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BY

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Hot-Ion Bernstein Wave with Large $k_{\parallel}$

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Abstract: The complex roots of the hot plasma dispersion relation in the ion cyclotron range of frequencies have been surveyed. Progressing from low to high values of perpendicular wave number $k_{\perp}$ we find first the cold plasma fast wave and then the well-known Bernstein wave, which is characterized by large dispersion, or large changes in $k_{\perp}$ for small changes in frequency or magnetic field. At still higher $k_{\perp}$ there can be two hot plasma waves with relatively little dispersion. The latter waves exist only for relatively large $k_{\parallel}$, the wave number parallel to the magnetic field, and are strongly damped unless the electron temperature is low compared to the ion temperature. Up to three mode conversions appear to be possible, but two mode conversions are seen consistently.

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I. Introduction

Studies of wave phenomena in the ion cyclotron range of frequencies have been of interest for heating tokamaks[1] and for non-inductive current drive.[2–4] There has been recent interest in rf waves to channel the energy of alpha particles into current drive or ion heating.[5–7]

Physical analysis in this frequency range often makes use of the fact that some important features can be found from cold plasma equations, while other features require the consideration of hot plasma physics, at least in some of the elements of the dielectric tensor.[8] For example, the fast magnetosonic wave is often studied with equations from cold plasma theory, plus hot plasma effects to obtain absorption from cyclotron damping or electron Landau damping.[1] However, the Bernstein wave,[9] which has been the subject of research on direct excitation at the boundary of the plasma,[10] requires hot plasma effects for both propagation and damping; cold plasma physics does not tell anything about the Bernstein wave.

The original mathematical derivation of the Bernstein wave dispersion relation[9] explored the implications for small wave number parallel to the magnetic field, $k_{||}$, without the complications associated with coupling to cold plasma waves. Later generalizations[11,12] treated a complete electromagnetic dispersion relation, but retained the limit of small $k_{||}$. The work on the directly launched Bernstein wave[10] has concentrated on ray tracing on the hot plasma equations from the boundary of the plasma in a situation where $k_{||}$ was constantly evolving, and even changing sign. Therefore, consideration of the effects of $k_{||}$ alone did not arise.

We surveyed the solutions that can be expected to exist in a hot plasma by varying the mix of ions, the frequency, the temperatures and densities, and, particularly, $k_{||}$. The process attempts to ignore the distinction between results that can be found from cold plasma equations and results that need hot plasma terms. The general intent is to identify the important points to be considered in a computational model of rf heating and current drive in the ion cyclotron range of frequencies.

In the survey we found two short wavelength propagation modes between cyclotron harmonics with little dispersion. These waves apparently have not been recognized previously in the literature. The lower-$k_\perp$ wave is typically coupled to the highly dipersive Bernstein wave by mode conversion.

II. Hot plasma dispersion relation

Following standard notation we use a coordinate system (x, y, z) such that the magnetic field is in the z-direction, and the wave vector is in the xz plane: $\mathbf{k} \equiv k_x\hat{\mathbf{x}} + k_z\hat{\mathbf{z}} \equiv k_\perp\hat{\mathbf{x}} + k_{||}\hat{\mathbf{z}}$. The more important hot plasma dielectric tensor elements are then:[8]

$$
\begin{aligned}
K_{xx} &= 1+\frac{\omega_{\rm pe}^2}{\omega_{\rm ce}^2} \\
&\quad +\sum_s \frac{\omega_{\rm ps}^2}{\omega^2}\sum_n \xi_{0,s} Z(\xi_{n,s})\, n^2\, \frac{W_n(\lambda_s)}{\lambda_s} &(1)\\
K_{xy} &= -K_{xy} \\
&= i\frac{\omega_{\rm pe}^2}{\omega\omega_{\rm ce}} + i\sum_s \frac{\omega_{\rm ps}^2}{\omega^2}\sum_n \xi_{0,s} Z(\xi_{n,s})\, n\, W_n'(\lambda_s) &(2)\\
K_{yy} &= K_{xx} - 2\,\lambda_e \frac{\omega_{\rm pe}^2}{\omega^2}\xi_{0,e} Z(\xi_{0,e}) W_0'(\lambda_e) \\
&\quad -2\sum_s \lambda_s \frac{\omega_{\rm ps}^2}{\omega^2}\sum_n \xi_{0,s} Z(\xi_{n,s}) W_n'(\lambda_s) &(3)\\
K_{zz} &= 1+\frac{\omega_{\rm pe}^2}{\omega^2}\, 2\,\xi_{0,e}^2\ (1+\xi_{0,e} Z(\xi_{0,e})) - \sum_s \frac{\omega_{\rm ps}^2}{\omega^2} \quad . &(4)
\end{aligned}
$$

