the results of Bernard *et al.* [19] one finds $M_N = 0.97 + 0.24 - 0.27$ respectively for the first three terms in Eq.14. Thus, the loop corrections in individual masses are large and one could question whether χPT is applicable at all to baryons. On the other hand χPT results for mass differences and the Gellmann-Okubo formula work very well, just as in the quark model. So, it is possible that the loop effects somehow conspire to just shift the overall scale, in which case χPT is useful and it is worthwhile examining the consequences of the quenched approximation.

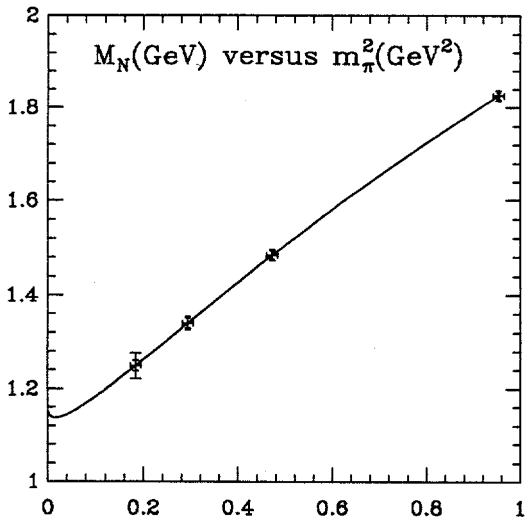
Labrenz and Sharpe [16] have extended the Lagrangian approach of Bernard-Golterman to baryons using the "heavy-quark" formalism of Jenkins and Manohar [17]. They show that along with a modification of the c_i in Eq.14 one gets a $m_0^2 M_\pi$ term due to η' loops. The quenched expression for degenerate quark masses is (assuming $\alpha_0 = \gamma = 0$, where γ is a parameter in the baryon sector of $\mathcal{L}_{cpt}$ and defined in [16])

$$M_B = \bar{M} + c^{(1)}\delta M_\pi + c^{(2)} M_\pi^2 + c^{(3)} M_\pi^3 + \ldots \quad (15)$$

where $c^{(1)} \sim -2.5$, $c^{(2)} \sim 3.4$, and $c^{(3)} \sim -1.5$ using the full QCD parameters.

Fits to lattice data using Eq. 15 are not very reliable because the number of light quark masses

Figure 5. Fit to the LANL nucleon mass data.



explored are typically $3-4$ and only the point at the heaviest mass (typically $m_q \gtrsim 2m_s$) shows any significant deviation from linearity. I find such 4-parameter fits to 4 points very unstable, for example in the case of LANL data [12] even the different JK samples give completely different values of $c^{(i)}$. The best I could do was to fix one of the parameters and make a 3-parameter fit and then vary the fixed parameter to minimize χ^2. The best fit (fixing any one of the less well determined coefficients, $c^{(1)}$, $c^{(2)}$ or $c^{(3)}$, works and gives the same final result on minimizing χ^2) to the LANL data expressed in units of GeV is shown in Fig.5 and gives

$$M_B = 1.16 - 0.36M_\pi + 1.6M_\pi^2 - 0.5M_\pi^3. \qquad (16)$$

Assuming $c^{(1)} = -2.5$, Eq.eq:BaryLANL gives $\delta \approx 0.14$. The same method applied to $\beta = 5.93$ (012 sink) data obtained by the GF11 collaboration[20] gives (this is an updated version of the fit presented in Ref.[16])

$$M_B = 1.18 - 1.0M_\pi + 3.0M_\pi^2 - 1.3M_\pi^3 \qquad (17)$$

which implies $\delta \approx 0.4$, and $c^{(2)}$ and $c^{(3)}$ have values close to those for full QCD.

7. THE KAON B PARAMETER

The kaon B parameter is a measure of the strong interaction corrections to the $K^0 - \bar{K}^0$ mixing. It is one of the best measured lattice quantities. For details of the phenomenology and of the lattice methodology I refer you to Refs.[21][22][23]. Here, I present a summary of just the chiral behavior.

Sharpe has calculated the chiral behavior of B_K in both the full and quenched theories [23]. The full QCD result is

$$B_K = B\left[1-(3+\frac{\epsilon^2}{3})y\mathrm{Ln}y+by+c\epsilon^2y+O(y^2)\right] \quad (18)$$

where $y = m_K^2/(8\pi^2 f^2) \approx 0.2$ and $\epsilon = (m_s - m_d)/(m_s+m_d)$ measures the degeneracy of s and d quarks. B is the leading order value for B_K, which is an input parameter in χPT, and b and c are unknown constants. The quenched result [18]

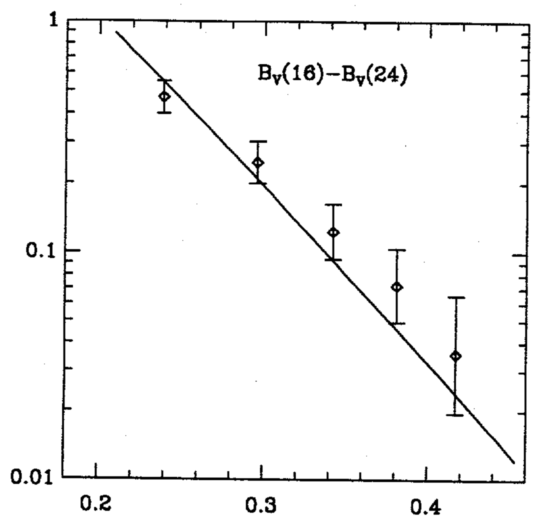
$$\begin{aligned} B_K^Q &= B^Q\Big[1-(3+\epsilon^2)y\mathrm{Ln}y + b^Qy + c^Q\epsilon^2y \\ &\quad +\delta\left(\frac{2-\epsilon^2}{2\epsilon}\mathrm{Ln}\frac{1-\epsilon}{1+\epsilon}+2\right)\Big] \end{aligned} \quad (19)$$

has exactly the same form except for the additional term proportional to δ, which is an artifact of quenching. The term proportional to δ is singular in the limit $\epsilon \to 1$, therefore it will not be possible to extrapolate quenched results to the physical non-degenerate case. For $\epsilon = 0$ this term vanishes, and since there is little incentive to calculate B_K for $\epsilon \to 1$ in the quenched approximation, therefore it is unlikely that we will, in the foreseeable future, be able to extract δ using Eq.19.

The constants B, b, c are different in the full and quenched theories and cannot be fixed by χPT. Assuming $B = B^Q$, then the coefficient of the chiral log term is the same for $\epsilon = 0$. This is the best agreement one can expect between the two theories, therefore, Sharpe advocates that B_K for degenerate quarks has the possibility of being a "good" quantity to calculate using the quenched theory.

Using full QCD values $3y\mathrm{Ln}y \approx 1$; so one can ask whether this normal chiral log is visible in the present data and whether it should be included in the extraction of B_K? With existing data it is hard to distinguish this term from the one linear in y as the range of m_K is not large

Figure 6. Evidence of finite size effects in enhanced chiral logs in B_V.



enough to significantly effect the logarithm. Also, there exist data for $m_q \approx m_s/2$, so for degenerate quarks (which, as explained above, is the best one can do with the quenched theory) there is no need for an extrapolation. However, for staggered fermions B_K can be written as the sum of two terms, $B_K = B_V + B_A$, each of which can be analyzed using χPT. These quantities are defined in Ref. [3] and are explicitly constructed such that they do not diverge as $1/m_K^2$ in the chiral limit. Both B_V and B_A have enhanced logs (terms proportional to Lny and not suppressed by powers of y) that have nothing to do with quenching, *i.e.* are not due to the η'. It is these logs, or more precisely the volume dependence of these logs, that has been seen in lattice data. Sharpe has shown that this volume dependence is of the form

$$\begin{aligned} B_V(L) - B_V(\infty) &= -(B_A(L) - B_A(\infty)) \quad (20) \\ &\approx -b_2 \sqrt{\frac{2\pi}{m_K L}} \frac{6\mu^2 e^{-m_K L}}{8\pi^2 f^2}. \end{aligned}$$

The constant b_2 is not well determined, however, the shape of the m_K dependence is. The staggered fermion data at $\beta = 6.0$ on 16^3 and 24^3 lattices [24] are shown in Fig. 6, and qualitatively confirm the expected finite size effects in the chiral logs.