The tensor elements which are zero in the cold plasma limit can be ignored for many purposes, but are included in our calculations for completeness. Those elements are:

$$
\begin{aligned}
K_{yz} &= -K_{zy} \\
&= -i\frac{k_\perp}{k_\|}\omega_{\rm pe}^2 \frac{\omega}{\omega_{\rm ce}} (1+\xi_{0,e} Z(\xi_{0,e})) \\
&\quad +i\frac{k_\perp}{k_\|}\sum_s \omega_{\rm ps}^2 \frac{\omega}{\omega_{cs}}\sum_n (1+\xi_{n,s} Z(\xi_{n,s}))\, W_n'(\lambda_s) &(5)\\
K_{xz} &= K_{zx} \\
&= \frac{k_\perp}{k_\|}\sum_s \omega_{\rm ps}^2 \frac{\omega}{\omega_{cs}}\sum_n (1+\xi_{n,s} Z(\xi_{n,s}))\ n\,\frac{W_n(\lambda_s)}{\lambda_s} \quad . &(6)
\end{aligned}
$$

In the above equations ω is the frequency of a wave with wave numbers $k_\|$ and $k_\perp$; ω_{pe}, ω_{ps} are the plasma frequencies of electrons and ions of species s; ω_{ce}, ω_{cs} are the signed cyclotron frequencies of electrons and ions of species s; $\xi_{0,e} = \omega/(\sqrt{2}k_\| v_e)$ and $\xi_{n,s} = (\omega - n\omega_{cs})/(\sqrt{2}k_\| v_s)$; $v_{e,s}$ are the thermal velocities $\kappa T_{e,s}/m_{e,s}$; $\lambda_s = (k_\perp v_s/\omega_{cs})^2$; $\lambda_e = (k_\perp v_e/\omega_{ce})^2$. The Boltzmann constant is κ, and $m_{e,s}$ is the electron or ion mass. We assumed that λ_e and $k_\| v_i/\omega$ are small, but allowed λ_i and $k_\| v_e/\omega$ to be large. The index n extends over all positive and negative integers, and zero.

The function Z is the plasma dispersion function. The function $W_n(\lambda)$ represents $\exp(-\lambda)I_n(\lambda)$ where I_n is the modified Bessel function, and is invariant under a change of sign of n. The power series for W_n is

$$W_n(\lambda) = \frac{\lambda^n}{2^n\, n!}\left(1-\lambda\left(1-\lambda\frac{n+(2)-\frac{1}{2}}{n+(2)^2/2}\left(1-\lambda\frac{n+(3)-\frac{1}{2}}{n+(3)^2/2}(1\ldots)\right)\right)\right) \quad (7)$$

where we have written (2), $(2)^2$, (3), $(3)^2$, in the second (2) and third (3) correction terms to show how the series develops. Inspection of Eq. (7) shows that the number of terms must be considerably greater than $|\lambda|$ (and also the order of the Bessel function n for $|\lambda|$ above 1). The power series is therefore cumbersome for $|\lambda|$ approaching unity, but the hundreds of terms needed if $|\lambda|$ is in the range of 10 or more are tractable with a computer. At large argument, meaning $|\lambda|$ above the greater of 5 and $n^2/2$, the asymptotic expansion[13]

$$W_n(\lambda) \approx \frac{1}{\sqrt{2\pi\lambda}}\left(1 - \frac{4n^2-1}{8\lambda} + \frac{(4n^2-1)(4n^2-9)}{1!(8\lambda)^2} - \ldots\right) \tag{8}$$

is accurate.

The dispersion relation D_0 arises from setting to zero the determinant of the matrix arising from the dielectric tensor elements and the Maxwell equations.

$$D_0 \equiv \left| \, \mathbf{kk} - \mathbf{I}k^2 + \kappa_0^2\mathbf{K}(\mathbf{k},\omega) \, \right| = 0 \tag{9}$$

The free space wave number is $\kappa_0 \equiv \omega/c$, where c is the velocity of light. The expansion of D_0 is straightforward. An equivalent form, D_1, obtained by dividing D_0 by $(\kappa_0^2 K_{zz} - k_\perp^2)(\kappa_0^2 K_{yy} - k_\perp^2 - k_\parallel^2)$, is sometimes more useful for the purposes of studying Bernstein waves.[10]

Omitting the tensor elements of Eq. (5) and Eq. (6) simplifies the expression for D_1 as follows:

$$D_1 \approx K_{xx} \; - k_\parallel^2 \frac{K_{zz}}{\kappa_0^2 K_{zz} - k_\perp^2} - \kappa_0^2 \frac{K_{xy}^2}{\kappa_0^2 K_{yy} - k_\perp^2 - k_\parallel^2} \quad . \tag{10}$$

The utility of Eq. (10) is that the fast wave arises from the first and third terms balancing each other, whereas the Bernstein wave arises from a balance of the first and second terms. If all three terms are important in a solution, then the parameters are in a region of mode conversion involving the fast wave and the Bernstein wave. In the limit $k_\parallel = 0$, which is often studied, the Bernstein wave is governed by $K_{xx} = 0$.