8. ME OF SINGLET AXIAL CURRENT IN THE PROTON

Ever since the measurement of the spin structure of protons using deep inelastic muon scattering from protons by the EMC collaboration[25], there has been much interest in the calculation of the forward matrix elements of the singlet axial current in the proton, $\langle \vec{p}, s | \bar{q} i \gamma_\mu \gamma_5 q | \vec{p}, s \rangle$. There are two possible Wick contractions that contribute to this matrix element (ME). These connected and disconnected diagrams are discussed in [27]. Since the disconnected diagram is hard to measure, Mandula [26] used the anomaly condition to derive the relation

$$\begin{aligned} \langle \vec{p}, s | A_\mu | \vec{p}, s \rangle s_\mu &= N_f \frac{\alpha_s}{2\pi} \lim_{\vec{q} \to 0} \frac{-i|\vec{s}|}{\vec{q} \cdot \vec{s}} \times \\ & \langle \vec{p}', s | \mathrm{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu}(\vec{q}) | \vec{p}, s \rangle \quad (21) \end{aligned}$$

where $\vec{q} = \vec{p} - \vec{p}'$ and s is the proton's spin vector. The hope was that it would be easier to measure the ME of this purely gluonic operator. Since the η' propagator contributes to this ME at tree level, the question arises whether Eq.21 is valid in the quenched approximation. The answer is NO [27]. Consider the Fourier transform of the anomaly relation

$$\begin{aligned} i q_\mu \langle \vec{p}', s | A_\mu(q) | \vec{p}, s \rangle = 2 m_q \langle \vec{p}', s | P | \vec{p}, s \rangle + \\ N_f \frac{\alpha_s}{2\pi} \langle \vec{p}', s | \mathrm{Tr}\, F\tilde{F} | \vec{p}, s \rangle \quad (22) \end{aligned}$$

Each of the three ME in Eq.22 can be parameterized in terms of form-factors as

$$\begin{aligned} \langle \vec{p}', s | A_\mu(q) | \vec{p}, s \rangle &= \bar{u} i \gamma_\mu \gamma_5 u G_1^A - i q_\mu \bar{u} \gamma_5 u G_2^A, \\ \langle \vec{p}', s | P | \vec{p}, s \rangle &= \bar{u} \gamma_5 u G^P, \\ \langle \vec{p}', s | \mathrm{Tr}\, F\tilde{F} | \vec{p}, s \rangle &= \bar{u} \gamma_5 u G^F. \quad (23) \end{aligned}$$

In the quenched approximation the singularities in these form factors for forward ME with respect to q^2 and due to the η' propagators are

$$\begin{aligned} G_1^A(q^2) & \quad \text{no } \eta' \text{ poles} \\ G_2^A(q^2) &= \frac{a_2}{(q^2 - m_{\eta'}^2)^2} + \frac{a_1}{(q^2 - m_{\eta'}^2)} + \tilde{G}_2, \\ G^P(q^2) &= \frac{p_2}{(q^2 - m_{\eta'}^2)^2} + \frac{p_1}{(q^2 - m_{\eta'}^2)} + \tilde{P}, \\ G^F(q^2) &= \frac{f_1}{(q^2 - m_{\eta'}^2)} + \tilde{F}. \quad (24) \end{aligned}$$

Equating the single and double pole terms gives two relations. Using these and taking the double limit, $q^2 \to 0$ and $m_q \to 0$, gives

$$\begin{aligned} 2M_P G_1^A(q^2=0) &= -a_1 + N_f \frac{\alpha_s}{2\pi}\tilde{F} \qquad (25) \\ &= \frac{2m_q}{m_{\eta'}^2}\left(\frac{p_2}{m_{\eta'}^2} - p_1\right) + N_f\frac{\alpha_s}{2\pi}(\tilde{F} - \frac{f_1}{m_{\eta'}^2}). \end{aligned}$$

The term proportional to $N_f \alpha_s/2\pi$ diverges in the chiral limit and there is no obvious way of extracting the physical answer from it alone. Thus the method fails in the quenched theory.

In the full theory, there are no double poles and an analogous analysis gives

$$\begin{aligned} 2M_P G_1^A(q^2=0) &= -a_1 + N_f \frac{\alpha_s}{2\pi}\tilde{F}, \\ &= N_f\frac{\alpha_s}{2\pi}(\tilde{F} - \frac{f_1}{m_{\eta'}^2}), \qquad (26) \end{aligned}$$

which justifies the use of the anomaly relation.

9. MASSES OF LIGHT QUARKS

In order to extract light quark masses from lattice simulations we use an ansatz for the chiral behavior of hadron masses. Theoretically, the best defined procedure is χPT which relates the masses of pseudoscalar mesons to m_u, m_d, m_s.

The overall scale μ in the mass term of Eq.1 implies that only ratios of quark masses can be determined using χPT. The predictions from χPT for the two independent ratios are [6] [32]

	Lowest order	Next order
$(m_u+m_d)/2m_s$	$\frac{1}{25}$	$\frac{1}{31}$
$(m_d-m_u)/m_s$	$\frac{1}{44}$	$\frac{1}{29}$.

In Lattice QCD it is traditional to make fits to the pseudoscalar spectrum assuming $m_{12}^2 = A_\pi(m_1+m_2)$ and using either m_ρ or f_π to set the scale. (The expression in Eq.5 is not relevant for this discussion since most quenched simulations have $m_q \geq m_s/2$.) A consequence of using just the linear term is that the ratio $m_s/\overline{m} = 25$, *i.e.* these fits can be used to extract either $\overline{m} = (m_u+m_d)/2$ or m_s by using the physical masses for m_π or m_K but not both. (One would get a different number if $O(m_q^2)$ and chiral log terms are included in the relation.) Furthermore, since lattice calculations are done in the isospin limit, $m_u = m_d$, therefore χPT can be used to predict only one quark mass. The mass I prefer to extract, barring the complications of quenched χPT, is $\overline{m}$ as it avoids the question whether lowest order χPT is valid up to m_s. Akira Ukawa reviewed the status of $\overline{m}$ at LATTICE92 [28] and I present an update on it.

To convert lattice results to the continuum $\overline{MS}$ scheme I use

$$m_{cont}(q^*) = m_{latt}(a)\ [1 - \frac{g^2}{2\pi^2}(log(q^*a) - C_m)] \qquad (27)$$

where q^* defined in [29] is chosen to be π/a, $C_m = 2.159$ for Wilson [30] and 6.536 for staggered fermions [31], and the rho mass is used to set the scale. (I have not used the tadpole improvement factor of U_0 [29] in C_m and m_{latt} as this factor cancels in perturbation theory and is a small effect otherwise.) The value of g^2 used in Eq.27 is given by [29]

$$\frac{1}{g^2} = \frac{\langle plaq \rangle}{g_{latt}^2} + 0.025 \qquad (28)$$

to relate it to the continuum $\overline{MS}$ scheme at $q^* = \pi/a$. Finally, all the results are run down to $Q = 2\ GeV$ using

$$\begin{aligned} \frac{m(Q)}{m(q^*)} &= \left(\frac{g^2(Q)}{g^2(q^*)}\right)^{\frac{\gamma_0}{2\beta_0}} \times \\ &\left(1 + \frac{g^2(Q) - g^2(q^*)}{16\pi^2}\left(\frac{\gamma_1\beta_0 - \gamma_0\beta_1}{2\beta_0^2}\right)\right) \qquad (29) \end{aligned}$$