III. Propagation for large wave number

Our interest is in $k_\parallel \neq 0$. The numerical work described below in Sec. IV shows that, for a single set of plasma parameters not particularly near a cyclotron resonance, it is possible to have solutions at very large $k_\perp$, as well as the electromagnetic solution at low $k_\perp$ and the Bernstein solution at intermediate $k_\perp$. Before showing the actual results of the computation, we employ approximate algebraic formulas to suggest how these results come about.

It is helpful to write K_{xx} as

$$K_{xx} \approx 1 + \frac{\omega_{\rm pe}^2}{\omega_{\rm ce}^2} + \sum_s \frac{\omega_{\rm ps}^2}{\omega_{cs}^2} \sum_{n=1}^{\infty} \frac{W_n(\lambda_s)}{\lambda_s/2}$$
$$+ \sum_s \frac{\omega_{\rm ps}^2}{\omega_{cs}^2} \frac{\omega^2}{\omega_{cs}^2} \left\{ \sum_{n=1}^{\infty} \left[\frac{1}{n^2 - \omega^2/\omega_{cs}^2} \right] \frac{W_n(\lambda_s)}{\lambda_s/2} \right\} \quad , \tag{11}$$

where the Z functions have been expanded with their asymptotic form $Z(\xi) \approx -1/\xi$. Use of a Bessel function identity[13] $\sum_n I_n(\lambda) = e^{\lambda}$ leads to a considerable simplification, since

$$\sum_{n=1}^{\infty} \frac{W_n(\lambda_s)}{\lambda_s/2} \equiv \frac{1 - W_0(\lambda_s)}{\lambda_s} \quad . \tag{12}$$

The asymptotic form of $W_0(\lambda)$ from Eq. (8) provides the following expansion:

$$\sum_{n=1}^{\infty} \frac{W_n(\lambda_s)}{\lambda_s/2} \approx \frac{1}{\lambda_s} \left(1 - \frac{1}{\sqrt{2\pi\lambda_s}} \right) \quad ; \lambda_s > \approx 5 \quad . \tag{13}$$

Next, consider the sum inside the curly brackets in Eq. (11). At large $k_\perp$ the W_n are nearly independent of n, as shown by Eq. (8). At the same time, the large-n terms (which we can assume to be non-resonant because we are in the ion cyclotron range of frequencies) are reduced in importance by the $1/n^2$ factor in square brackets. As a result, the term in curly brackets can be collapsed to the few low-n terms which are near resonance, $n \approx \omega/\omega_{cs}$. Using the asymptotic forms for the individual functions $W_n(\lambda_s)/(\lambda_s/2)$ at large λ_s gives the behavior of K_{xx} at $k_\perp^2$ large enough that the λ_s are much greater than unity:

$$K_{xx} \approx 1 + \frac{\omega_{\rm pe}^2}{\omega_{\rm ce}^2} + \sum_s \frac{\omega_{\rm ps}^2}{\omega_{cs}^2} \frac{1}{\lambda_s} (1 - \alpha_s) \tag{14}$$

$$\alpha_s \equiv \frac{1}{\sqrt{2\pi\lambda_s}} \left(1 - 2\frac{\omega^2}{\omega_{cs}^2} \sum_{n=1}^{\rm few} \left\{ \frac{1}{n^2 - \omega^2/\omega_{cs}^2} \right\} \right) \quad . \tag{15}$$

Note that the quantity α_s is small compared to unity whenever ω is not very near a multiple of one of the ω_{cs} (also assuming λ_s is large compared to unity).

We seek the a high-$k_\perp$ solution to the equation $D_1 = 0$ in which the first term of D_1 in expression 10 [given approximately by Eq. (14)] balances the second term (involving K_{zz}). Setting $K_{zz} \approx -\omega_{pe}^2/\omega^2$ gives

$$1 + \frac{\omega_{\rm pe}^2}{\omega_{\rm ce}^2} + \frac{1}{k_\perp^2} \sum_s \frac{\omega_{\rm ps}^2}{T_s/m_s} (1 - \alpha_s) = k_\parallel^2 \frac{\omega_{pe}^2/\omega^2}{k_\perp^2 + \omega_{pe}^2/c^2} \quad , \tag{16}$$

which yields a quadratic equation in $k_\perp^2$ if one ignores the extremely weak $k_\perp$ dependence in $(1 - \alpha_s)$.

One can then find two approximate expressions for the wave number of the propagating wave assuming the two solutions are well-separated. We have for the lower-$k_\perp$ solution

$$k_\perp^2 = \eta \, \frac{\omega_{pe}^2}{c^2} \, \frac{m_e c^2}{\kappa T_i} \left[n_\parallel^2 - (1 + \omega_{pe}^2/\omega_{ce}^2) - \eta \frac{m_e c^2}{\kappa T_i} \right]^{-1} \quad , \tag{17}$$

where we have introduced a parallel index of refraction $n_\parallel \equiv k_\parallel c/\omega$, an average ion temperature T_i, and the quantity η,

$$\eta \equiv \sum_s \frac{\omega_{ps}^2}{\omega_{pe}^2} \frac{m_s}{m_e} \frac{T_i}{T_s} (1 - \alpha_s) \quad , \tag{18}$$

written to show that it is approximately unity. The higher-$k_\perp$ solution is

$$k_\perp^2 = \frac{\omega_{pe}^2}{c^2} \frac{1}{1 + \omega_{pe}^2/\omega_{ce}^2} \left[n_\parallel^2 - (1 + \omega_{pe}^2/\omega_{ce}^2) - \eta \frac{m_e c^2}{\kappa T_i} \right] \quad . \tag{19}$$

In thinking about equations 17 and 19 one should keep in mind that the asymptotic forms used in the argument mean that $\lambda_s >\approx 5$, and that $m_e c^2 \approx$ 500 keV. Taking η to be about 1 we see that $n_\parallel^2 >\approx 500/T_i$; however, the phase velocity of a wave along the magnetic field overlaps the electron thermal velocity distribution sufficiently to allow electron Landau damping whenever $n_\parallel^2 >\approx 50/T_e$, so these waves can be strongly absorbed unless the electron temperature is considerably smaller than the ion temperature.

It is interesting that Eq. (19) gives a dispersion relation similar to the high frequency lower hybrid wave, even though the frequency is far below the lower hybrid range.

We have identified two short wavelength waves propagating between cyclotron harmonics. The dispersion is small. Since cyclotron harmonics were ignored in the derivation, one should expect some additional dispersion from a more accurate treatment, particularly near a harmonic where strong cyclotron damping would occur. It is conceivable that the two solutions of Eq. (16) could join with each other, and also join the traditional Bernstein wave. Therefore, there could be three mode conversions: (1) fast wave to the Bernstein wave in existence at small $k_\parallel$; (2) then to the low-$k_\perp$ solution of Eq. (17); (3) finally, to the high-$k_\perp$ solution of Eq. (19).

The existence of the second, high-$k_\perp$, mode conversion can be expected to affect the propagation and damping characteristics of the Bernstein wave even in cases of $T_e \sim T_i$. Details of this effect need to be explored with accurate solutions of the dispersion relation. The search for actual solutions should be prepared for heavily complex values of $k_\perp$, whether or not one is near a cyclotron resonance.

IV. Numerical results

In computing solutions, W_0 was found from Eq. (7) and Eq. (8) with a transition at $\lambda = 13$. The other W_n, where n runs to 40, were derived with downward recursion normalized to W_0.[15] A look-up table of the real part of plasma dispersion function $Z(\xi)$ was filled with an algorithm based on the sampling theorem.[16] A table of the complex values of D_0 and D_1 at many regularly-spaced values of entirely real $k_\perp^2$ was constructed. Trial solutions came from examining that table for changes in sign of the real part of either D_0 or D_1. The solutions were improved with one application of Newton's method and up to five applications of the quadratic generalization, Muller's method.[17,18] In the course of the iterations tentative solutions would either converge rapidly, change gradually with each iteration, or move toward heavily imaginary solutions. Only rapidly converging solutions with the magnitude of the imaginary part of $k_\perp$ less than 40% of the magnitude of the real part of $k_\perp$ ($|\mathrm{Im}\, k_\perp^2| < |\mathrm{Re}\, k_\perp^2|$) are reported in what follows.

We study three cases, chosen to represent conditions of thermonuclear interest in the Tokamak Fusion Test Reactor (TFTR) program[19] while illustrating features discussed in this paper. We choose a toroidal field $B = 40$ kG, major radius $R = 3$ meter, minor radius $a = 1$ meter. We give the solutions from the inner major radius ($r = -1$ m; $R = 2$ m; $B = 60$ kG) to the outer major radius ($r = +1$ m; $R = 4$ m; $B = 30$ kG). The plasma density and temperature are taken to be independent of position to simplify the interpretation of the results.

The cases are:
(A) ion-ion hybrid in a pure deuterium-tritium (D-T) plasma;
(B) second harmonic tritium in a hot D-T plasma;
(C) ion-ion hybrid in a cooler, but still thermonuclear-grade, D-^{3}He plasma with ^{4}He and carbon impurities.

We present graphs of the real part of $k_\perp$ and the absolute value of the imaginary part of $k_\perp$ in inverse meters. The solution with a positive real part is chosen; a positive imaginary part represents absorption of a forward wave, and a negative imaginary part represents absorption of a backward wave. A forward wave (graphed with the heavier points) has the real part of $k_\perp$ rising toward low field, or the right in our figures; a backward wave (graphed with the lighter points) has the real part of $k_\perp$ rising toward high field, or the left in our figures.

The logarithmic scale covers the range of solutions found, except that small imaginary parts are not represented. No solutions are sought with a wave number greater than 10^4 per meter (10 per millimeter).