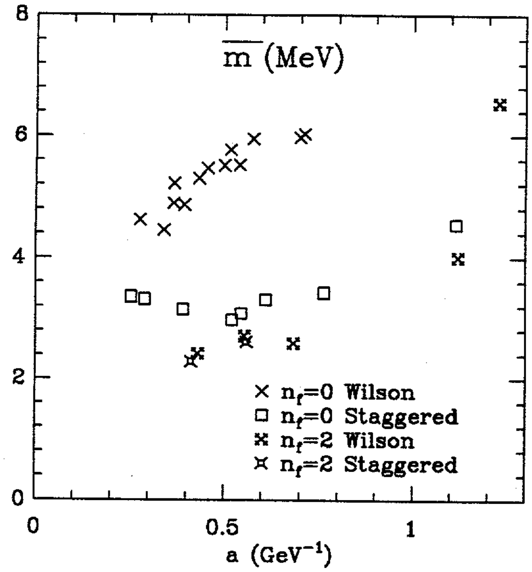
with

$$\frac{g^2(Q)}{16\pi^2} = \frac{1}{\beta_0 \mathrm{Ln}(\frac{Q^2}{\Lambda^2})}\left(1 - \frac{\beta_1 \mathrm{Ln}[\mathrm{Ln}(\frac{Q^2}{\Lambda^2})]}{\beta_0^2 \mathrm{Ln}(\frac{Q^2}{\Lambda^2})}\right). \qquad (30)$$

The values of Λ used for the two cases, $n_f = 0$ and 2, are 245 MeV and 190MeV respectively.

The status of calculations of $\overline{m}(2\ GeV)$ for quenched Wilson [35] [20] [36] [37] [12] [38], quenched staggered [39] [9] [40] [7] [8], dynamical ($n_f = 2$) Wilson [41] [42], and dynamical ($n_f = 2$) staggered fermions [40] [43] [44] is shown in Fig. 7. I have suppressed error bars as I want to first emphasize key qualitative features. The quenched staggered, and $n_f = 2$ Wilson and staggered give $\overline{m} = 2-3\ MeV$ and are roughly consistent; however, the quenched Wilson results seem

Figure 7. $\overline{m}$ extracted using m_π data with the scale set by m_ρ.



to approach that value from above and even at $\beta = 6.4$ are significantly higher. (The recent result $m_s = 128(18)$ MeV by Allton *et al.* [45] for both Wilson and Sheikholeslami-Wohlert actions at $\beta = 6.0$ and 6.2 is consistent with the results in Fig. 7 once one notes that $\overline{m} = m_s/25$.) I believe that, at this stage, it is important to understand why the quenched results with the Wilson action are so different from the rest!

An alternative to using $m_q = Z_{mass} m_q^L = Z_s^{-1} m_q^L$ to calculate the quark masses with Wilson fermions is to use the Ward identity [34][15]

$$m_q = \frac{Z_A}{Z_P} \frac{m_\pi}{2} \frac{\langle A_4(\tau) P(0) \rangle}{\langle P(\tau) P(0) \rangle} \tag{31}$$

Using the perturbative values for Z_A and Z_P (with $q^* = \pi/a$ and boosted g^2 defined in Eq.28) I get, for the LANL Wilson data, [12], $\overline{m} = 3.53(10)$ MeV in contrast to $\overline{m} = 5.15(15)$ MeV as shown in Fig. 7. The statistical errors are calculated using a single elimination JK with a sample of 100 lattices of size $32^3 \times 64$, so the difference is significant. The Rome collaboration [45] has found a similar discrepancy and argue that it can be resolved if one uses the non-perturbative value for Z_P, which they advocate calculating using matrix elements of the operators within quark states in a fixed (Landau) gauge. Their results indicate that perturbation theory (including tadpole improvement) fails for Z_P, and the two methods for extracting $\overline{m}$ give consistent results once the non-perturbative value of Z_P is used.