A. Ion-ion hybrid in a D-T plasma

A frequency of 25 MHz puts the ion-ion hybrid for a plasma with equal parts of deuterium (D) and tritium (T) near the center $R = 3$. In that case the fundamental resonances are at 0.6 m (D) and -0.6 m (T). Taking $T_i = T_D = T_T = 10$ keV, $T_e = 1$ keV and $k_{\|} = 5\ \mathrm{m}^{-1}$ ($n_{\|} \approx 10$) serves to illustrate the points developed above. For Fig. 1 we take the electron density to be 10^{19} m^{-3}.

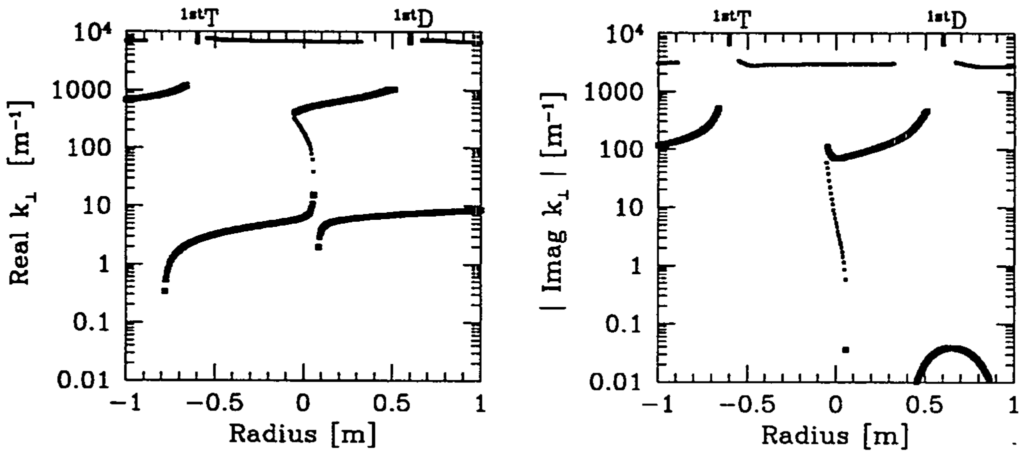

Figure 1: Fast wave, Bernstein wave, high-$k_\perp$ mode converted wave, and a heavily damped mode at very short wave length at a D-T ion-ion hybrid resonance with low density and low electron temperature of 1 keV.

At this low electron temperature (chosen to illustrate relatively small electron Landau damping) and low density (chosen to illustrate the high-$k_\perp$ root of Eq. (19) four solutions are found at some points, including the two weakly dispersive solutions as suggested by equations 17 and 19. At the left edges of Fig. 1 are strongly damped waves corresponding to Eqs. (17) and (19) having, respectively: real $k_\perp$ around 10^3 per meter ($\lambda \approx 15$), and imaginary $k_\perp$ over 10^2 per meter; and real $k_\perp$ around 6000 per meter ($\lambda \approx 600$), and imaginary $k_\perp$ around 3000 per meter. Note that the Bernstein wave cannot exist at the left edges of the figure because the 25 MHz frequency is less than any cyclotron frequency in that region. The high-$k_\perp$ waves are repeated in the center of the graph. There is a mode conversion with the Bernstein wave, which itself is connected to the fast wave at the ion-ion hybrid resonance near $r = 0$, for the lower-$k_\perp$ wave. The highest-$k_\perp$ wave is repeated at the right edges of the graph.

For these parameters Eq. (17) yields $k_\perp \approx 600$ and Eq. (19) yields $k_\perp \approx 4200$, in rough agreement with the numerical calculations.

The fast wave shows weak absorption near the D-resonance at $r = 0.6$ meter; absorption at the T-resonance is below the range of the graph of imaginary parts.

Mode conversions with the lower-$k_\perp$ solutions are suggested by the zero-crossings of the real parts of D_0 and D_1 for real $k_\perp$, but such a feature does not survive the iteration.

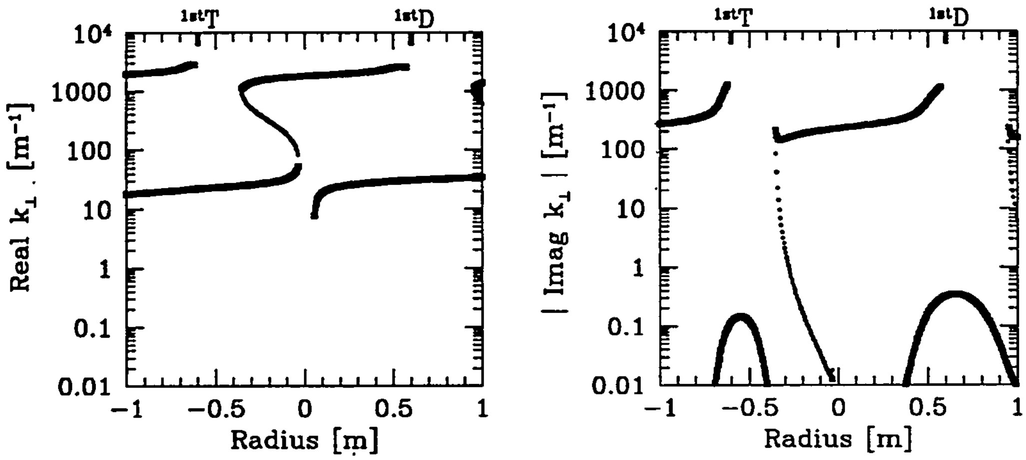

Figure 2: Fast wave, Bernstein wave, high-$k_\perp$ mode converted wave at a D-T ion-ion hybrid resonance with high density and low electron temperature.

Figure 2 shows the change if density is raised to 10^{20} m^{-3}, keeping everything else the same. The highest $k_\perp$ wave has risen out of the range of search, or acquired too large an imaginary part, or both. The mode conversion between the Bernstein wave and the high-$k_\perp$, low-dispersion wave has moved to higher field, and a similar mode conversion has appeared on the low-field edge of the figure. The latter mode conversion involves a Bernstein wave which couples to the fast wave at the second harmonic of T, which is at lower magnetic field than covered by the graph. Absorption of the fast wave at the D and T first harmonics is evident.

Figure 3 gives the result of raising the electron temperature to 10 keV, so $T_e = T_i = T_D = T_T$. At this electron temperature the parallel phase velocity of the wave (having $k_\| = 5$ m^{-1} and $n_\| = 10$) is in the body of the electron velocity distribution, K_{zz} has a *positive* imaginary part and is heavily complex. Equation (17) could be replaced by

$$k_\perp^2 = \eta \frac{\omega_{pe}^2}{c^2}\frac{m_e c^2}{\kappa T_i}\left[n_\|^2 + \eta \frac{m_e c^2}{\kappa T_i}\frac{(\omega_{pe}^2/\omega^2)}{K_{zz}}\right]^{-1} \quad . \tag{20}$$

The disappearance of resonance in the denominator of Eq. (20) plus the significant complex part of K_{zz} are consistent with the computed result of Fig. 3: that the real part of $k_\perp$ is much smaller than in Fig. 2 while the imaginary parts are comparable.

The algorithm used has difficulty near mode conversions with highly complex solutions, which leads to the missing points in Fig. 3 near the second mode conversion. Similar missing points are a feature of Fig. 4 and Fig. 7,

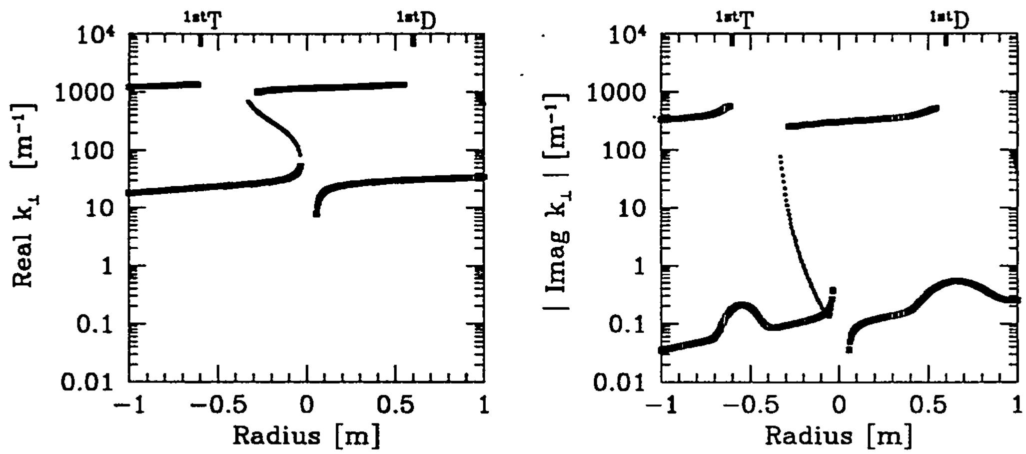

Figure 3: Fast wave, Bernstein wave, high-$k_\perp$ mode converted wave at a D-T ion-ion hybrid resonance with high density and high electron temperature of 10 keV.

below. The difficulty stems from two sources: failure to find a proper starting point from the zero crossings of either D_0 or D_1; and slow convergence of the iteration. A root-tracing (similar to ray-tracing) technique can be expected to fill in the missing points.

B. Second harmonic tritium in D-T plasma

Assume the frequency is raised to 40 MHz, and the $k_{||}$ is raised to 8 m^{-1} to keep $n_{||}$ constant at 10. Holding the electron density at 10^{20} m^{-3} and the temperatures at 10 keV, yields the results of Fig. 4.