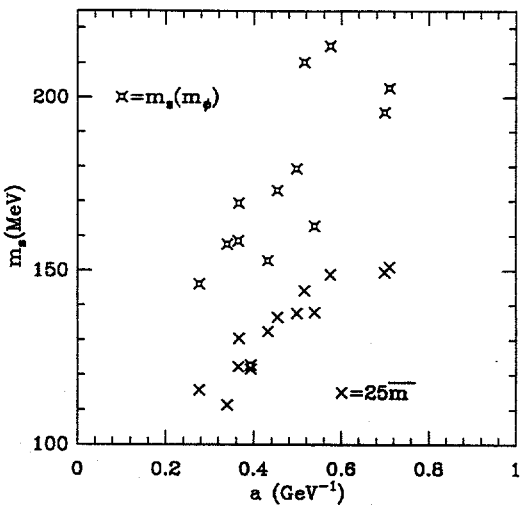
Having fixed $\overline{m}$ one can extract m_s, m_c and m_b using, for example, K^*, D, and B meson masses provided it is assumed that these masses are linear in the light quark mass, and in the heavier quark mass around the physical value. Alternately, one can use m_ϕ, J/ψ and Υ spectrum to get at these quark masses directly without needing to extrapolate in the light quark mass. The results for m_c and m_b have been reviewed by Sloan [33] at this conference so I will only analyze the data for m_s and compare these estimates to $25\overline{m}$. Note that the same data used to compile Fig. 7 is used to calculate m_s from m_{K^*} and m_ϕ, and the procedure for translating the value to 2 GeV in the $\overline{MS}$ scheme is also the same. The results are shown in Fig. 8. The estimate of m_s from m_ϕ is systematically higher by $15-20\%$ compared to $25\overline{m}$.

To convince you that the systematic errors due to choice of hadron used to set the scale of the strange quark are now a dominant source of error I compare the various estimates using the LANL data[12]. We find that, in the $\overline{MS}$ scheme at 2 GeV, $m_s = 25\overline{m} = 129(4)$ MeV using M_K, $m_s = 151(15)$ MeV using M_{K^*}, and $m_s = 157(13)$ MeV using M_ϕ. Note that the latter two estimates give $m_s/\overline{m} \sim 30$, which is much closer to the "Next Order" prediction of χPT. The larger errors in this case reflect the fact that on the lattice masses of pseudoscalar mesons are measured with much better statistical accuracy than those of vector mesons.

10. CONCLUSIONS AND COMING ATTRACTIONS

The analysis of various quenched quantities show that the parameter δ characterizing the hairpin vertex in the η' propagator lies in the range $0.1-0.2$. Also, for $m_q \lesssim m_s/2$ one sees significant deviations from the lowest order chiral behavior in m_π^2/m_q. On the basis of these,

Figure 8. Comparison of m_s extracted using m_ϕ and $m_s = 25\overline{m}$. The data are for quenched Wilson simulations.



I conclude that extrapolation of quenched data obtained for $m_q \leq m_s/2$ to the chiral limit cannot be done simply using full QCD formulae for quantities which have large contribution from enhanced logs. For quantities like the matrix element of the singlet axial vector current using the Adler-Bell-Jackiw anomaly, the quenched approximation fails altogether.

The alarmists are busy calculating 1-loop corrections to other quantities to determine what can be extracted reliably from quenched simulations. Bernard and Golterman have extended the results presented at LATTICE93 [46] and calculated chiral corrections to the energy of two pions in a finite box as derived by Lüscher [47]. They find terms at $O(1)$ and $O(1/L^2)$, whose contribution could be substantial, in addition to modifications of the $O(1/L^3)$ term which is related to the pi-pi scattering amplitude [48]. Sharpe and Labrenz have extended the analysis of baryons to include the Δ decuplet [49]. Booth [50] and Zhang and Sharpe [18] have calculated corrections to heavy-light meson properties like f_B and B_B. These new results and more data should provide a clearer picture of what is possible with quenched QCD by LATTICE 95.

In the calculations of light quark masses we need to understand the factor of two difference between the quenched Wilson and staggered data. On the other hand, the quenched staggered data is consistent with the $n_f = 2$ Wilson and staggered data. The analysis presented here leaves open the question — is the agreement between quenched Wilson (and $O(a)$ improved SW action) data with the phenomenologically favored estimates of $\overline{m}$ (or equivalently m_s) fortuitous and an artifact of strong coupling? If so, then the $n_f = 0, 2$ staggered and $n_f = 2$ Wilson data give an estimate of $\overline{m}$ that is $2-3$ times smaller than the commonly accepted phenomenological value. Using m_{K^*} or m_ϕ to extract m_s gives a $\sim 20\%$ larger value than that obtained from m_K, and provides information beyond the lowest order χPT result $m_s = 25\overline{m}$. The statistical errors are, however, larger in this case.

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