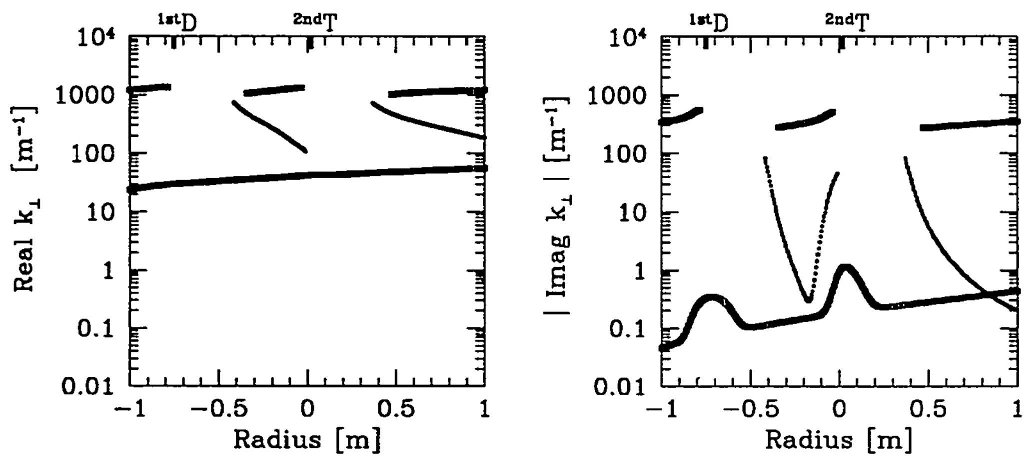

Figure 4: Fast wave, Bernstein wave, high-$k_\perp$ mode converted wave at second harmonic T- resonance with high density and high electron temperature of 10 keV.

The fast wave shows weak absorption at $r = 0$ for second harmonic T and at $r = -0.75$ for first harmonic D. A Bernstein wave shows mode conversion with the fast wave for second harmonic T, but the connection is suppressed by our limit on imaginary parts. Mode conversion with the fast wave at second harmonic D would appear on the low field (right) side of the graph. The high-$k_\perp$, weak dispersion wave of Eqs. (17) or (20) is seen in three places.

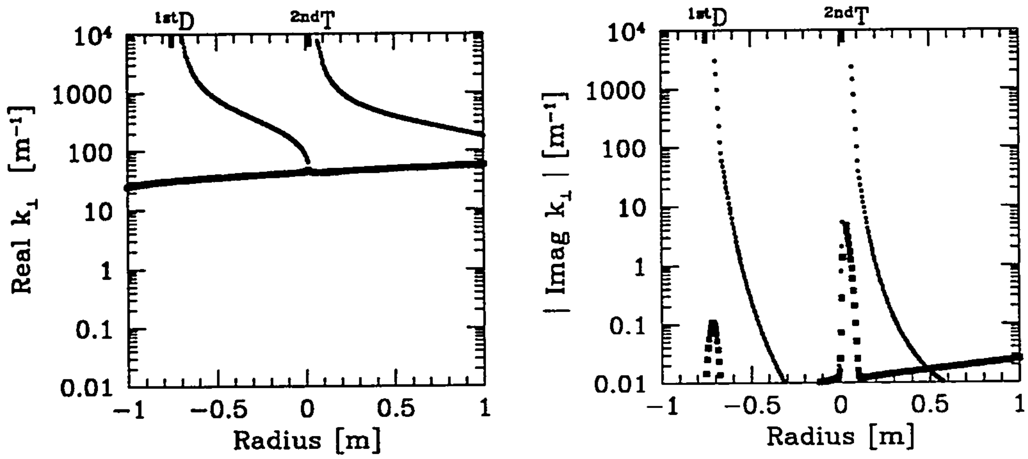

Figure 5: Fast wave and Bernstein wave with $k_{||} = 2$ at second harmonic T-resonance with high density and high electron temperature of 10 keV.

Figure 5 is like Fig. 4 except that $k_{||}$ is reduced to 2 m^{-1}. The second mode conversion is no longer present, consistent with Eq. (17) where now $n_{||}^2 \approx 4$ but $\eta\, m_e c^2/(\kappa T_i) \approx 40$. The mode conversion at the fast wave is now suggested in the figure because the absorption at the second harmonic T resonance is narrower and not so strong as previously.

C. Ion-ion hybrid in a multi-species D-^{3}He plasma

We now turn attention to a complex mixture of species suggested by some TFTR experiments:[4,19] ^{3}He = 25%, D = 12%, ^{4}He = 7%, C = 4%. The temperature and density area more moderate at 5 keV and $5\,10^{19}$ m^{-3}. The $k_{||}$ is 6 m^{-1}, and the frequency 40 MHz, giving $n_{||} \approx 7.5$.

Figure 6 shows a mode conversion at the ion-ion hybrid resonance with some unusual properties. First, the spatial motion of the mode-converted wave is insignificant while the real part of the wave number changes by about two orders of magnitude, and the imaginary part increases similarly. Second, the behavior seems to reveal three mode conversions all at essentially one location.

The horizontal scale is expanded in Fig. 7 such that only the 0.1 meter-wide space near the mode conversions is examined. In fact, we find a fast-to-Bernstein conversion at $r = -0.36$ meter and $k_\perp \approx 10^2$ m^{-1}; a Bernstein-to-forward wave conversion at $r = -0.37$ meter and $k_\perp \approx 5\,10^2$ m^{-1}; and a forward-to-backward conversion at $r = -0.36$ meter and $k_\perp \approx 10^3$ m^{-1}. The

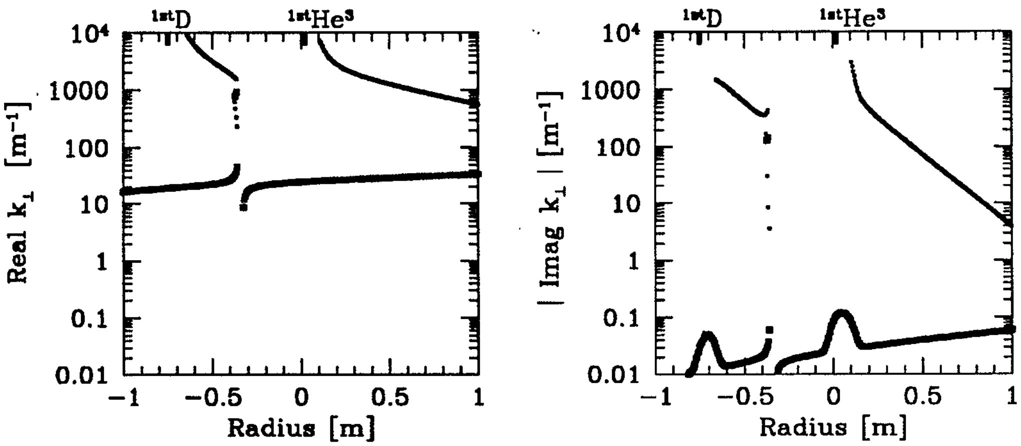

Figure 6: Fast wave and Bernstein wave with $k_{||} = 6$ at D-^{3}He ion-ion hybrid with ^{4}He and C impurities, and moderate density and temperature ($4\,10^{19}$ and 5 keV).

high-$k_\perp$ conversions might be related to Eq. (17) in some fashion, but not in a straightforward manner since the dispersion is high and λ is only of order 3 – 10 where the high-$k_\perp$ mode conversions are taking place. Note that in Fig. 6 behavior similar to the low-$k_{||}$ Bernstein wave is seen between $r = -0.6$ and $r = -0.4$.

V. Conclusions

We have examined the hot plasma dispersion relation in the ion cyclotron range of frequencies, and allowed the search to extend to very short perpendicular wavelengths, moderately short parallel wavelengths, and hot plasma. A feature has appeared which is not believed to be widely known: there can be multiple mode conversions and changes in direction in a plasma with varying magnetic field.

From an approximate formula, and from examining several cases, it appears that the effect will commonly be accompanied by strong electron Landau damping unless electrons are much colder than the ions.

It is possible that the effects of moderately large $k_{||}$ can have an effect on the location of energy deposition because of the multiple mode conversions and changes of direction. This can be studied with a ray-tracing approach in which the geometric effects present in fusion plasma can be taken into account. Such studies may have to develop some new techniques because of the rapid changes of wavelength with position, for example as seen in Fig. 6.

It is our belief that the present analysis identifies all solutions of the dispersion relation that could have physical interest: other solutions will have the imaginary part of $k_\perp^2$ greater than the real part. Since the $W_n(\lambda)$ functions

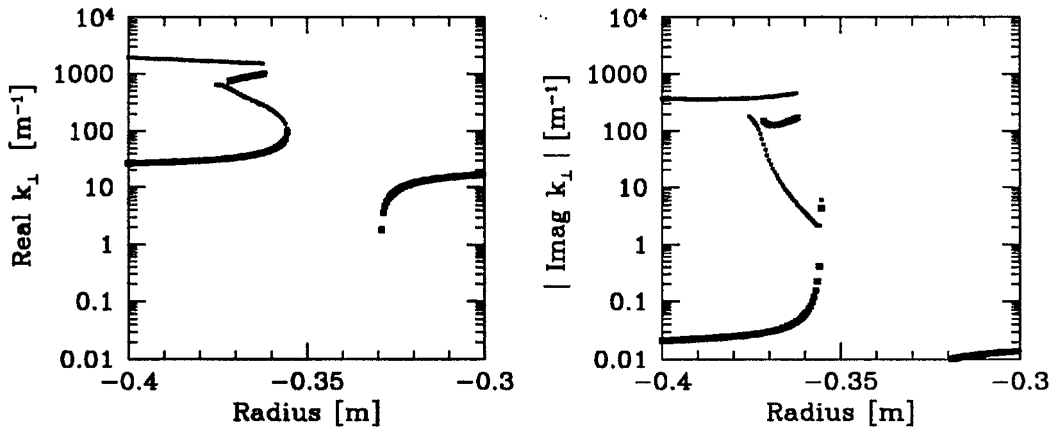

Figure 7: Same as the previous figure, but from $r = -0.4$ meter to $r = -0.3$ meter.

become oscillatory for λ imaginary, it is plain that many other solutions can be expected for λ near the imaginary axis.

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